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The Problem of Counterpossibles

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Introduction In the first chapter of *Counterfactuals* ([7]), David Lewis raises the question of evaluating counterfactuals with impossible antecedent conditions. Under his initial proposal, all such counterfactuals are vacuously true. The derivatively defined 'might counterfactuals', accordingly, are all false. While Lewis himself is "fairly content" with this, he recognizes that it does not have universal appeal and so proposes alternative sets of truth conditions for counterfactuals which do not have these untoward consequences. The alternatives are a strengthened 'would' and a weakened 'might' for which the values are reversed when the antecedent is impossible: unless "impossible worlds" are added to the semantic framework, stronger 'woulds' with impossible antecedents all become false and the weaker 'mights' become true. Lewis expresses some preference for the mixed pair of the weaker (original) 'would' and the weaker (second) 'might' but thinks the simple interdefinability of either the original or second pair outweighs whatever might be gained. He ultimately opts for the original pair, but notes that the others can be recovered, if not quite so simply.

It will be argued here that Lewis's motivation for vacuously true "counterpossibles" suggests an interpretation using the logic of conditional assertion in the metalanguage of the semantics, and that such an interpretation satisfies all the major desiderata.

1 The problem That there should be any controversy over the matter of counterfactual conditionals with antecedent clauses that describe logically impossible states of affairs might itself be thought odd. There are, to be sure, reasons why *some* theories of conditionals would need to be concerned with impossible antecedents. Any theory of the logical relation "follows-from", such as Relevant Implication, ought to be able to handle the unfortunately all-too-possible case of reasoning from inconsistent data. A theory of counterfactuals, however, is not offered as either an information processing calculus or a theory of some

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objective, extralinguistic phenomenon. Rather, it is to explicate our linguistic practice and intuitions.

In addition to this, because the subject matter is primarily linguistic practice and not some objective logical phenomenon, there is another reason for thinking impossible antecedents would be a nonissue: no one really loses any sleep worrying about what would be the case or what might be the case if something that logically could not be the case were the case. Impossible antecedents would appear to represent a limiting case for counterfactual locutions: just as a counterfactual is inappropriate when the antecedent is accepted as true, so too the locution seems to require that the antecedent be entertainable.

Against this are three good reasons in favor of dealing with the matter—aside from simply rounding out one's theory. In the first place, there are impossible counterfactuals—"counterpossibles"—that we do want to count as assertable and there are distinctions among such counterfactuals that we do want to make. For example,

(1) If only 8×7 were 58, I'd have had an A on my arithmetic test,

would certainly be assertable in the appropriate circumstances even though the antecedent describes an impossibility—and we may be well aware of its impossibility. Lewis ([8], p. 25) offers the following as "sensible things to say":

(2) If there were a largest prime p, $p! + 1$ would be prime,

and

(3) If there were a largest prime p, $p! + 1$ would be composite.

But

(4) If there were a largest prime p, pigs would have wings,

is not. But perhaps, as he suggests, this is a question concerning the assertability conditions, not the truth conditions.

Even so, this points to a second justification for confronting the problem: counterfactual reasoning, which is to say, reductio proofs. Examples (2) and (3), unlike (4), are ways of expressing steps that might appear in a reductio proof. The connection between conditionals and reasoning is just what a deduction theorem establishes. But one has to be careful here. It is easy to confuse conditional kinds and conclude that reductio proofs are sequences of counterfactuals. That would be a serious error for two reasons. One, counterfactuals are not subject to some of the laws of logical implication, e.g. weakening, contraposition, and transitivity. Two, it would reduce any such argument to a *petitio principii*: counterfactuals presuppose the falsity of the antecedent—which is exactly what is to be proved. (This argument is from [10], pp. 200-201.) Rather, indirect reasoning can be understood as a sequence of logical implications. But if A logically implies B, then A "counterfactually implies" B, too (and the "if . . . then . . ." just used is the logical one, hence the counterfactual would also be acceptable). The concept of a counterfactual, therefore, is an integral part of our notion of indirect proof insofar as counterfactuals are immediate consequences. That is, mixing the notation of Anderson and Belnap's relevant implication with Lewis's for counterfactuals, $(A \rightarrow B) \rightarrow (A \mathrel{\Box\!\!\rightarrow} B)$ and $(A \rightarrow$

$B) \mathbin{\Box\!\!\rightarrow} (A \mathbin{\Box\!\!\rightarrow} B)$, but neither's converse, are valid conditional forms. So, if we cannot make sense of counterfactuals with impossible antecedents, the same would be true of the corresponding logical implications and, by implication, of indirect proofs.

