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# The Distribution of Terms: A Defense of the Traditional Doctrine

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In traditional logic the validity of immediate inferences and of categorical syllogisms is determined by applying various rules, one of which is the familiar rule of distribution ([5], p. 180; cf. [1], p. 69):

(RD) No term may be distributed in the conclusion which was not distributed in one of the premises.

However, the very idea of distribution has come in for considerable criticism of late. John Neville Keynes (1887) explained it in a section headed "The Distribution of Terms in a Proposition" in the following way:

> A term is said to be distributed when reference is made to *all* the individuals denoted by it; it is said to be undistributed when they are only referred to partially, i.e., information is given with regard to a portion of the class denoted by the term, but we are left in ignorance with regard to the remainder of the class. ([5], p. 68)

This doctrine has been criticized in several ways. Geach [2] and Barker [1] have attacked Keynes's formulation directly, while Katz and Martinich [4] propose to replace Keynes's criterion with another, quite different, criterion. It is the purpose of this note to defend Keynes against the criticisms of Geach and Barker, and to show that the criterion of Katz and Martinich presupposes Keynes's.

The objection that Geach raises is that there are two semantical relations that a term has according to the statement of the rule of distribution, namely, that of denoting and that of *referring*. Thus, 'man' always denotes each and every man, but refers to some men only in one context (when it is not distributed) and in another refers to all men (when it is distributed). The problem is the semantical relation of *referring*, and how it is to be distinguished from denoting. "The whole doctrine hinges on this distinction, but neither Keynes's nor any later exposition tells us what this distinction is" ([2], p. 6). Since the notion of "referring" remains unexplained, the doctrine of distribution is worthless, and

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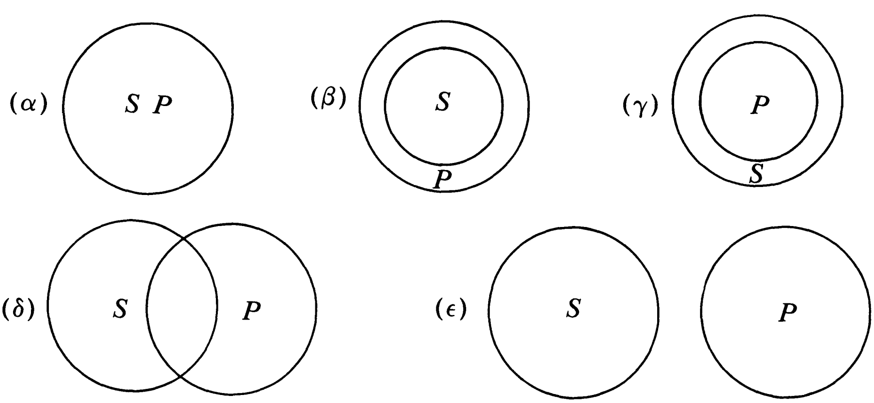
the rule for evaluating inferences that is based on it, that a term distributed in the conclusion must be distributed in the premises, is worthless.

Now, it is true that Keynes does not explicitly explain what he has in mind. Yet it is not so unclear as Geach suggests. Indeed, it is sufficiently clear for one to recognize that Keynes does not think of reference as a relation between a term and objects in the world. To be sure, denotation is a semantical relation in which terms stand to objects; but reference is not.

Keynes tells us: "a term is said to be distributed when reference is made to all the individuals denoted by it". This *may* be read in such a way that it is the term that has reference, but it *need not* be so read. Keynes continues, and explains that when a term is undistributed "*information* is given with regard to a portion of the class denoted by the term". It is, however, not a term that gives information—it merely denotes—*it is the proposition in which it occurs that gives information.* Thus, when Keynes tells us that "reference is made to all the individuals" it is the proposition that makes reference, rather than the term. That is, in Keynes' statement 'reference' is to be ascribed not to the word 'term' that comes earlier in the statement but to the word 'proposition' that occurs in the immediately preceding section title, "The Distribution of Terms in a Proposition". Thus, *Keynes's notion of "reference" has to do with the truth conditions of propositions rather than the semantical relations of a term to its denotata.*

Keynes makes this clear when he explains the truth-conditions of categorical propositions, and the distribution of terms, by means of the Euler diagrams. He first lays out the standard five relations of inclusion and exclusion between two classes *S* and *P* that are a priori possible; these five possibilities are mutually exclusive and jointly exhaustive. Keynes then states the truth conditions of categorical propositions by listing for each form those among the five possibilities under which the form is true: "the force of the different propositional forms is to exclude one or more of (the five) possibilities" ([5], p. 117).

