Mindful Logic: How to Resolve
Some Paradoxes of Identity

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It seems a tame and unexciting contention that logic ought not be mindless. After all, one cannot construct a logical theory or apply logic without thinking. But we mean more than this. Contrary to the common assumption that logical theory can be successfully constructed without so much as mentioning thoughts or anything like them, and in fact is better off utterly detached from considerations of the mental, it seems to us that some logical problems are insoluble if one insists on approaching them with such self-imposed restrictions. Our purpose here, however, is not to argue this conviction at length, but to illustrate the value of permitting oneself to mention the mind and its acts when working on logical theory. We shall discuss a problem concerning the principle of the indiscernibility of identicals. After stating the problem, we shall propose a way of understanding the principle which preserves both self-evidence and utility. In particular, we shall argue that this understanding dissolves certain apparent counterexamples to the principle. These counterexamples arise from application of the principle of indiscernibility to epistemic, modal, and definitional contexts.

1 The problem   Consider the standard formulation of the principle:

$$(\forall F)(\forall x)(\forall y)\,[(x = y \cdot Fx) \to Fy]\,.$$

What does it mean?

Sometimes the principle is thought of in a realistic way as stating something about the world: "If two things are identical then they have the same attributes" (see, e.g., [2], VII.1; [1], I, Q.40, a.1). One typically uses the phrase "indiscernibility of identicals" when thinking of the principle in this way.

On other occasions one thinks of the principle as stating a truth about lan-

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guage: "Given a true statement of identity, one of its two terms may be substituted for the other in any true statement and the result will be true" (see, e.g., [13], p. 139). One generally uses the phrase "substitutivity of identity" when thinking about the principle in this way.[1]

Of course, the first thing that strikes one about the realistic formulation is its apparent incoherence. As has been often noted, it seems self-contradictory to refer to "two things" and then assert their identity (see, e.g., [15], 5.5305).

So, in order to avoid incoherence it seems necessary to understand the realistic formulation as asserting that if something is identical with itself then it has whatever attributes it has. This formulation avoids self-contradiction. Indeed, it is a necessary truth. But the necessity is purchased at the cost of apparent uselessness. It is true to say that if something has an attribute then it has it. But how could there ever be any point in saying so?

The second way of understanding the principle seems useful. It purports to be a method for generating truths from other truths by substituting terms. But the appearance of utility is offset by the lack of self-evidence. If the substitutivity principle is necessarily true, it is not obviously so (see, e.g., [8], p. 89).

In fact, not only does the substitutivity principle lack self-evidence, it seems false. Consider one of Quine's examples.[2] Suppose that Cicero = Tully and that, although Philip knows that Cicero denounced Cataline, he does not know that Tully denounced Cataline. Then the substitutivity principle seems to lead from the true propositions:

(1)   Tully = Cicero;
(2)   Philip is unaware that Tully denounced Cataline;

to the false proposition:

(3)   Philip is unaware that Cicero denounced Cataline.[3,4]

Thus, we seem forced to choose between an understanding of the principle which renders it self-evident but useless, and one which promises utility but forfeits truth. Is there a way of understanding the principle which preserves both self-evidence and usefulness?

We believe that there is. In Sections 2, 3, and 4 we shall introduce certain distinctions necessary for formulating such an understanding. After summarizing these matters we shall propose a version of the principle which seems to satisfy both requirements.

2 Intentional subjects and predicates   Although the mind is occasionally a merely passive recipient of impressions and ideas, it usually functions as an agent whose acts are intentional. The mind's activity is referential; its acts are about things.

With respect to the act of judgment, intentionality involves at least three kinds of selectivity. First, the mind focuses upon certain things rather than others. Second, given such an object, the mind focuses upon it in certain ways rather than others. Third, having focused upon something in a selective way, the mind asserts about that something certain things rather than others.

These points can be made in terms of reference and predication. The mind

refers selectively. It selectively refers to its objects of reference. And it makes selective predications.

Suppose, for example, that Smith notices a small apple tree which seems to be dying. He thinks to himself, "That apple tree is dying". He has acted selectively in all three ways. He has focused upon (referred to) this particular tree, as opposed to all of the other things upon which he could have focused. He has focused upon it as an "apple tree", as opposed to focusing upon it as "a tree", or "a small apple tree", or "that thing", or "Mrs. Dunstan's experiment". And he has predicated of the referent the attribute of dying, as opposed to all of the other attributes he might have chosen to predicate.

We shall call that upon which the mind focuses in such an act the *intentional subject* of the act.[5] Assuming that one has just expressed a judgment about something, the intentional subject is that to which one would refer in response to the question, "What are you talking about?"[6,7]

The concept of intentional subject is person-relative; things are not intentional subjects by themselves. To say that something is an intentional subject is to say, or at least imply, that it is an intentional subject for someone and for one of his judgments.[8]

Intentional subjects have characteristics; they instantiate universals. The mind focuses upon an intentional subject by grasping one or more of its instantiated universals. And, as we have observed, this grasping is selective. The mind gains access to a thing by apprehending something less than all of its instantiated universals. The universal apprehended by the mind in making the initial reference is the *intentional subject attribute*.[9] The understanding the mind has of the intentional subject attribute is the *intentional subject concept*.[10] The linguistic vehicle, if any, the mind uses to make the initial reference to the intentional subject is the *intentional subject expression*.