The third reason for confronting the problem of counterpossibles as part of the larger question of counterfactuals is that this is just what this kind of theory is supposed to do. The process of explicating a concept manifest in linguistic practice (and this *is* the case with counterfactuality, even if not for logical implication) is one of both formally capturing it and sharpening the concept. This may in part involve doing away with some of the rich fuzziness around the edges. This is not done, however, merely to resolve peripheral ambiguities. Rather, it is for the light this can shed on the entire body of counterfactual talk. In the case at hand, the extreme case of impossible antecedents is exploited as the exception that proves (i.e., tests) the theory. One ignores or dismisses the odd case at peril to one's grasp of all the others.

2 The originals Some familiarity with Lewis's treatment of counterfactuals is presupposed. Only the barest essentials and points of particular relevance to this discussion will be presented here.

Let $A \mathbin{\Box\!\!\rightarrow} B$ represent the 'would' counterfactual 'If it were the case that A, then it would be the case that B', and let $A \mathbin{\Diamond\!\!\rightarrow} B$ represent the 'might' counterfactual 'If it were the case that A, it might be the case that B'. They are to be interdefinable as follows:

$$A \mathbin{\Box\!\!\rightarrow} B = {\sim}(A \mathbin{\Diamond\!\!\rightarrow} {\sim}B)$$

and

$$A \mathbin{\Diamond\!\!\rightarrow} = {\sim}(A \mathbin{\Box\!\!\rightarrow} {\sim}B).$$

(I prefer this symbolization to ">" since that has been used generically in discussions of conditional logics.) The truth conditions provided by Lewis's semantics involve reference to a set of possible worlds ordered by a relation of relative similarity. The result can be understood as systems of nested spheres around each world, each of whose inner spaces contains worlds more similar to the center world than any worlds outside the given sphere. Finally, $A \mathbin{\Box\!\!\rightarrow} B$ is true at a world w, iff either there is some sphere centered on w that contains some A-world (a world in which A is true) and $A \supset B$ is true at every world in that sphere, or else no sphere centered on w has any A-worlds. Derivatively, $A \mathbin{\Diamond\!\!\rightarrow} B$ is true at w iff there is some sphere centered on w that has an A-world and every sphere with an A-world has an A & B-world. Intuitively, $A \mathbin{\Box\!\!\rightarrow} B$ is true at a world iff *all* of the closest A-worlds are B-worlds; and $A \mathbin{\Diamond\!\!\rightarrow} B$ is true at a world iff *some* of the closest A-worlds are B-worlds.

What happens if the antecedent describes an impossibility? If there are no A-worlds (i.e., if A is not possible), then there are no closest A-worlds. Then $A \mathbin{\Box\!\!\rightarrow} B$ is true by its truth conditions. The intuitive formulation, as an Aristotelian A-proposition, has an empty subject class and these are commonly counted as vacuously true. But $A \mathbin{\Diamond\!\!\rightarrow} B$ comes out false. The I-proposition form of the intuitive truth conditions is interpreted as making an existential claim.

3 In favor of vacuous truth One always has the option of ignoring problems at the extremes provided that the main body is well-handled. This seems to be Lewis's strategy: cover the middle ground and let the chips fall where they may for the curiosities. Lewis describes himself as "fairly content" with the consequences that all 'would' counterfactuals with impossible antecedents come out true on his theory but he does offer some reasons why this is as it should be.

The first defense Lewis offers is precisely the attitude characterized above:

> Confronted by an antecedent that is not really an entertainable supposition, one may react by saying, with a shrug: If that were so, anything you like would be true! ([8], p. 24)

Let us call this the "shrug defense". It is not without some weight, for hypothesizing an impossibility does violate some conversational conventions. The limits of what can be set up as a counterfactual situation have been transgressed. Accordingly, one might reasonably expect anarchy to ensue and tolerate it when it does. The problem of distinctions is relegated to the larger question of assertability rather than truth.

The second line of reasoning implicitly recognizes the validity of the schemata noted in Section 1 above, or at least the validity of the inference from logical implications to counterfactual conditionals:

> Further, it seems that a counterfactual in which the antecedent logically implies the consequent ought always to be true. . . .