> Any information given with respect to two classes limits the possible relations between them to one or more of the following:
>
> 
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>
> Such information may in all cases be expressed by means of the propositional forms *A*, *E*, *I*, *O*.[1]

Note how Keynes here speaks of "information" being conveyed by the proposition. This confirms our reading of his statement on distribution.

The four categorical forms have the following truth-conditions:

*A*: All *S* are *P*: true on either ($\alpha$) or ($\beta$)
*E*: No *S* are *P*: true on ($\epsilon$)
*I*: Some *S* are *P*: true on ($\alpha$), ($\beta$), ($\gamma$) or ($\delta$)
*O*: Some *S* are not *P*: true on ($\gamma$), ($\delta$) or ($\epsilon$).

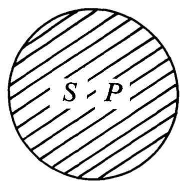
Thus, the semantics of the quantifier 'all' and the 'is' of class inclusion are such that any proposition insofar as it exemplifies the syntactical form 'All *S* are *P*' of *A* propositions is true if and only if the class denoted by the term substituted for '*S*' stands in either the relation ($\alpha$) or the relation ($\beta$) to the class denoted by the term substituted for '*P*'. Again, the semantics of 'some', 'not', and 'is' are such that any proposition insofar as it exemplifies the syntactical form 'Some *S* are not *P*' of *O* propositions is true if and only if the *S* and *P* classes stand either in the relation ($\gamma$) or in the relation ($\delta$) or in the relation ($\epsilon$). Similarly for the other two categorical forms.

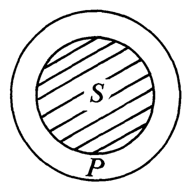
Keynes uses the Euler diagrams "to illustrate the distribution of the predicate in a proposition".

> In the case of each of the four fundamental propositions we may shade the part of the predicate concerning which knowledge is given us. ([5], p. 119)

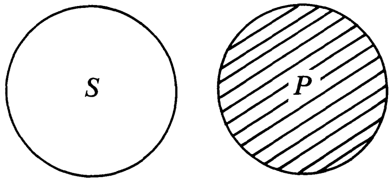
Note the word 'knowledge', a clear synonym for 'information'. This again confirms our reading of the passage on distribution.

The *A* proposition is an inclusion: the whole of *S* is included in *P* as the whole of *P* or as part of *P*. In the former case we have information about all of *P*, as in ($\alpha$), and in the latter case we have information about only part of *P*, as in ($\beta$):

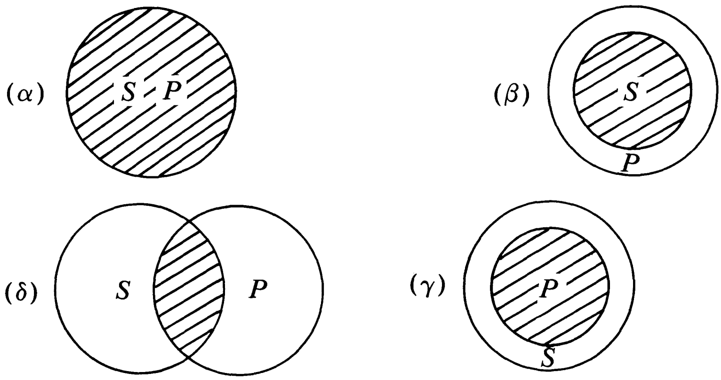
($\alpha$)



($\beta$)