Thus, an act of referring with language involves at least four things: the object of reference,[11] the characteristic (or characteristics) of the object grasped by the mind in making the reference, the mind's concept of that characteristic, and the linguistic expression used to make the reference.

Analogous distinctions can be drawn with respect to the predicative element of the act of judgment. The universal which the mind asserts is instantiated by the intentional subject is the *intentional predicate attribute*. The understanding the mind has of the intentional predicate attribute is the *intentional predicate concept*. The linguistic vehicle, if any, used to make the predication is the *intentional predicate expression*.[12]

3 The information conveyed by presuppositions   We find it useful to rely upon the truism that part of the information conveyed by the assertion of a proposition consists of its presuppositions.

The common sense of the matter is easily stated. In asserting propositions one generally presupposes others. One of the purposes for asserting propositions is conveying information. Part of the information conveyed by the assertion of a proposition P is what P itself states. But another part of the information conveyed is what P presupposes. For example, when John says that yesterday he and Judy spoke with the red-haired vice-president of the First National Bank,

he conveys more information than he asserts. He conveys at least the information that he was at the First National Bank yesterday, that Judy was there with him, that Judy was alive yesterday, that he knows Judy, that there was a First National Bank, that it had a vice-president, that the vice president had red hair, and so on, indefinitely.

But, as is usually the case in philosophy, the ease with which the common sense of the matter is stated contrasts with the difficulty of making the common sense precise. Because we do not think that a deep analysis is necessary for this paper we offer only a few preliminary suggestions.

In a relatively weak sense a proposition P presupposes a proposition Q if the truth of Q is a necessary condition for the truth of P. In this sense the presuppositions of a proposition P are P's entailments.[13,14,15]

In a stronger and more interesting sense, P presupposes Q if a rational speaker would have to believe Q in order to sincerely assert P. Thus, a presupposition in this sense is a necessary condition for believing something, as contrasted with the weaker sense in which a presupposition is a necessary condition for something being true. A presupposition in the strong sense is a proposition by means of which a speaker grasps and asserts another proposition.

Presuppositions in the weak sense are not necessarily presuppositions in the strong sense. For example, one can sincerely assert that the cat is on the mat without believing, or even thinking about, the proposition that the cat is on the mat or there is no decision procedure for quantification theory.

On the other hand, all presuppositions in the strong sense are presuppositions in the weak sense. For example, the proposition that Smith has hair is a presupposition in the strong sense of the proposition that Smith has red hair. A fortiori, it is a presupposition of the same proposition in the weak sense.

We shall use "presuppose" in the strong sense. Of course, the word as so defined is vague to some degree. There will sometimes be controversies over whether certain propositions are presupposed by others. And whatever vagueness attaches to "presupposition" carries over to "information". But vagueness, though present, does not frustrate our purpose. We shall not stray from the paradigmatic core of the extensions of these terms.[16]

4 What do identity statements mean?   Because identity statements are involved in every application of the principle of indiscernibility it is important to understand their meaning.

We set aside senses of identity less strict than numerical identity. It seems obvious that the principle of indiscernibility is not intended to apply to such modes of identity as generic identity (e.g., A and B are both animals) or specific identity (e.g., A and B are both horses).

Thus, we confine our attention to numerical identity. Now, if there is only one sense of numerical identity then that is the sense in which identity is used in the principle. In that case the principle would have an invariant meaning in all applications, at least insofar as the identity component is concerned. On the other hand, the existence of different senses of numerical identity would raise the possibility of the principle meaning different things on different occasions, depending on the sense of numerical identity used in any particular case. We

shall argue that although there are different senses of numerical identity they share a common presuppositional core which is all the principle of indiscernibility requires.

Suppose that John is sitting in his living room with his dog, Pepper, lying beside him. Suppose that John says, "Pepper is the only dog in this room". What might he mean?

Sense A: John may be talking about Pepper and predicating of him a certain relationship to certain linguistic expressions. That is, he may mean that Pepper can be correctly designated by the expressions "Pepper" and "the only dog in this room". The intentional subject is Pepper. The intentional predicate attribute is ⌜_____ is the referent of "Pepper" and "the only dog in this room".⌝[17]

Sense B: John may be talking about linguistic entities. That is, he may be saying that he is using the expressions "Pepper" and "the only dog in this room" to refer to the same thing. The intentional subject is the pair of expressions and the intentional predicate attribute is ⌜_____ and _____ have the same referent.⌝

Sense C: John may be talking about Pepper and predicating of him a certain attribute. That is, John may mean that Pepper has the attribute of being the only dog in this room. The intentional subject is Pepper and the intentional predicate attribute is ⌜_____ is the only dog in this room.⌝

Sense D: John may be talking about the only dog in this room and predicating of it a certain attribute. John might mean to say that the only dog in this room has the attribute of having the name "Pepper". The intentional subject is the only dog in this room and the intentional predicate attribute is ⌜_____ has the name "Pepper".⌝

Sense E: Although perhaps unlikely, John might simply mean that the thing to which he refers with the expressions "Pepper" and "the only dog in this room" is self-identical. Here the intentional subject is Pepper and the intentional predicate attribute is ⌜_____ is self-identical.⌝[18]

These examples can be generalized. A sentence of the form "$x = y$" can be used to express any of the following propositions:

(A) That some entity, E, is correctly designated by "x" and "y" (where the intentional subject is E and the intentional predicate attribute is ⌜_____ is the referent of "x" and "y"⌝)

(B) That the expressions "x" and "y" have the same referent (where the intentional subject is the pair of expressions and the intentional predicate attribute is ⌜_____ and _____ have the same referent⌝)

(C) That x has the attribute of being identical with y (where the intentional subject is x and the intentional predicate attribute is ⌜_____ is identical with y⌝)

(D) That y has the attribute of being identical with x (where the intentional subject is y and the intentional predicate attribute is ⌜_____ is identical with x⌝)

(E) That the referent of "x" and "y" is self-identical (where the intentional subject is the referent and the intentional predicate attribute is ⌜_____ is self-identical⌝).

