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Syllogisms with Reduplication in Aristotle

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Prior Analytics I.38 is a difficult text that offers a way of handling qua propositions in formal syllogistic. By 'qua proposition' I mean a proposition that contains a qualifying term, phrase, or clause. Many such propositions have a qua connector like 'qua', 'insofar as', 'in virtue of the fact that', 'with respect to', although in some cases a construction like an accusative of respect occurs instead of an explicit connective.[1] Still, all qua propositions may be paraphrased by explicit qua connectives. So the class of qua propositions is a grammatical class of propositions of the form '*S* is *P* qua *M*'. The *Prior Analytics* chapter deals with a specific logical type of qua propositions, and its syllogistic properties.[2]

In the first part of this chapter (49a11-26) Aristotle considers how a conclusion of the form '*S* is *P* qua *M*' is to be argued for syllogistically. The question is how to construct two premises that have a qua proposition as a conclusion.

Aristotle deals here only with the question of which terms in the premises the qua phrase should be associated with. He does not think there is a difficulty in determining what the middle term should be. Indeed, the same term (*M*) is the middle term, and is present in the qua phrase. ('Reduplicated' means 'repeated' ([1], 367.17).) When Aristotle says, "what is reduplicated in the premises must be attached to the first extreme" (49a11-2), he is implying that some term is repeated in the premises. However, as the examples in this part of chapter 38 confirm, the *M* term differs from both the *S* and *P* terms. For example, in 'justice is known, that it is good', the *M* term, 'good', differs from 'known' and 'justice'. Still, Aristotle says that the *M* term is repeated in the premises and that 'qua *M*' is to be attached to the major extreme *P*. The remaining candidate for the *M* term that is repeated in the premises is the middle term. If the *M* term is also the middle term, there will be repetition of

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the same term in at least one premise: the major premise will be '*M* is *P* qua *M*'. Alexander confirms this interpretation when he says, "Now what is reduplicated in the premises is the middle term. For this, when it is predicated additionally, has been reduplicated" ([1], 367.19-20).

This interpretation is supported on two other grounds. First, it agrees with Aristotle's analysis of the example, 'justice is known, that it is good'. Second, it agrees with the Greek commentators' analyses of the three other examples given in this part of chapter 38 (49a22-5) that Aristotle does not analyze (see [1], 268.28ff; [2], 138.12ff; and [5], 345.12ff).

The main point is clear: In constructing the premises for a conclusion of form '*S* is *P* qua *M*', the qua phrase must be attached to the major extreme. So Aristotle argues for the following construction:

(1)	Every *M* is *P* qua *M*	The good is known, that it is good
	Every *S* is *M*	Justice is good
	Therefore, every *S* is *P* qua *M*	Justice is known, that it is good.

Attaching 'qua *M*' to the middle term will not work:

(2)	Every *M* qua *M* is *P*	The good, that it is good, is known
	Every *S* is *M* qua *M*	Justice is good that it is good
	Therefore, every *S* is *P*	Justice is good.[3]

The most obvious reason is that if the qua phrase forms part of the middle term, there will be no qua phrase in the conclusion of (2). There are also difficulties with the particular example that Aristotle uses; Aristotle says that the minor premise in (2) is "false and unintelligible". Note that he does not consider the possibility that the qua phrase is attached to the minor extreme. This omission is due to the fact that he regards qualification by a qua phrase as a type of supplementary predication. So, then, a qua phrase cannot be attached to the subject, and hence Aristotle will have argued exhaustively that the qua phrase must be attached to the major extreme.