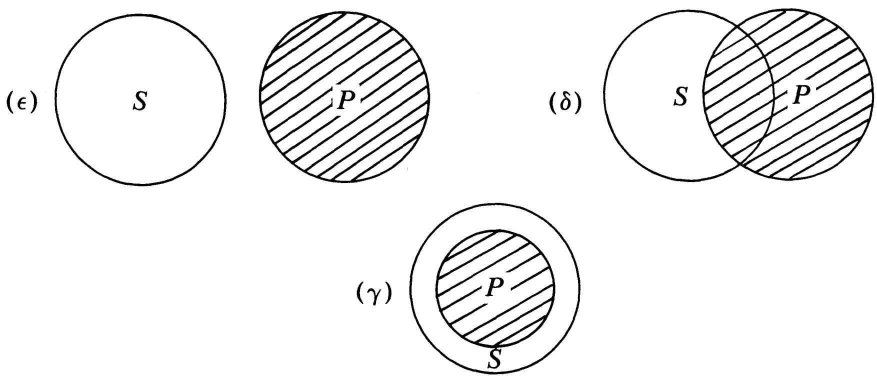

The *E* proposition is an exclusion: the whole of *S* is excluded from the whole of *P*. We thus have information about the whole of *P*, as in ($\epsilon$):

($\epsilon$)


The *I* proposition is an inclusion: the whole of *S* is included in *P* as the whole of *P* (case ($\alpha$)), or as part of *P* (case ($\beta$)), or part of *S* is included in *P* as part of *P* (case ($\delta$)), or as the whole of *P* (case ($\gamma$)):




The *O* proposition is an exclusion: the whole of *S* is excluded from the whole of *P* (as in ($\epsilon$)), or part of *S* is excluded from the whole of *P* (as in ($\gamma$) and ($\delta$)):




As Keynes now says:

> The result is that with *A* and *I* there are cases in which only part of *P* is shaded; whereas with *E* and *O*, the whole of *P* is in every case shaded; and it is made clear that negative propositions distribute, while affirmative propositions do not distribute, their predicates. ([5], p. 130)

A similar exercise with subjects can establish that universal propositions do, while particular propositions do not, distribute their subjects.

The notions of 'inclusion' and 'exclusion' among classes are perfectly clear, as are the notions that the 'whole' or a 'part' of a class are included or not included in another class; the Euler diagrams provide a model that easily succeeds in making these notions clear. It follows that there is nothing problematic about Keynes's explanation of the doctrine of distribution. Geach's complaints are without substance, and stem from his failure to read Keynes with sufficient care to recognize that, when "reference" is made in the case of a distributed term to all the individuals denoted by it, the reference is made by the proposition, not the term.

But, *if* the notions of 'whole', 'part', 'inclusion', and 'exclusion' are not sufficiently clear, they can in fact be clarified in two distinct, but equivalent, ways.

Before turning to these, however, we should look at two criticisms that Barker has made of Keynes's criterion of distribution. He objects to the claim that "a term in a categorical sentence is distributed if and only if the sentence 'refers to' all members of the class of things to which the term applies".

> . . . this explanation is obscure and misleading. The sentence "All equilateral triangles are equiangular triangles" 'refers to' all equilateral triangles, and since necessarily these and only these are equiangular triangles the sentence would appear to 'refer to' all equiangular triangles also. Thus according to the old-fashioned account, it would seem that the predicate ought to count as distributed in this *A* sentence. The predicate is not considered to be distributed, however, and this illustrates one unsatisfactory aspect of the old-fashioned explanation of distribution. ([1], p. 44)

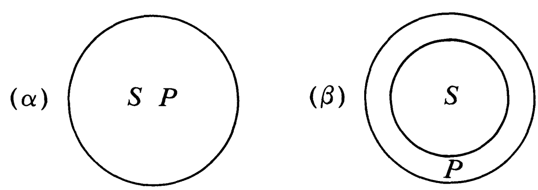
Barker's argument is this. Suppose both

(1) All equilateral triangles are equiangular triangles

and

(2) All and only equilateral triangles are equiangular triangles

are true: then, since all of the subject is "referred to" by (1), so must all the predicate also be referred to by (1); but in that case the predicate is distributed in (1), contrary to the traditional doctrine. However, as we saw, "refer to" is to be understood in terms of truth-conditions. Now, the truth-possibilities under which (1) is true are




while (2) is true under ($\alpha$) alone. It is certainly true that if (2) is true then *P* is distributed—after all, (2) entails