In light of these alternatives the impropriety of asking for *the* meaning of identity is obvious. One should rather ask, in each instance, "What does *this* speaker mean?"

What are the implications for the principle of indiscernibility? Does it mean different things on different occasions depending on the sense of identity used in any particular case? Before accepting this conclusion it seems prudent to look for some core of meaning present, at least as a presupposition, in all identity judgments. If there is such a core, and if that core is all the principle of indiscernibility requires, then we can ignore the other senses of identity for purposes of understanding the principle.

It seems clear enough that an assertion of identity in sense D presupposes an assertion of identity in sense A. The proposition that y has the attribute of being identical with x entails the proposition that y is correctly designated by the expressions "x" and "y". Part of the information conveyed by an assertion of identity in sense D is identity in sense A.

Similarly, it is easy to see that assertions of identity in sense C presuppose assertions of identity in sense A.

Sense B presupposes sense A. Asserting that two expressions have the same referent presupposes that the referent is correctly designated by those expressions.

Finally, sense E presupposes sense A. The proposition that the referent of "x" and "y" is self-identical presupposes the proposition that the referent is correctly designated by "x" and "y".

Thus, all the senses of numerical identity presuppose identity in sense A. Hence, we can say that sense A is a common presuppositional core of the others. Part of the information conveyed by an assertion of identity in sense B, C, D or E is identity in sense A.[19]

5 Summary   We have introduced several concepts concerning reference and predication.

With respect to the act of reference, every such act is intentionally tied to an object of focus, the intentional subject. This subject is apprehended as an instance of a universal, the intentional subject attribute. That attribute, in turn, is grasped through an intentional subject concept, which is expressed with an intentional subject expression.

With respect to the act of predication, we have drawn analogous distinctions between the intentional predicate attribute, the intentional predicate concept and the intentional predicate expression.

Finally, we have defined a strong sense of "presupposition" and have argued that there is a sense of numerical identity presupposed in this sense by the other senses of numerical identity. Hence, with respect to the sense of identity at issue in the principle of indiscernibility, we need concern ourselves only with this presupposed sense.

6 What does the principle of indiscernibility mean?   What is the common-sense understanding of the principle? It seems best to begin with an application. Suppose that, while looking at Pepper, John says, "If something is true of Pep-

per then it does not matter what expression (and hence what concept) I use to refer to him in stating that truth". This formulation, which borrows from both the realistic and linguistic understanding of the principle, seems to provide both self-evidence and substitutability.

The formulation is realistic insofar as it says something about the world. It is because Pepper, as a self-identical entity, is what he is that the formulation bears the mark of self-evidence. The formulation is linguistic insofar as it refers to the possibility of different modes of linguistic (and hence conceptual) access to the extra-linguistic entity, and to the legitimacy of substituting these modes of access for each other in assertions about the entity.

Thus, the apparent dichotomy between a realistic understanding of the principle of indiscernibility which is self-evident but useless, on the one hand, and a linguistic understanding which is useful but not self-evident, on the other, is a false dichotomy. The commonsense understanding of the principle is both realistic and linguistic, and yields both self-evidence and utility.

Notice that John's formulation relies upon identity in sense A. He makes Pepper the intentional subject of his assertion and, among other things, says (or at least implies) that Pepper can be designated in more than one way. It follows, in general, that the intentional subject of the identity statement in such an application of the principle is the referent of the two expressions flanking the identity sign.

These observations can be stated more formally and more generally. If x is identical with y (in sense A), and if something, F, is true of x (where x is the intentional subject of the judgment Fx), then F is also true of y (where y is the intentional subject of Fy). Thus, if "x" and "y" are expressions designating the same thing, E, then, in stating truths about E, it does not matter whether those truths are stated with "x" or with "y". And insofar as "x" and "y" may carry differing modes of conceptual access to E, it does not matter whether those truths are stated with one or the other mode.

Putting it schematically, we can say that if (i) $x = y$, and (ii) the intentional subject of Fx is x, and (iii) the intentional subject of Fy is y, then Fy.

This formulation arises from a certain intuitive picture of how the principle of indiscernibility is applied. In a single act the mind focuses upon some particular thing and moves from the propositions that "x" and "y" designate that thing, and that some truth about that thing can be stated with "x", to the proposition that the same truth can be stated with "y".

7 Conditions for the application of the principle   This background picture brings to light certain requirements for the correct application of the principle, requirements which are important in connection with the apparent counterexamples we shall consider in the next section.

Requirement 1. Preserve subject continuity. Do not change the subject of the conversation. This condition, in turn, consists of two more specific conditions: *(1a) Preserve subject continuity between the predicative statements.* That is, make sure that the intentional subject of Fx is the intentional subject of Fy. *(1b) Preserve subject continuity between the identity statement and the predicative*

statements. That is, make sure that "*x*" and "*y*" of the identity statement refer to the intentional subject of *Fx* and *Fy*.

