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# A Note on the Principle of Predication

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Let $A$ be a well-formed formula of first-order modal logic whose only free variable is $\underline{x}$. We shall use the following abbreviations:

$\underline{Mat}(A)$ for $(\underline{x})[\Diamond A \wedge \Diamond{\sim}A]$
$\underline{Form}(A)$ for $(\underline{x})[\Box A \vee \Box{\sim}A]$
$\underline{Ban}(A)$ for $((\underline{x})\Box A) \vee ((x)\Box{\sim}A)$
$\underline{Pred}(A)$ for $\underline{Form}(A) \vee \underline{Mat}(A)$.

We read $\underline{Mat}(A)$, $\underline{Form}(A)$, and $\underline{Ban}(A)$ respectively as: "$A$ is material", "$A$ is formal", and "$A$ is banal"; *Pred*$(A)$ is the assertion of the Principle of Predication for $A$.

We prove that if $F$ and $M$ are formulas whose only free variable is $\underline{x}$ such that $\underline{Ban}(F)$, $\underline{Form}(F)$, and $\underline{Mat}(M)$ are true in any suitable $T$-model, then $\underline{Pred}(F \wedge M)$ and $\underline{Pred}(F \vee M)$ are not acceptable as axioms.

**Theorem** *The formulas:*

(1) $\sim\underline{Ban}(F) \wedge \underline{Form}(F) \wedge \underline{Mat}(M) \supset {\sim}\underline{Pred}(M \wedge F)$
(2) $\sim\underline{Ban}(F) \wedge \underline{Form}(F) \wedge \underline{Mat}(M) \supset {\sim}\underline{Pred}(M \vee F)$

*are T-valid.*

*Proof:* Let $\langle W, R, D, Q, V\rangle$ be a $T$-model ([1], p. 171) and $w_i \in W$. If $V({\sim}\underline{Ban}(F) \wedge \underline{Form}(F) \wedge \underline{Mat}(M), w_i) = 1$ then $V({\sim}\underline{Ban}(F), w_i) = 1$ and there exist $a, b \in D_i$ such that:

(3) $V^a(\Diamond{\sim}F, w_i) = 1$ and $V^b(\Diamond F, w_i) = 1$

where $V^a$ and $V^b$ are just like $V$ except for assigning $a$ and $b$, respectively, to $\underline{x}$.

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Since $V(\underline{Form}(F), w_i) = 1$, we have $V^a(\Box F \vee \Box{\sim}F, w_i) = 1$ and $V^b(\Box F \vee \Box{\sim}F, w_i) = 1$; hence by (3),

(4) $V^a(\Box{\sim}F, w_i) = 1$ and $V^b(\Box F, w_i) = 1$.

By hypothesis, $V(\underline{Mat}(M), w_i) = 1$ and therefore $V^b(\Diamond M \wedge \Diamond{\sim}M, w_i) = 1$. Then there exist $w_h, w_k \in W$ such that $w_iRw_h$, $w_iRw_k$, $V^b(M, w_h) = 1$, $V^b({\sim}M, w_k) = 1$; and by (4):

(5) $V^b(M \wedge F, w_h) = V^b({\sim}(M \wedge F), w_k) = 1$.

Now if we assume that $V(\underline{Mat}(M \wedge F), w_i) = 1$, it follows that $V^a(\Diamond(M \wedge F) \wedge \Diamond({\sim}(M \wedge F)), w_i) = 1$ and therefore $V^a(\Diamond F, w_i) = 1$, which contradicts (4). If we assume that $V(\underline{Form}(M \wedge F), w_i) = 1$, it follows that $V^b(\Box(M \wedge F) \vee \Box({\sim}(M \wedge F)); w_i) = 1$ which contradicts (5). Consequently, $V({\sim}\underline{Mat}(M \wedge F) \wedge {\sim}\underline{Form}(M \wedge F), w_i) = V({\sim}\underline{Pred}(M \wedge F), w_i) = 1$, which completes the proof of (1).

The $T$-validity of (2) follows from that of (1) in view of the fact that $\underline{Ban}(A)$, $\underline{Form}(A)$, $\underline{Mat}(A)$, and $\underline{Pred}(A)$ are respectively equivalent to $\underline{Ban}({\sim}A)$, $\underline{Form}({\sim}A)$, $\underline{Mat}({\sim}A)$, and $\underline{Pred}({\sim}A)$.

In [2] it is proved that if $\underline{Pred}(A)$ is an open formula, then $\underline{Pred}(A)$ is not acceptable as an axiom. Consequently the author proposes the following formalization of Von Wright's Principle of Predication:

**(PP*)** *If $\underline{Pred}(A)$ is closed then $\underline{Pred}(A)$ is valid.*

Now the formulas $\underline{Pred}(F \wedge M)$ and $\underline{Pred}(F \vee M)$ of the above theorem are closed. It follows that the principle PP* is also unsound.

## REFERENCES


[1] Hughes, G. E. and M. J. Cresswell, *An Introduction to Modal Logic*, Methuen, London 1972.

[2] Tichy, P., "On de dicto modalities in quantified S5," *Journal of Philosophical Logic*, vol. 2 (1973), pp. 387-392.



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