(3) All equiangular triangles are equilateral triangles

and universals distribute their subject terms. But from the fact that (2) is true, it does not follow that ($\beta$) is *not* among the truth-possibilities for (1). The traditional doctrine counts *P* as undistributed in (1) because ($\beta$) is among its truth-possibilities. Barker's conclusion follows only if one thinks the truth of (2) entails that ($\beta$) is not one of truth-*possibilities* for (1). This Barker attempts to secure by using an example (2) which is "necessarily true". Since (2) entails (1), the latter, too, will be necessarily true. If we let '$\nabla$' represent the exclusive 'or', then we have

(1) = $(\alpha \nabla \beta)$
(2) = $(\alpha)$.

With the modalities in place, we have

(4) $\Box(\alpha \nabla \beta)$
(5) $\Box(\alpha)$

as Barker's premises. From these it of course follows that

(6) $\Box \sim (\beta)$

or, equivalently,

(7) $\sim \Diamond (\beta)$.

Thus, given that (1) and (2) are necessarily true, it is not possible that the situation represented by $(\beta)$ obtain. From this Barker concludes that we have in (1) a proposition of the *A* form in which the predicate is distributed contrary to the traditional doctrine in which propositions of affirmative form do not distribute their predicates.

However, Barker's conclusion that the predicate is distributed in (1) under the condition that (2) is necessarily true follows only if one can infer from the fact that $(\beta)$ is not possible that $(\beta)$ is not one of the *truth-possibilities* of (1). But this does not follow. (1) exemplifies the *A* form

All *S* are *P*

The semantics of this form are such that $(\beta)$ *is* among the truth-possibilities of (1). It may be that one of the truth-possibilities that (1) has by virtue of exemplifing the *A* form may for some other reason be impossible of fulfillment; but it does not follow that it is not among truth-possibilities of (1) insofar as the latter are determined by the semantics of the *A* form that (1) exemplifies. The semantics of the *A* form makes $(\beta)$ one of the truth-possibilities of (1), and Barker's argument does not change that semantics. Indeed, if we are even to state that argument, we must reckon $(\beta)$ among the truth-possibilities of (1). That, after all, is how one obtains the premise (4); if $(\beta)$ is *not* among the truth-possibilities of (1), then (1) = (2) and Barker's argument is

$(4^1)$ $\Box(\alpha)$
(5) $\Box(\alpha)$

from which the desired conclusion about $(\beta)$ does not follow. Hence, Barker's argument gets started only if it presupposes what it aims to refute.

Perhaps Barker's error lies in supposing that, by virtue of the necessity of (2), (1) exemplifies the propositional form

$(AA_1)$ All *S* are *S*.[2]

The semantics of the quantifier 'all' and the 'is' of the class inclusion are such that any proposition insofar as it exemplifies the $AA_1$ form is true if and only if the relation $(\alpha)$ obtains; and it is evident that the $AA_1$ form guarantees that any proposition insofar as it exemplifies it will distribute both its subject and predicate terms. But of course, even though (2) is necessary, it does *not* follow that (1) exemplifies the $AA_1$ form *rather than* the $A$ form. To suppose otherwise is akin to supposing that since

$$(p \vee q) \equiv [\sim(\sim p \cdot \sim q)]$$

is a necessary truth, any proposition exemplifying the logical form

$$p \cdot (p \vee q)$$

somehow does *not* exemplify that form but rather the form

$$p \cdot [\sim(\sim p \cdot \sim q)].$$

However, if Barker's example (1) will not secure his point, perhaps another example will, one in which it is explicit in the form of the proposition itself that subject and predicate are coextensive; e.g.,

(8) All men are men.

This is of the $A$ form, but distributes its predicate; so we have an $A$ proposition that distributes its predicate, contrary to the traditional doctrine. But, while (8) exemplifies the $A$ form, and also distributes its predicate, it does not do the latter by virtue of the former. Rather, it distributes its predicate by virtue of being of the form $AA_1$; insofar as (8) exemplifies the $A$ form, $(\beta)$ *is* among its truth-possibilities.