Requirement 2. Preserve predicate continuity. That is, do not change the intentional predicate attribute. The attribute represented by "*F*" in *Fx* must be the attribute represented by "*F*" in *Fy*. This requirement follows from the basic idea that an application of the principle involves only a change in the mode of access to the intentional subject; the thing predicated of the intentional subject remains the same. To speak metaphorically, the principle in application pivots on its predicative leg, moving only its referring leg.

Now the question arises as to how one should enforce this condition of predicate continuity. Since applications of the principle of indiscernibility involve continuity of predicate attributes, it might seem pointless to insist upon continuity of predicate concepts or predicate expressions. Surely attributes can generally be expressed with different expressions and different concepts.

The problem with concentrating solely upon continuity of predicate attributes is that the task of formulating a criterion of identity for attributes is difficult and controversial. There is no consensus on the proper way to count universals. Focusing exclusively upon identity of attributes would compel building into the principle of indiscernibility a criterion of identity for attributes.

This seems an unreasonable burden to impose upon defenders of the principle. We do not deny the interest and importance of the project of fashioning a criterion of identity for universals. But we do deny that that project is identical with that of explicating the principle of indiscernibility. It is not the proper role of the principle of indiscernibility to furnish such a criterion. The principle *presupposes* continuity of predicate attribute. The only thing that should change in an application of the principle is the mode of access to the intentional subject. It is the holding constant of the predicate attribute, while allowing the focus on the intentional subject to change, which gives applications of the principle the self-evidence we have stressed.

Hence, it seems reasonable to insist upon continuity of predicate *expression*. If one's favorite criterion of identity for universals dictates that "*G*" and "*F*" express the same universal, then that universal can be adequately expressed with either "*G*" or "*F*" alone. The moral is that one should make whatever identifications of predicate attribute one thinks are indicated *before* applying the principle of indiscernibility. Once such an identification has been made, the principle should be applied using a single form of linguistic expression for that attribute.

Thus, although the ultimate concern of Requirement 2 is continuity of intentional predicate attribute, we shall enforce that condition by insisting upon continuity of intentional predicate expression. The gain is that one can usually recognize continuity of linguistic expression. We thereby prevent the difficulties inherent in counting universals from casting doubt on the meaning or applicability of the principle of indiscernibility.

8 *Some apparently falsifying instances of the principle*   We turn to consider some examples which seem to falsify the principle of indiscernibility. Specifi-

cally, we shall consider the use of the principle in epistemic, modal, and definitional contexts.

8.1 Epistemic contexts

Suppose that last night Pepper killed the neighbor's cat but that John does not know this. John does know that Pepper has black hair. Then the following propositions are true:

(a) Pepper = the dog who killed the neighbor's cat last night
(b) John knows that Pepper has black hair.

But this proposition seems false:

(c) John knows that the dog who killed the neighbor's cat last night has black hair.

Thus, we seem to have a case in which the antecedent of an application of the principle of indiscernibility is true while the consequent is false, thereby falsifying the principle. (For a similar example, see [13], p. 141.)

But closer examination dispels the appearance of falsification. Consider the possible cases.

Case 1. Suppose that the intentional subject of both (b) and (c) is Pepper. Then both (b) and (c) are true and there is no counterexample. Indeed, this is a correct application of the principle. There is subject continuity between the two predicative statements; the intentional subject of both is the thing designated by "Pepper". There is subject continuity between the identity statement and the predicative statements; Pepper is the intentional subject of all the statements. And there is predicate continuity. The intentional predicate expression of both predicative statements is "John knows that ____ has black hair".

Case 2. Suppose that the intentional subject of both (b) and (c) is John. Then (b) is true; it is true of John that he knows that Pepper has black hair. What about (c)?

If (c) means that John is an entity x such that x knows that the referent of the "the dog who killed the neighbor's cat last night" has black hair, then (c) is true. For John is acquainted with that referent and knows that it has black hair. But if (c) is true there is no counterexample.

On the other hand, if (c) means that John is an x such that x knows that a dog killed the neighbor's cat last night and knows that this dog has black hair, then (c) is false. But there is no counterexample because Requirement 1b has not been satisfied; there is no subject continuity between the identity statement and the predicative statements. The referent (Pepper) of the terms in the identity statement is not the intentional subject (John) of the predicative statements. In addition, Requirement 2 has not been met; there is no continuity of intentional predicate expression. The predicate expression of (b) is "____ knows that Pepper has black hair". The predicate expression of (c), under this interpretation, is "____ knows that a dog killed the neighbor's cat last night and knows that this dog has black hair".

Case 3. Suppose that the intentional subject of (b) is Pepper and that the intentional subject of (c) is John. Then (b) is true and (c) is false, (where (c) is interpreted in the second of the ways outlined in Case 2). But there is no

counterexample because Requirement 1a has not been satisfied. There is no subject continuity between the predicative statements.

Case 4. Suppose that the intentional subject of (b) is John and that the intentional subject of (c) is Pepper. Then (c) is true and we need not worry about a counterexample. (In any case, the example violates Requirement 1a; there is no subject continuity between the predicative statements.)

Case 5. Suppose that the intentional subject of (b) is the proposition that Pepper has black hair and that the intentional subject of (c) is the proposition that the dog who killed the neighbor's cat last night has black hair. Then (c) is false but there is no counterexample. The case violates both Requirements 1a and 1b; there is failure of subject continuity between the predicative statements and between the identity statement and the predicative statements.