The second part of Chapter 38 discusses what conditions there will be on the middle term, for a sound syllogism, when a 'qua *M*' phrase appears in the conclusion, besides those conditions for soundness that hold for a simple Barbara syllogism. So it concerns finding a middle term suitable for a Barbara syllogism with a qua proposition as conclusion. But, in fact, the topic here is more specific. In the first part, Aristotle strongly hints that the middle term should be the reduplicated term (*M*). So why is there any need to worry about what the middle term should be? The answer is revealed by the sort of example that Aristotle considers here: the qua proposition that is the conclusion has the form '*S* is *P* qua *S*', a variant of '*S* is *P* qua *M*', where *M* = *S*. So here there is reduplication of the minor term in the conclusion of the syllogism. But try to apply the analysis of the first part to this example; the syllogism would be

(3)	Every *S* is *P* qua *S*	The good is known, that it is good
	Every *S* is *S*	The good is good
	Therefore, every *S* is *P* qua *S*	Therefore, the good is known, that it is good

—an obvious case of *petitio principii*. So, although the first part still informs us that the qua phrase is to be attached to the major extreme, it does not offer a way of finding a middle term for '*S* is *P* qua *S*' propositions. Thus, although what is said in this part about middle terms in syllogisms with qualifications may hold for all such syllogisms, in fact a very special case of those syllogisms is being dealt with. Indeed, it is only for the '*S* is *P* qua *S*' case that there is difficulty about the middle term. (Notice that the conclusion of (3) is the major premise of (1).)

Although the text is in detail quite difficult, the main point can be clearly seen. Aristotle says that when a qua phrase appears in the conclusion the middle term must be more particular than it need be in the case of a simple syllogism:

(4)	Every *B* is *P* qua *S*	The desirable is known, that it is good
	Every *S* is *B*	The good is desirable
	Therefore, every *S* is *P* qua *S*	The good is known, that it is good[4]

To what must *B* be more particular? *P* is not a good candiate, since it is present in the same position in the simple syllogism. It is *S*, then, to which *B* must be particular. But surely *S* in the minor premise, as the minor term, should not make a new requirement for *B*, since the minor premise is the same in the simple syllogism. It is, thus, the *S* in the qua phrase that makes this new condition. As the particular example suggests, this new condition arises because *S* is predicated of *B*. Hence, every *S* must be *B*, and every *B* must be *S*. So *S* and *B* must be commensurately universal, or coextensive. As Aristotle says in *Prior Analytics* (49a36) the middle term "is a sign of the proper essence" of *S*. Consequently, although in a simple Barbara syllogism, the middle term may be much more general than the minor term, in the case of a Barbara syllogism with a qua proposition as conclusion, the middle term must be coextensive with the qualifying term (*M*). This condition is met trivially in *S* is *P* qua *M* propositions where $S \neq M$.

In order to arrive at a satisfactory symbolization for a qua proposition of the type dealt with in Chapter 38, let me review some of the logical features that such a qua proposition has. First, the *M* term should have some direct relation to the *S* and *P* terms, since, although it is part of the predicate for the purpose of constructing the premises of the syllogism, it refers grammatically to the subject. For example, in 'justice is known, that it is good', 'good', although attached to the predicate as a specification of 'known', is connected directly to the subject via the antecedent of 'it'. Second, as Aristotle seems to view the qua phrase as a sort of supplementary predication that does not change the truth value of the corresponding unqualified proposition, the inference from '*S* is *P* qua *M*' to '*S* is *P*' should be valid.

Finally, as the close connection between the middle term and the additional predicate (*M*) suggests, the additional predicate has many of the features of the middle term. This should become clearer below. When it is asserted that *S* is *P* qua, i.e., in respect of being, *M*, it is claimed that predicating *P* of *S* is bound up in some way with *S*'s being *M*. *M* is some property or aspect of *S*, and the relation of *M* and *P*, namely that *P* is said of *M*, provides grounds for

predicating P of S. In other words, 'S is P qua M' asserts that S is P, since S is M and M is P.

Thus, an 'S is P qua M' proposition (of the type in the *Analytics*) is in effect a condensed syllogism. It yields the two premises, 'S is M' and 'M is P', and the conclusion, 'S is P'. 'M' is the middle term in this syllogism; indeed, when the conclusion desired is 'S is P qua M', 'M' becomes the middle term of this syllogism, when $S \neq M$. For example, take 'justice is known, that it is good'. The corresponding syllogism would be:

(5) The good is known
Justice is good
Therefore, justice is known

(Note that since 'good' can be used as the middle term in this syllogism it can be taken causally, to mean 'because', although, as we have seen, Aristotle does not unambiguously take it in this way.)