The traditional doctrine of distribution as stated by Keynes holds that a proposition *insofar as it exemplifies the A form* does not distribute its predicate or, what is the same, does not exclude $(\beta)$ from among its truth-possibilities. One can't use examples like (1) or (8) to show that this is wrong, any more than one can show that it is wrong to hold that a disjunction is true if and only if one or the other or both disjuncts is true because one can find disjunctions like

$$p \vee (q \cdot \sim q)$$

that are true if and only if the first disjunct is true.

It thus seems that Barker's first objection to the traditional doctrine of distribution is unsound.

Barker's second objection to Keynes's account of distribution

> . . . is that it is unclear in its treatment of the predicate in the $O$ form. To claim that the sentence "some seamen are not prohibitionists" 'refers to' all prohibitionists is to make an obscure and unsatisfactory claim. ([1], p. 43)

But this is simply to say that it is unclear what is meant when one holds that a proposition of the $O$ form

Some $S$ are not $P$

asserts that *either the (nonempty) whole or a (nonempty) part of S is excluded from the whole of P.* I myself do not find this either obscure or unclear, especially in light of the explanation in terms of the Euler diagrams. If, however, it is obscure or unclear, then one can do no less than attempt to clarify such crucial notions as 'whole' 'part', 'inclusion', and 'exclusion'. I have already suggested that there are two, equivalent, ways of doing this. Let us turn to the first. As it turns out, it is precisely this that Barker proposes as a "replacement" for Keynes' doctrine of distribution.

The *first* way to clarify the crucial notions consists in rephrasing each categorical proposition in an equivalent form which uses the notions of 'class' and 'subclass' (or "portion of a class") and then, using these notions and the relations of identity and nonidentity, unpacks the notions of 'whole', 'part', 'inclusion', and 'exclusion':

$A$: All $S$ are $P$
$A^1$: For every subclass $f$ of $S$ there is a subclass $g$ of $P$ such that $f = g$
$E$: No $S$ are $P$
$E^1$: For every subclass $f$ of $S$ and every subclass $g$ of $P$, $f \neq g$
$I$: Some $S$ are $P$
$I^1$: There is a subclass $f$ of $S$ and there is a subclass $g$ of $P$ such that $f = g$
$O$: Some $S$ are not $P$
$O^1$: There is a subclass $f$ of $S$ such that for every subclass $g$ of $P$, $f \neq g$.

If $T$ is a term in one of the categorical propositions, then reference is made to the whole of the class $T$ just in case $T$ is attached to a universal quantifier, while reference is made to a portion only of $T$, i.e., a proper subclass, just in case $T$ is attached to a particular quantifier. 'Inclusion' and 'exclusion' are explicated in terms of the relations of identity and nonidentity among the classes and subclasses denoted by $T$ and the other term $T'$ of the proposition.

Distribution now clearly falls out quite nicely: a term $T$ is distributed just in case that $T$ is attached to a universal quantifier in the primed forms.

The unprimed and primed forms are logically equivalent. In order to see this clearly one must have recourse to the symbolic techniques of modern logic. Consider the $A$ form. This transcribes as

$$(x)(Sx \supset Px)$$

and $A^1$ transcribes as

$$(f)[(f \subset S) \supset (\exists g)[(g \subset P) \cdot (f = g)]].$$

The other primed statements have similar translations. It is easy to prove that the members of each pair entail each other. The unprimed and the primed forms of the categorical propositions are thus logically equivalent. Thus, what the primed formulas do is *provide an explicit syntactical criterion for which terms are distributed.* But, since the primed formulas are logically equivalent to the corresponding unprimed formulas, and *cannot differ from the latter in their truth-possibilities, this syntactical criterion adds nothing that is not already implicit in the truth-possibilities of the categorical propositions in standard form.* In short, the reformulation achieved by the primed formulas may add clarity, but, *far from being shown Keynes's doctrine of distribution is inadequate, what we see is why that doctrine is correct.*

Barker proposes to redefine the notion of distribution ([1], p. 43):

> A term $S$ occurring as the subject of a categorical sentence is said to be distributed in that sentence if and only if the sentence, in virtue of its form, says something about *every kind of S*. Similarly, a term $P$ occurring as the predicate is said to be distributed in that sentence if and only if the sentence, in virtue of its form, says something about *every kind of P*.