Other cases. The other possible cases can be similarly handled. Requirement 1b is violated by the case in which the intentional subject of both (b) and (c) is the ordered pair (John, Pepper). Requirement 1a is violated by the case in which the intentional subject of (b) is the ordered pair (John, Pepper) and the intentional subject of (c) is John. And so on.

8.2 Modal contexts   Having argued the value of allowing oneself reference to the mind in clarifying the use of the indiscernibility principle in epistemic contexts, we turn to its use in modal contexts.

These propositions seem true:

(d)   9 = the number of planets
(e)   It is necessarily true that 9 is greater than 7.

But this proposition seems false:

(f)   It is necessarily true that the number of planets is greater than 7.

Once again, we seem to have a counterexample. (This example is from [13], p. 143.)

As before, it seems possible to dispel the appearance of falsification.

Case 1. Suppose that the intentional subject of both (e) and (f) is the number 9. Then (e) and (f) are true and there is no counterexample. This case is a correct application of the principle. There is subject continuity between the predicative statements, and between the identity statement and the predicative statements. The intentional subject of all of them is the number 9. And there is predicate continuity between the predicative statements. The predicate expression of both is "It is necessarily true that _____ is greater than 7".

Case 2. Suppose that the intentional subject of both (e) and (f) is the number 7. Then (e) is true. Whether (f) is true depends upon how it is construed.

If (f) is construed as asserting that the number 7 is an entity x such that it is necessarily true that the referent of "the number of planets" is greater than x then (f) is true and there is no counterexample.

On the other hand, if (f) is construed as stating that the number 7 is an x such that it is necessarily true that the class of planets has more than x mem-

bers, then (f) is false. But Requirement 1b is violated. The intentional subject of the predicative statements is 7 while that of the identity statement is 9.

(In addition, Requirement 2 is violated. The predicate expression of (e) is: "It is necessarily true that 9 is greater than _____". That of (f) is: "It is necessarily true that the class of planets has more than _____ members".)

Case 3. Suppose that the intentional subject of (f) is the class of planets. (The idea here is to interpret (f) as asserting that it is necessarily true that the class of planets has more than 7 members, a claim which is false.) Now, the intentional subject of (e) cannot be the class of planets, since that entity is not mentioned in (e). Hence, the intentional subject of (e) must be either the number 9 or the number 7. Under either interpretation (e) is true. On the other hand, (f) is false, as we have noted. But there is no counterexample because Requirement 1a has not been satisfied. There is no subject continuity between the predicative statements.

Case 4. If the intentional subject of (e) is the proposition that 9 is greater than 7 and the intentional subject of (f) is the proposition that the number of planets is greater than 7, then both Requirements 1a and 1b are violated. There is failure of subject continuity between the predicative statements, and between the identity statement and the predicative statements.

Other cases. The other cases can be similarly analyzed. For example, the case in which the intentional subject of (e) is the number 9 and the intentional subject of (f) is the number 7 violates Requirement 1a. And so on.

8.3 Paradox of Analysis   Finally, we consider definitional contexts. The following propositions seem true:

(g)   The universal ⌜humanity⌝ = the universal ⌜rational animality⌝
(h)   The proposition that the universal ⌜humanity⌝ = the universal ⌜rational animality⌝ is illuminating.

But application of the principle of indiscernibility seems to lead to this falsehood:

(i)   The proposition that the universal ⌜humanity⌝ = the universal ⌜humanity⌝ is illuminating.[20]

Once again, analysis shows that there is no falsification of the principle.

Case 1. Suppose that the intentional subject of (h) is the proposition that the universal ⌜humanity⌝ = the universal ⌜rational animality⌝, and that the intentional subject of (i) is the proposition that the universal ⌜humanity⌝ = the universal ⌜humanity⌝. Both Requirements 1a and 1b are violated. There is no subject continuity between the identity statement and the predicative statements and no subject continuity between the predicative statements. Hence, there is no counterexample.

Case 2. The intentional subject of (h) is the proposition that the universal ⌜humanity⌝ = the universal ⌜rational animality⌝, and the intentional subject of (i) is the universal ⌜humanity⌝, where the intentional subject of (i) is referred to by the first occurrence in (i) of the words "the universal ⌜human-

ity⌐¬". This violates Requirement 1a; there is no subject continuity between the predicative statements.

Case 3. The intentional subject of (h) is the proposition that the universal ⌐humanity¬ = the universal ⌐rational animality¬, and the intentional subject of (i) is the universal ⌐humanity¬, where the intentional subject of (i) is referred to by the second occurrence in (i) of the words "the universal ⌐humanity¬". Requirement 1a is again violated; there is no subject continuity between the predicative statements.

Case 4. The same argument serves to dispose of the case in which the intentional subject of (h) is the proposition that the universal ⌐humanity¬ = the universal ⌐rational animality¬, and the intentional subject of (i) is the universal ⌐humanity¬, where the intentional subject of (i) is referred to by both occurrences in (i) of the words "the universal ⌐humanity¬".

In similar fashion, Cases 5 through 7 can be handled by observing that each involves violation of Requirement 1a, a failure of subject continuity between the predicative statements.

Case 5. The intentional subject of (i) is the proposition that the universal ⌐humanity¬ = the universal ⌐humanity¬, and the intentional subject of (h) is the universal ⌐humanity¬, where the intentional subject of (h) is referred to by the words "the universal ⌐humanity¬".