But Lewis explicitly adopts as his theory of logical implication, not *R*, but the "material conditional":

> . . . and one sort of impossible antecedent, a self-contradictory one, logically implies any consequent. ([8], p. 24)

The argument for this "material conditional defense" is certainly valid: if, one, the horseshoe of the material conditional is the proper account of logical implication, and, two, logical implications entail counterfactuals, then counterpossibles are all (vacuously) true. Soundness is another matter. The second premise seems unexceptionable; the first has been controverted with regularity.

Finally, Lewis notes that at least some counterpossibles are asserted, hence, already counted as (vacuously) true in the course of reductio arguments. Uniform treatment suggests his conclusion. Of course, not all counterpossibles are tolerated as vacuous, but that too might be a matter of assertability.

Taken separately, none of these arguments is terribly persuasive. There are more or less obvious refutations available for anyone so inclined. Lewis is well aware of this but no great weight is placed on either these arguments or the issue itself. Curiously, taken together these arguments may present an even weaker case for Lewis's treatment of counterpossibles insofar as they operate under inconsistent presumptions and insofar as they are at odds with other parts of Lewis's theory. For example, while the shrug defense suggests that we may be arbitrary about assigning truth values to counterpossibles, the argument from actual reductio practice suggests the incompatible view that this is a question whose answer can be empirically decided by linguistic investigation, and the material conditional account suggests the dramatically opposed view that this is an issue that is nonempirically and definitively settled by logic alone. Further, it surely seems as though 'would' counterfactuals should imply the corresponding—and weaker—'might' counterfactuals. This, however, is not universally the case in the Lewis system: the implication holds *except* for counterpossibles. Nevertheless, some of the arguments for the truth of 'would' counterpossibles apply *mutatis mutandis* to the weaker 'might' case. In particular, the shrug defense so translates and Lewis himself makes the translation:

> confronted by an antecedent that is not really entertainable, one might say, with a shrug: If that were so, anything you like *might* be true! ([8], p. 25)

If anything, the shrug defense is stronger for 'might' counterpossible. 'Might' counterpossibles, then, should also be vacuously true, but they are not. Lewis provides no specific motivation for having 'might' counterpossibles all false.

To be fair to Lewis, he offers the above translation in the course of providing alternative truth conditions for counterfactuals that avoid the consequence that all 'would' counterpossibles are vacuously true. It is to these that we now turn.

4 Mighty 'woulds' and wooden 'mights' One way to preserve the system-of-spheres semantic framework for evaluating counterfactuals without making counterpossibles vacuously true is to include "impossible worlds", worlds that are not closed under deductive consequence (assuming one is still laboring under a material conditional reading of implication). Lewis balks at this: the hesitancy to reify impossible worlds is the flip side of his distaste for any instrumentalist account of genuinely possible worlds (see [8], pp. 84–91, [7], and [9]).

Instead, Lewis offers the following proposals (using, as he does, the box-and-double-arrow for this alternative): $A \mathrel{\Box\!\!\Rightarrow} B$ is true at a world iff some sphere around that world has an A-world and $A \supset B$ holds at every world in that sphere. This strengthened 'would' cannot be vacuously true. If there are no A-worlds in any of the spheres around the world at which the counterfactual is to be evaluated, i.e., if the counterfactual is a counterpossible, then it is false.

As before, this 'would' can be used to define a 'might' counterfactual. However, this strengthened 'would' gives rise to a weakened 'might', $A \mathrel{\Diamond\!\!\Rightarrow} B$, which is true at a world iff every sphere around that world that has an A-world also has an A & B-world. If there are no impossible worlds, then all of these weakened 'might' counterpossibles would be vacuously true.

It is clear from his remarks that Lewis thinks the optimal combination would be the original 'would' with the alternative 'might'. That is, he prefers the weaker of each pair, those having vacuously true counterpossibles. The shrug defense, which he offers for each separately, would then be uniformly applicable. But, he adds,

> the simple interdefinability of 'would' and 'might' seems plausible enough to destroy the appeal of the mixed pair of $\Box\!\!\rightarrow$ and $\Diamond\!\!\Rightarrow$.

Either pair can be defined by the other, but not so simply. In the next sections, a way of obtaining the mixed pair will be suggested.

5 Aristotle, Frege, and Lewis The appeal of the mixed pair of weak counterfactuals stems in part, I think, from two different, sometimes incompatible, traditions in logical thought. Each of these separate traditions provides a desideratum for the theory, and in light of Lewis's own contributions, the two are compatible only with some effort.