He offers a "more rigorous" formulation of the definition as follows:

> Suppose that $T$ is a term which occurs as a subject or predicate in a categorical sentence $s$. Where $T^1$ is any other term, let $s^1$ be the sentence that is exactly like $s$ except for containing the compound term $T^1$ and $T$, where $s$ contains $T$. Now, $T$ is said to be distributed in $s$ if and only if, for every term $T$, $s$ logically implies $s^1$.

Katz and Martinich have objected to this criterion that, while one can indeed infer

(8a) Some seamen are not rich prohibitionists

from

(8) Some seamen are not prohibitionists

one cannot infer

(8b) Some seamen are not sham prohibitionists

from (8), and so the predicate of $O$ propositions cannot be distributed according to Barker's criterion but contrary to the traditional doctrine ([4], p. 117). But this is surely unjust. Barker restricts his "$T$" and "$T^1$" to *terms*, where terms "apply to . . . individual things" ([1], p. 37), many in the case of general terms and one only in the case of singular terms—a fairly traditional procedure, followed, for example, by Keynes ([5], pp. 10, 53). The role of 'sham' is to negate; a sham prohibitionist is a *non*prohibitionist, *not* a prohibitionist. Thus, the use of 'sham' is akin to that of 'non-' in "non-$P$". It is, in short, *syncategorematic*, and, as Keynes has said, "using the word term in the sense in which it was defined . . . it is clear that we ought not to speak of syncategorematic terms" ([5], p. 10).

The point to be noticed is that Barker's conjunctive terms "$T^1$ & $T$" denote *subclasses of T*. Thus, upon Barker's criterion, a term $T$ is distributed in $s$ just in case that what $s$ says about $T$ it also says about *every subclass* of $T$, or as Barker puts it in his "less rigorous" formulation, *every kind* of $T$. That is, $T$ is distributed in $s$ just in case that $s$ can be reformulated as a statement that makes an assertion about every subclass of $T$. It is evident that the relevant reformulations are the primed formulas above. Indeed, the latter show why, precisely, it is that Barker's criterion works: it works because the formulas which make clear the terms to which it applies are logically equivalent to the standard categorical forms. But if we see why Barker's criterion works, we also see that it is unnecessary: it invokes nothing that is not already implicit in the truth-possibilities of the categorical proposition in standard form. Far from replacing Keynes's criterion, Barker's is merely a reformulation of it.

Katz and Martinich propose a criterion apparently rather different from Barker's. In the end, however, as we shall see, it is, like Barker's, essentially

Keynes's criterion, and therefore, despite appearances, is not after all radically different from Barker's.

The criterion is this ([4], p. 281): Let $C(T_1, T_2)$ be a categorical proposition. Then $T_1$ is distributed in $C(T_1, T_2)$ if and only if either

$$\begin{array}{l} C(T_1, T_2) \\ \underline{T_1 a} \\ \therefore (\exists x)(T_2 x \cdot x = a) \end{array}$$

or

$$\begin{array}{l} C(T_1, T_2) \\ \underline{T_1 a} \\ \therefore (\exists x)(T_2 x \cdot x \neq a) \end{array}$$

is valid. The general idea behind this criterion goes back well into medieval logic. Peter of Spain[3] described a term $T$ in a proposition $s$ as distributed if we can validly infer $s^1$ from $s$, where $s^1$ is obtained from $s$ as replacing '$T$', or '$T$' with its quantifier, by 'This $T$'. For example, from

Every man is an animal

we may deduce

This man is an animal

but not

Every man is this animal.

Hence, the subject is, but the predicate is not, distributed in an $A$ proposition. Similarly, we may deduce from

No man is a horse

both

This man is not a horse

and

No man is this horse

and conclude that both subject and predicate are distributed in an $E$ proposition. Peter of Spain did not use the doctrine of distribution to test syllogisms for validity. That idea came only later, perhaps with Pseudo-Scot ([7], pp. 272–273). Be that as it may, the point is that the medievals recognized that when a term $T$ is distributed, one can deduce a conclusion about an *arbitrary member* of $T$, e.g., '*this* man' in the case of 'Every man is an animal'. It is this idea about drawing a conclusion about an arbitrary member that the criterion of Katz and Martinich is designed to capture.