Case 6. The intentional subject of (i) is the proposition that the universal ⌐humanity¬ = the universal ⌐humanity¬, and the intentional subject of (h) is the universal ⌐humanity¬, where the intentional subject of (h) is designated by "the universal ⌐rational animality¬". [Additional Comment: The case fails for another reason as well. The interpretation falsifies (h), rendering the example useless as a counterexample. For, under this interpretation (h) states that the universal ⌐humanity¬ is an entity x such that x satisfies the predicate "The proposition that the universal ⌐humanity¬ = x is illuminating". That is, (h) asserts that it is illuminating to say of an entity that it has the attribute of being itself, an assertion which is plainly false.]

Case 7. The intentional subject of (i) is the proposition that the universal ⌐humanity¬ = the universal ⌐humanity¬, and the intentional subject of (h) is the universal ⌐humanity¬, where the intentional subject of (h) is designated by the words "the universal ⌐humanity¬" and by the words "the universal ⌐rational animality¬". [Additional Comment: A comment analogous to that made under Case 6 applies here. This interpretation falsifies (h). For, (h) asserts that the universal ⌐humanity¬ is an x which satisfies the predicate "The proposition that $x = x$ is illuminating".]

The additional cases can be similarly handled.[21]

Thus, analysis shows that the Paradox of Analysis does not falsify the principle of indiscernibility. (i) does not follow from (g) and (h). Incidentally, the analysis also shows that there is no Paradox of Analysis.[22]

9 Conclusion   In conclusion, we have tried to show how bringing into the picture the way in which the mind uses the principle of indiscernibility leads to

the formulation of a self-evident and useful version of the principle and to a dis-
solution of some of the familiar paradoxes of identity.

NOTES

1. For convenience, we shall use the phrase "principle of indiscernibility" when refer-
   ring to the principle. But this should not be taken to mean that we see nothing of
   value in the linguistic version. We shall later argue that a proper understanding of
   the principle incorporates both a realistic and a linguistic component.

2. See [13], p. 141. We assume that quotational contexts present no problem for the
   principle of indiscernibility in light of the possibility of refusing to classify instances
   of referring expressions inside quotation marks as genuine occurrences of those
   expressions.

3. Since the principle of indiscernibility is formulated as a universal generalization, a
   refutation would have to show at least that the generalization is not universally true.
   This, in turn, would require specification of at least one instance of the principle
   with a true antecedent and a false consequent. Thus, a refutation would have to be
   a falsifying interpretation in which, for some particular a, b, and F, $a = b$ and a is
   F but b is not F.

4. Quine's use of the phrase "failure of substitutivity" might be taken to suggest that
   he thinks that such examples falsify the principle of indiscernibility. This is appar-
   ently not his meaning. He says elsewhere, for example, that the principle "cannot
   properly be challenged" (see [12], p. 174). And, indeed, an overall reading of [13]
   suggests that for Quine the choice is between the truth of the principle of indiscer-
   nibility and the meaningfulness of referentially opaque contexts. If the analysis we
   offer in this paper is correct then that dilemma is a false one.

5. We use the adjective "intentional" throughout this section in order to suggest the
   factor of selectivity variously involved in the act of judgment.

6. We assume that referring is done by persons, not by words. Words are used by per-
   sons to refer. See, e.g., [7], p. 89.

7. Our concept of intentional subject is not the same as Donnellan's concept of the
   object of a referential, as opposed to an attributive, use of a definite description.
   (For the referential-attributive distinction, see [4].) The objects of attributive uses
   of definite descriptions are candidates for intentional subjects in our sense. For
   example, consider the sentence "The man who killed Smith is insane", where the
   definite description is used attributively in Donnellan's sense. The intentional subject
   of the proposition expressed by the sentence is the intentional subject in our sense,
   despite the fact that the subject is accessed attributively.

8. Notice that although the concept of intentional subject is person-relative in the sense
   that to be an intentional subject is to be the object of some intellect's attention, it
   does not follow that the subject itself is person-relative. The object of focus is what
   it is, independent of any mode of referential access. The object is always more than
   the instantiated attribute seized by the mind in making the reference. Although one
   cannot gain referential access to an entity without selective reference, it does not
   follow that the existence or nature of the object itself is a function of any mode of
   referential access. (We do not deal here with the question of self-referential expres-
   sions.)

9. Although we cannot argue the matter here, we think that even the case of reference with proper names can be accommodated by this approach. If something has a name then one of its attributes is having that name. It does not necessarily follow, however, that proper names have a "sense" in Frege's sense.

10. Although we cannot deal adequately with the matter here, perhaps a few words are in order about our usage of the word "concept". We do not identify concepts with linguistic usages. Although a concept may be expressed in words, it does not have to be. As Price has shown, many concepts are pre-verbal. Even with respect to concepts acquired and used after language acquisition, it is the concept, not the words, which is epistemologically fundamental (see [11]).

On the other hand, we do not identify concepts with universals. Suppose that Smith thinks that humans have souls while Jones does not. Here there is only one universal, viz., humanity, but two concepts. Hence, a concept could not *be* a universal.

The basic idea is that the mind uses concepts to apprehend universals. Universals do not enter the mind bodily; they are grasped in one way or another through concepts. There is a sense in which concepts are on the mind's side of things whereas universals are independent of the mind's activity.