On the one hand, there is the tradition that can be called "Fregean" which has it that general claims whose subject classes are empty are true, even if only "vacuously". This falls right out of the reading of A-propositions as quantified conditionals. The truth conditions for the original 'would' counterfactual can be put into an explicit A form, taking advantage of thus:

> $A \mathrel{\Box\!\!\rightarrow} B$ is true iff *all* the closest A-worlds, i.e., all A-worlds in the smallest A-permitting sphere, are B-worlds,

where an A-permitting sphere is a sphere with at least one A-world. Consequently, it becomes desirable to have 'would' counterpossibles true.

On the other hand, there is the even more venerable "Aristotelian" tradition in which an I-proposition, or 'some'-sentence, is subaltern to an A-proposition and so can be had by an immediate inference. Sure enough, the truth conditions for the original 'might' counterfactual can be expressed as an I-proposition:

> $A \mathrel{\Diamond\!\!\rightarrow} B$ is true iff *some* of the closest A-worlds, A-worlds in the smallest A-permitting sphere, are B-worlds.

Only the quantifier changes in these formulations. (The alternatives merely relocate the existential claim.) Thus, a 'would' counterfactual should imply a 'might' counterfactual.

Obviously, the only way to satisfy both: (1) the Fregean desideratum that all 'would' counterpossibles be true, and (2) the Aristotelian desideratum that 'would' counterfactuals imply 'might' counterfactuals, is by: (3) some contrivance in which 'might' counterpossibles are also true. This is impossible in light of what might be called (4) the "Lewis desideratum" that 'would' and 'might' be interdefined as above.

6 Nonassertiveness The obvious solution is not, of course, the only possible response. The Aristotelian and Fregean desiderata can be reinterpreted, modified, or rejected outright. Indeed, these have all been done in other contexts. Of particular relevance to this discussion are contributions made by Strawson and Belnap.

In his *Introduction to Logical Theory* ([11], pp. 173ff), Strawson argues that the Aristotelian square of opposition might best be understood as applying only to "statements", not sentences, with the question of the truth or falsity of instances of the Aristotelian propositional forms arising only when the subject class is not empty. If this is applied to the A- and I-proposition formulations of the truth conditions for 'would' and 'might' counterfactuals, then the class of counterpossibles would count as outside the realm of true-or-false statements. That is, they would not be statements. Entertaining the impossible, or trying to, would constitute, in the Austinian jargon, a misfire in the act of assertion (or, more exactly, a "hitch" or "misexecution") ([1], p. 18). Counterpossibles would be neither-true-nor-false; they would simply be nonassertive. Insofar as they are not false, the Fregean desideratum might be considered satisfied; and insofar as that whenever the 'would' is true, the 'might' is also, the Aristotelian desideratum is also satisfied. Finally, if the negation of a sentence that does not assert is itself nonassertive, then the Lewis desideratum might also be satisfied.

On the negative side of this treatment, the nonassertiveness Strawson considers arises from a failure of presupposition. Accordingly, if understood as a third value for formulas, it would have to be nondesignated. But this violates the spirit of the Fregean desideratum and there certainly does not seem to be anything wrong with the following example:

> If Fermat's last theorem were false, a counterexample would have been produced by now.

(This subjunctive might not be a counterfactual in all contexts, but it certainly could be in, say, a discussion of the power of computers as tools in mathematical research. Remember that the actual truth-value of the antecedent is not relevant; the presupposed value is. Presumably, if the antecedent is believed false, it could be believed necessarily false.)

Furthermore, Strawson's suggestions are, in a sense, just that: suggestions from without, in this case, from the metalanguage. Full, formal development in the object language is lacking. But there is reason for this. The concern is with a logic of statements (propositions), not well-formed formulas. The language of that logic, however, has the expressive power to formulate meaningful expressions outside the purview of that logic. Consequently, distinguishing the assertive elements from the nonassertive ones is not seen as a purely formal task. That a sentence expresses a statement is not solely a matter of logical form. Thus, the traditional semantic job—supplying truth-conditions—arises only after the appropriate elements, the statements among the formulas, have been isolated. The import of all this is that a consistent reading of the *A*- and *I*-proposition formulations of the truth-conditions for counterfactuals along the lines drawn by Strawson would place counterpossibles beyond the range of such logical relations as contrariety, contradictoriness, and even implication. This would seem to violate the spirit of the Aristotelian desideratum.