The rule (RD) prohibits inferring a conclusion where $T$ is distributed from premises where it is not. It is easy enough to see why we must accept this rule if we accept the Katz and Martinich criterion. Suppose that $T_1$ is not distributed in

(i) $C_1(T_1, T_2)$.

Then, from (i) and

(ii) $T_1 a$,

the conclusion

(iii) $(\exists x)(T_1 x \cdot y = a)$

does not follow. Suppose, however, that $T_1$ *is* distributed in

(iv) $C_2(T_1, T_2)$

because (iii) follows from (iv) and (ii). If (iv) *were* to follow from (i) then we could deduce (iii) from (iv) and (ii) and therefore from (i) and (ii). But in that case, $T_1$ would, contrary to our original supposition, be distributed in (i). A violation of (RD) must therefore be invalid.

Nonetheless, to leave it at this, as Katz and Martinich do, is hardly satisfactory. We are told ([4], p. 281) that

> 'All men are mortals and Socrates is a man' logically implies 'For some mortal, Socrates is the same as that mortal'. Hence the subject term is distributed.

But we are not told *why* the relation of logical implication holds as it is asserted to hold. We are told nothing about the logical form that determines the truth-possibilities of categorical propositions so that we could see that the conclusion is indeed, as it is asserted to be, contained in the premises; that is, that every truth-possibility for the premises is also a truth-possibility for the conclusion. Nor are we told anything about the logical form that would enable us to see why the Katz–Martinich conclusions do *not* follow in the case of undistributed terms. Taking entailment to be an undefined term, as Katz and Martinich apparently do, is simply not helpful: one wants to know the logical feature that distributed terms have in the logical forms of categorical propositions that validates the Katz–Martinich inferences.

Katz and Martinich ([4], p. 281) hint at what this form may be.

> Whether the statement does say or imply something about each particular depends on whether the term is distributed in the statement; if the term is distributed, it does, and if the term is undistributed, it does not.

If $T$ occurs in $s$, then one can draw an inference from $s$ about an *arbitrary* member $a$ of $T$ only if $s$ makes an assertion about *all* of $T$. The hook-up with Barker's criterion is evident enough: one can draw an inference about an arbitrary member of $T$ only if, as Barker says, the proposition $s$ makes the same claim about every subclass of $T$ that it makes of $T$. But the hook-up with Keynes's criterion is also evident enough: one can draw an inference about an arbitrary member of $T$ only if $s$ makes reference to *all* the individuals denoted by $T$; and one cannot draw an inference about an arbitrary member of $T$ if $s$ refers to them *only partially*. It thus appears that Katz and Martinich are not so far from either Keynes or Barker as appearances might suggest.

What we must do, of course, is attempt to make explicit those features of logical form that Katz and Martinich only hint at. This brings us to the *second* way to clarify the notions of 'whole', 'part', 'inclusion', and 'exclusion' that we, following Keynes, used to state the doctrine of distribution. This way consists in rephrasing each categorical proposition in an equivalent form which explicitly uses the relation of denotation of a term, and which unpacks 'whole', 'part', 'inclusion', and 'exclusion' in terms of that relation and the notions of identity and nonidentity among individuals. The equivalent forms are these:

$A$: All $S$ are $P$
$A^*$: For every $x$ such that $S$ denotes $x$ there is a $y$ such that $P$ denotes $y$ and $x = y$
$E$: No $S$ are $P$
$E^*$: For every $x$ such that $S$ denotes $x$ and for every $y$ such that $P$ denotes $y$ and $x \neq y$.
$I$: Some $S$ are $P$
$I^*$: There is an $x$ such that $S$ denotes $x$ and there is a $y$ such that $P$ denotes $y$ and $x = y$
$O$: Some $S$ are not $P$
$O^*$: There is an $x$ such that $S$ denotes $x$ and for every $y$ such that $P$ denotes $y$, $x \neq y$.