This explains why certain things can be meaningfully said about concepts which cannot be said (in the same sense) about universals. For example, concepts can be acquired, rejected, constructed, modified, taught, applied, misused, complicated, simple, incoherent, vague, and inadequate. Such things cannot be said (in the same sense) about universals.

This distinction between concepts and universals is the same as that drawn by some Thomists between "mental" and "objective" concepts (see, e.g., [9], pp. 17–20).

Both concept and universal should be distinguished from the mental act of conception.

In order to forestall ontological misgivings about the nature of concepts we hasten to add that they should not be understood in a Lockean or Kantian spirit as opaque entities standing *between* the mind and reality, making it necessary for the mind to *infer* things about the world from the nature of its concepts. We understand concepts in the traditional Thomistic way as formal rather than material signs. A concept is an intentional tool of the mind. Its entire being, so to speak, is the grasping of its intended universal (see, in general, [10], pp. 16–20).

Of course, despite their intentional and formal nature, concepts can be more or less adequate to their task of apprehending reality. Any theory of concepts must do justice to the fact of error and misunderstanding.

Medieval philosophers spoke of the *res* and the *ratio*. The *res* is the thing (whether at the moment existing or not) stripped of the relationship it bears to the apprehending mind. The *ratio* is that of the *res* which the mind grasps. It is the *res* as presenting this or that feature of itself to the mind. One can identify our "concept" with "ratio" so long as one remembers that the *ratio* is a function of the thing *and* the apprehending mind. Otherwise, as just remarked, there would be no way to account for the *ratio* which is inadequate to the reality it purports to grasp.

The phenomenon of having a concept is complex. An adequate account would have to consider several things. Among them are the capacity to recognize instances of a universal, the method or methods used to identify instances of a universal, propositions understood as defining a universal, the capacity to formulate such propositions, propositions understood as describing a typical instance of a universal, the capacity to formulate such descriptions, images of typical instances of a univer-

sal, the capacity to call forth such images and the capacity to formulate an understanding of a universal in words.

11. The object of reference need not be a particular. Universals can be intentional subjects of propositions. In addition, the intentional subject need not exist in any sense other than being an object of thought. Golden mountains or round squares are just as qualified to be intentional subjects as horses and chairs.

12. We do not mean to beg any questions concerning the debate over whether relational propositions are reducible to propositions of the traditional *A*, *E*, *I*, and *O* forms. That is, we do not mean to suggest that the intentional subject of a proposition has to be a single entity. It might well be the case that the intentional subject of a proposition is an ordered pair or an ordered triplet.

   Neither do we want to be read as equating the concept of an intentional subject with what some have called an "ultimate subject" (see, e.g., [3], pp. 66–67). That is, we do not mean to suggest that the intentional subject of a proposition is necessarily what one would be left with after applying appropriate tools of metaphysical reduction.

13. In a well-known paper Strawson argued that *Q* is a presupposition of *P* only if the truth of *Q* is a necessary condition for the truth *or* falsity of *P*. That is, *Q* is a presupposition of *P* only if the truth or falsity of *P* does not even arise unless *Q* is true. If *Q* is false then *P* is neither true nor false (see [14]).

   Strawson's claim to have ordinary usage on his side seems at least partially supported. If I am instructed to answer either "true" or "false" to the question whether the King of France is bald, I naturally feel a measure of discomfort with either response. I would prefer an alternative permitting me to make clear that there is no King of France.

   But there is evidence cutting the other way as well. Suppose that I am testifying in a criminal trial on my own behalf as defendant. I have committed the crime but claim to be innocent and have invented an alibi involving an imaginary Mr. Smith. If, in the course of testifying, I state, "I remember the time of day because I glanced at Mr. Smith's watch", is it not clear that I have committed perjury? I have uttered a *falsehood*. Yet, on Strawson's theory, it seems that I have uttered a proposition which is *neither* true nor false.

   An additional difficulty with Strawson's theory is that it sacrifices some of the generality of the law of excluded middle.

14. A question here is whether "entailments" should be restricted to "deductive" entailments. It seems that the term should not be so restricted. Many of the propositions we are intuitively inclined to classify as presuppositions of a given proposition are not deductively entailed by the proposition. For example, it seems right to say that one of the presuppositions of the proposition that I won the ball game with a home run in the ninth inning is that I did not have a blindfold over my eyes when I swung the bat. Yet it seems clear that the first proposition does not deductively entail the second.

   With the admission of nondeductive implications it is perhaps better to introduce a comparative sense of "presupposition". That is, a proposition *P* might be said to presuppose a proposition *Q* to a greater or lesser extent than *P* presupposes a proposition *R*, depending upon the relative strength of the implicative connection between *P* and *Q*, on the one hand, and *P* and *R*, on the other.

15. One might also find useful a distinction between the "objective" and "subjective" presuppositions of a proposition. The subjective presuppositions of a proposition

P, as uttered by a speaker *S*, are those entailments of *P* which *S* recognizes as entailments of *P*. The objective presuppositions of a proposition *P* are all of *P*'s entailments, whether recognized as such by anyone or not.

16. At least three kinds of presupposition can be distinguished. First, some of the presuppositions of a proposition are carried by its intentional subject attribute. That is, some of the presupposed implications of a proposition are based on the mind's choice of a particular way of referring to something. For example, one of the presuppositions of the proposition that the dog in this room has black hair is the proposition that the dog is in this room. Second, some of the presupposed implications of a proposition are grounded in the mind's selection of a certain predicate attribute. For example, one of the presuppositions of the proposition that the dog in this room has black hair is the proposition that the dog in this room has hair. Third, some of the presuppositions of a proposition flow from the proposition taken as a whole. For example, another of the presuppositions of the proposition that the dog in this room has black hair is the proposition that something in this room is black.