Belnap is another who has considered the possibility of nonassertive (but not meaningless) sentences and their logical relations, but for wholly other reasons. In exploring the logic of conditional assertions ([2])—conditional sentences which fail to assert when the antecedent, or condition of assertion, is false—Belnap produces what he fairly describes as

> an analysis of restricted quantification which seems to do justice to both Aristotle and Frege-Peirce-Russell. ([2], p. 48)

The result is the basis for a formal system whose elements may be either assertive or not while still entering into the requisite logical relations. It should be noted here that the nonassertiveness of a failed conditional assertion differs from that arising from, say, a nondenoting singular term, which is Strawson's paradigm. While asserting "The present King of France is bald" might involve some pragmatic transgressions, saying "Provided there is a present King of France, he is bald" does not. In a conditional assertion, the presupposition is, as it were, recognized and given its due.

Conditional assertions can be outfitted with either a "two-tier" semantics, one separating the assertives from the rest and the second separating the truths from the falsehoods, as in [12] or [6], or else with a three-valued semantics, as in [4]. Either way, the relevance of conditional assertions for the problem of counterpossibles is that they can provide an established and elegant way of satisfying all three of the Aristotelian, the Fregean, and the Lewis desiderata.

*7 **Conditional assertions, quantification, and counterfactuals*** Perhaps the most elegant feature of an object language connective for conditional assertion is how it interacts with the quantifiers: this single connective can be used for *both* existential and universal generalizations with the delightful result that such generalizations are nonassertive when the subject classes are empty but otherwise admirably reconstitute the old Aristotelian square of opposition (see [2], pp. 66–69).

The Fregean desideratum is satisfied, as for Strawson, in that vacuous generalizations are nonfalse. However, as noted, this nonassertiveness, unlike Strawson's, would have to count as a designated value in a multivalued formalization of the logic of conditional assertions. As has been argued elsewhere (see [3], [4]), an appropriate formalization axiomatizes the never-falses, not the always-trues (but Dunn, in [6], shows that one successful axiomatization guarantees the other). The point of designating nonassertiveness is that nonassertions should be tolerated—which is exactly the attitude recommended by the shrug defense motivating true counterpossibles in the first place! And just as the shrug defense applies in both the 'would' and 'might' counterpossible cases, a conditional assertion connective makes both universal and existential generalizations with empty subject classes nonassertive.

Reading the A- and I-proposition forms of the truth conditions for 'would' and 'might' counterfactuals as generalized conditional assertions also satisfies the Aristotelian desideratum that A-propositions imply I-propositions and so that 'would' counterfactuals imply their 'might' counterparts. And Aristotle receives his due in the strongest possible way: when a connective for logical implication is added to the logic of conditional assertions, universal generalizations logically imply their existential counterparts. That is, in a formal system with both conditional assertion and, say, relevant implication, where "/" represents the object language connective for conditional assertion, so that $\forall x(Ax/Bx)$ and $\exists x(Ax/Bx)$ are the proposed translations of "All A's are B's" and "Some A's are B's", respectively, and "$\rightarrow$" represents the implication connective of the Relevance Logic $\boldsymbol{R}$, then

$$\forall x(Ax/Bx) \rightarrow \exists x(Ax/Bx)$$

is a "valid" (i.e., never-false) formula (see [5] for a multiconditional logic with both conditional assertion and relevant implication). Duly formalized, A-propositions do imply I-propositions.

Finally, the interdefinability of 'would' and 'might', for which Lewis is willing to sacrifice Aristotle, is also completely satisfied:

(1) $A \mathrel{\Box\!\!\rightarrow} B$ iff $\forall x$ (x is an A-world in the smallest A-permitting sphere/x is a B-world)

so

(2) $\sim(A \mathrel{\Box\!\!\rightarrow} \sim B)$ iff $\sim\forall x$ (x is an A-world in the smallest A-permitting sphere/x is a $\sim B$-world);

i.e.,

(3) $A \mathrel{\Diamond\!\!\rightarrow} B$ iff $\exists x$ (x is an A-world in the smallest A-permitting sphere/x is a B-world).

The only assumptions used in the move from (2) to (3) are: first, that $\sim\forall xF$ is equivalent to $\exists x \sim F$, second, that $\sim(A/B)$ is equivalent to $A/{\sim}B$; and third, that x is a B-world iff x is not a $\sim B$-world. The first is unassailable; the second is a property of conditional assertions; and the third, negation consistency and completeness for worlds, certainly would not upset Lewis (though some Relevantists might balk).

8 Further implementation Although the proposal to interpret the generalizations in the truth conditions for counterfactuals as quantified conditional assertions does indeed satisfy all of the desiderata set out, there are some potentially problematic features.