Much of what Aristotle says about the middle term is applicable almost without change to the additional predicate of a qua proposition. For example, Aristotle, in constructing a syllogism, says: "Let A stand for two right angles, B for triangle, C for isosceles triangle. A then belongs to C because of B" (48a34-5). So this Barbara syllogism can be summarized by saying that A is C in virtue of being B. Indeed, elsewhere Aristotle says that a triangle qua triangle has its angles equal to two right angles, and that an isosceles triangle has its angles equal to two right angles qua triangle (*Posterior Analytics,* 73b30-74a4). Thus there is here a straightforward congruence between the structure of a syllogism and the structure of a qua proposition. The conclusion of the syllogism follows because A belongs to C because of B; the qua proposition asserts that C is A *qua*, or because of, B.

Therefore, I propose the following as a symbolization of the logical type of qua proposition found in the *Analytics*:

(6) Every S is P qua M if and only if
$(x)((Sx \supset Mx) \,\&\, (Mx \supset Px))$

This symbolization reflects the syllogistic structure of these qua propositions. $(x)(Sx \supset Px)$ follows from the two conjuncts of (6).

The qua syllogism (in Barbara) can thus be symbolized:

(7)	Every B is P qua M	$(x)((Bx \supset Mx) \,\&\, (Mx \supset Px))$
	Every S is B	$(x)(Sx \supset Bx)$
	Therefore, every S is P qua M	$(x)((Sx \supset Mx) \,\&\, (Mx \supset Px))$

The proof for the symbolization on the right is:

(8)	$(x)((Bx \supset Mx) \,\&\, (Mx \supset Px))$	premise
	$(x)(Sx \supset Bx)$	premise
	$(Ba \supset Ma) \,\&\, (Ma \supset Pa)$	
	$Sa \supset Ba$	
	$Ba \supset Ma$	
	$Sa \supset Ma$	
	$Ma \supset Pa$	

$(Sa \supset Ma)$ & $(Ma \supset Pa)$
$(x)((Sx \supset Mx)$ & $(Mx \supset Px))$ conclusion

Professor Angelelli gives a different analysis of *Prior Analytics* I.38 in [3]. Working from material from the second scholastics he holds that the symbolization should be

(9) Every S is P qua M if and only if
$(x)((Sx \supset Px)$ & $(Px \supset Mx))$.

The syllogism (7) will work on (9). However, (9) does not handle the examples that Aristotle gives. For example, 'justice is known, that it is good' is false according to (9). As Aristotle generally assumes that the propositions he uses in his examples are true, I can conclude only that (9) does not accurately analyze the doctrine of Chapter 38. Nor does (9) capture the relation between a qua proposition and a Barbara syllogism. Indeed, (9) will make false many of the propositions that Aristotle holds to be true in the *Analytics.*

Finally, note that according to (6) the additional predicate need not be commensurately universal to the middle term in the syllogism. Indeed, this condition seems to make no difference for the formal validity of qua syllogisms. However, it does matter for the truth of the major premise. Since Aristotle requires the premises of a syllogism used in scientific demonstration to be true, the requirement of commensurate universality might be directed toward ensuring that the syllogisms constructed are demonstrations.

NOTES

1. E.g., *Sophistical Refutations,* 167a10.
2. There is more than one logical type of qua proposition. Cf. the fallacy of *secundum quid et simpliciter.* Details of this, as well as a fuller explication of the text now being considered, may be found in my forthcoming [4].
3. On the right are the examples in the text (49a12-22). I am treating them as instances of Barbara. The analysis that follows will consider universal affirmative qua propositions only.
4. 49a31-6. 'The desirable' is Alexander's suggestion for the middle term (cf., [1], 370.27-34).

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