17. We use the square-corner notation for designating attributes.

18. So understood, the proposition that Pepper is the only dog in this room is a necessary truth. (In Kripke's usage, both "Pepper" and "the only dog in this room" are functioning as rigid designators. See [5].)
      Note that this proposition is not equivalent to the necessary truth that the only dog in this room is identical with the only dog in this room, or to the necessary truth that Pepper is identical with Pepper. For, the first proposition presupposes something the latter two do not, viz., that the object of reference is correctly designated by "Pepper" and "the only dog in this room". Hence, the first proposition conveys information not conveyed by either of the others.

19. There are other relationships of presupposition among the listed senses. For example, sense A presupposes senses E and B; sense E presupposes sense B, etc. We shall not need to use these other relationships.

20. For a discussion of the paradox of analysis, see [6]. See also Moore's reply to Langford in the same volume.

21. Now we consider a group of cases in which the intentional subject of both (h) and (i) is the universal humanity. The cases differ with respect to the way in which that universal is designated.

     Case 8. The universal ⌜humanity⌝ is designated in (h) by the words "the universal ⌜humanity⌝" and in (i) by the first occurrence there of the words "the universal ⌜humanity⌝". This violates Requirement 2. The predicative expression of (h) is "The proposition that _____ = the universal ⌜rational animality⌝ is illuminating". The predicative expression of (i), on the other hand, is "The proposition that _____ = the universal ⌜humanity⌝ is illuminating".

     Case 9. The universal ⌜humanity⌝ is designated in (h) by the words "the universal ⌜humanity⌝" and in (i) by the second occurrence there of the words "the universal ⌜humanity⌝". Again, Requirement 2 is not satisfied. The predicative expression of (h) is "The proposition that _____ = the universal ⌜rational animality⌝ is illuminating", whereas the predicative expression of (i) is "The proposition that the universal ⌜humanity⌝ = _____ is illuminating".

Case 10. The universal ⌜humanity⌝ is designated in (h) by "the universal ⌜humanity⌝" and in (i) by both occurrences there of "the universal ⌜humanity⌝". The predicative expression of (h) is "The proposition that ____ = the universal ⌜rational animalty⌝ is illuminating". The predicative expression of (i) is "The proposition that ____ = ____ is illuminating".

Case 11. The universal ⌜humanity⌝ is designated in (h) by "the universal ⌜rational animality⌝" and in (i) by the first occurrence of "the universal ⌜humanity⌝". The predicative expression of (h) is "The proposition that the universal ⌜humanity⌝ = ____ is illuminating". The predicative expression of (i) is "The proposition that ____ = the universal ⌜humanity⌝ is illuminating". (Additional Comment: The additional comment made under Case 6 applies here as well.)

Case 12. The universal ⌜humanity⌝ is designated in (h) by "the universal ⌜rational animality⌝" and in (i) by the second occurrence of "the universal ⌜humanity⌝". Now here we have continuity of predicate expressions. The predicate of both (h) and (i) is "The proposition that the universal ⌜humanity⌝ = ____ is illuminating". But there is no counterexample for the reason stated in the additional comment under Case 6. The interpretation falsifies (h).

Case 13. The universal ⌜humanity⌝ is designated in (h) by "the universal ⌜rational animality⌝" and in (i) by both occurrences of "the universal ⌜humanity⌝". The predicate of (h) is "The proposition that the universal ⌜humanity⌝ = ____ is illuminating". The predicate of (i) is "The proposition that ____ = ____ is illuminating". (Additional Comment: See the additional comment under Case 6.)

Case 14. The universal ⌜humanity⌝ is designated in (h) by "the universal ⌜humanity⌝" and by "the universal ⌜rational animality⌝", and in (i) by both occurrences of "the universal ⌜humanity⌝". The predicate expressions of both (h) and (i) are the same, viz., "The proposition that ____ = ____ is illuminating". Hence, there is predicate continuity. But for the reason set forth in the additional comment under Case 7, there is no counterexample because (h) is false. (h) says that the universal ⌜humanity⌝ is an x which satisfies the predicate "The proposition that $x = x$ is illuminating". (This point can be made as well about Cases 15 and 16. But those cases also involve violations of Requirement 2 as indicated.)

Case 15. The universal ⌜humanity⌝ is designated in (h) by the expression "the universal ⌜humanity⌝" and by "the universal ⌜rational animality⌝". The same universal is designated in (i) by the first occurrence of "the universal ⌜humanity⌝". The predicate of (h) is "The proposition that ____ = ____ is illuminating", while the predicate of (i) is "The proposition that ____ = the universal ⌜humanity⌝ is illuminating".

Case 16. The universal ⌜humanity⌝ is designated in (h) by "the universal ⌜humanity⌝" and by "the universal ⌜rational animality⌝". In (i) it is designated by the second occurrence of "the universal ⌜humanity⌝". The predicate of (h) is "The proposition that ____ = ____ is illuminating", while that of (i) is "The proposition that the universal ⌜humanity⌝ = ____ is illuminating".

22. In [13] Quine discusses two additional examples. One involves attributes, the other, propositions. We believe that each of these can be handled in the way we have approached the examples discussed above.

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