The proposal is for the truth-conditions, which is to say that it is on the metalinguistic level. There are those who insist on a bivalent or extensional metalanguage even when dealing with a multivalued or intensional semantic system. Whether stubbornness or squeamishness, this amounts to a theoretical hypocrisy: the logic of conditional assertions can certainly be explored as an interesting excursion into multiconditional logic, but it has always been, at heart, a proposal for understanding at least some ordinary conditionals (see [13]).

Perhaps a more problematic concern involves the pragmatic motivation and interpretation accorded the formal apparatus of conditional assertion logics. Is the acceptable nonassertiveness of a failed conditional assertion appropriate for counterfactuals with impossible or unentertainable antecedents? The formal machinery, to be sure, can be exploited regardless. Still, if an antecedent, A, is not even entertainable, then there are no possible worlds in which A is true, hence no A-permitting spheres at all. In particular, there will be no smallest A-permitting sphere. Thus, it can be argued, if the truth-conditions given above point to the conclusion that counterpossibles are nonassertive, it is to the pragmatically unacceptable, Strawsonian kind insofar as there is the nondenoting singular term, "the smallest A-permitting sphere".

There is some validity to this criticism, but there is a ready response: the definite description can be dispensed with in favor of indefinite descriptions and quantification. For example, $A \mathrel{\Box\!\!\rightarrow} B$ could be cashed out as follows:

> Some A-world is in an $A \supset B$-sphere; i.e., a sphere all of whose worlds are $A \supset B$-worlds.

Formally (treating possible worlds as sets of sentences so that $A \in w$ iff A is true at w),

$$(\exists w)(A \in w/(\exists S)(w \in S/(\forall x)(x \in S/A \supset B \in x)))$$

this takes advantage of the concentricity, the "nesting" of spheres in the Lewis system. If any sphere with an A-world has only $A \supset B$-worlds, then the smallest sphere with an A-world does too. The 'might' counterfactual then becomes:

$$(\forall w)(A \in w/(\forall S)(w \in S/(\exists x)(x \in S/A \ \& \ B \in x)))$$

i.e., every A-world is in A & B-permitting spheres only.

They also preserve the interdefinability of 'would' and 'might'. On the other hand, these formulations do not obviously satisfy the Aristotelian desideratum that 'would' counterfactuals imply their 'might' counterparts. (That principle was so-called because of the Aristotelian thesis that A-propositions imply I-propositions. These reformulations don't fit into those patterns, so the name is less fitting. It is retained anyway.) In fact, now it is the weaker 'might' whose truth-conditions start with the universal quantifier, while the truth-conditions for the 'would' counterfactual begin with the existential. But, in spite of all appearances, 'might' counterfactuals are still logical consequences of 'would' counterfactuals under these interpretations—given but one nonlogical assumption, viz., that for any two spheres, S_1 and S_2, either $S_1 \leq S_2$ or $S_2 \leq S_1$. That is, what is needed to ensure the validity of the inference is precisely Lewis's requirement that spheres be nested, i.e., that there be some basis for judgments of comparative similarity of worlds. This is just what the spheres provided and just what was used in eliminating the definite description, "the smallest A-permitting sphere". (The actual derivation is left to the reader.)

9 An open question What the logic of conditional assertion supplies is the philosophical motivation, as well as the purely formal machinery, for an interpretation of the generalizations with which both "Aristotelians" and "Fregeans" can be comfortable, and which, when applied to the systems-of-spheres semantics offered by Lewis for counterfactuals, provides satisfactory solutions to the problems of counterpossibles.

There is, to be sure, another avenue worthy of exploration in connection with counterpossibles, viz., the addition of impossible worlds. Adding worlds for every possible—and impossible—antecedent would have the salutary effect of eliminating any problems peculiar to counterpossibles (as well as eliminating the entire category of vacuous truths). Further, since many of the semantics offered for implicational logics include impossible "worlds", if there is to be a single system within which such formulas as $(A \rightarrow B) \rightarrow (A \mathbin{\Box\!\!\rightarrow} B)$ are to be evaluated, they might be added anyway.

This is problematic, however, for more than just metaphysical reasons. Are the "possible worlds" of a semantics for counterfactuals (i.e., counterfactual situations) analogous to the "possible worlds" of a semantics for logical implication (i.e., informational set-ups or theories)? How might the (logical) accessibility and the (counterfactual) similarity relations interact? This is left as an open question in multiconditional logic.

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