If $T$ is a term in one of the categorical propositions, then "reference is made to all the individuals denoted by it" just in case in the equivalent starred form it is attached to a universal quantifier; "individuals denoted by it are only referred to partially" just in case it is attached to a particular quantifier. These quantifiers thus make explicit what it is for a proposition to refer to a whole or part of (the denotation of) $T$. As for the notions of 'inclusion' and 'exclusion', these are explicated in terms of the relations of identity and nonidentity among the individuals that are the denotata of $T$ and the other term $T'$ of the proposition.

In particular, a term $T$ is distributed just in case it is attached to a universal quantifier in the starred forms.

The starred and unstarred forms are logically equivalent. In order to see this clearly, one must, as before, have recourse to the symbolic techniques of modern logic. Consider again the $A$ form. This transcribes as:

$$(x)(Sx \supset Px)$$

while $A^*$ transcribes as

$$(x)[Sx \supset (\exists y)(Py \cdot x = y)].$$ [4]

The other starred statements have similar translations:

$E^*$: $(x)[Sx \supset (y)(Px \supset x \neq y)]$
$I^*$: $(\exists x)[Sx \cdot (\exists y)(Py \cdot x = y)]$
$O^*$: $(\exists x)[Sx \cdot (y)(Py \supset x \neq y)]$.

It is easy enough to construct formal proofs that the members of each (starred/unstarred) pair entail each other; but it is perhaps more revealing to think in terms of quantifier-free expansions, for when a transformation is made to the latter it becomes perfectly clear how the extra clauses in the starred formulas add nothing, that is, nothing that is nontautological, to the unstarred forms.

What the starred forms succeed in doing is providing an explicit syntactical criterion for which terms are distributed. But again, since the starred formulas do not differ from the unstarred formulas in their truth-possibilities, this syntactical criterion adds nothing that is not already implicit in the truth-possibilities of categorical propositions in standard from. Thus, far from showing Keynes's doctrine is wrong, we see that the proposed alternative of Katz and Martinich, like that of Barker—to which it turns out to be logically equivalent—is in fact logically equivalent to Keynes's. Once again we have an "alternative" to Keynes that in fact turns out to demonstrate that Keynes's doctrine is correct.

One final point. As is well-known, a number of traditional inferences are not *formally valid* unless they are treated enthymematically with additional premises to be included where needed. These include the traditional subalternation:

All *S* are *P*
∴Some *S* are *P*

which is not formally valid unless one adds the premise

There are *S*.

Another is conversion by limitation of *E*:

No *S* are *P*
Some *P* are not *S*

which is not formally valid unless one adds the premise

There are *P*.

Yet another is inversion of *A*:

All *S* are *P*
Some non-*S* are not *P*

which is not formally valid unless one adds the premise

There are non-*P*.

It is inversion of *A* which is of interest here. For, the predicate '*P*' is *not* distributed in the premise "All *S* are *P*", but *is* distributed in the conclusion "Some non-*S* are not *P*". Geach (in [2]) argues that here we have an exception to the rule (RD), a valid argument in which a term distributed in the conclusion is not distributed in the premise; and concludes the doctrine of distribution is worthless.

Keynes ([6], pp. 139ff) had already considered this objection and replied that inversion of *A* is formally valid only if "there are non-*P*" is added as a premise, that *P* is distributed in this added premise, and that we therefore do *not* have a case of a valid argument in which a term is distributed in the conclusion but not in the premises.

If we use the rules, that we used above to obtain the starred versions of the categorical propositions, to translate the premises needed to make the traditional inferences valid, then the statement that "There are $S$", that $S$ is nonempty, that is, the statement

$$(\exists x) Sx,$$

becomes

$$(\exists x)(\exists y)(Sy \cdot y = x)$$

and the statement "There are non-$P$", that is, the statement

$$(\exists x) \sim Px,$$

becomes

$$(\exists x)(y)(Py \supset x \neq y).$$

The criterion of distribution is that a term $T$ is distributed just in case it is attached to universal quantifier. Thus, in a statement that a term is nonuniversal, the term is distributed, while in a statement that a term is nonempty, the term is undistributed. Thus, Keynes's claim is clearly correct that the rule (RD) of distribution is not violated by inversion of $A$, once one takes into account the added premise that is required for validity.

Geach goes on ([2], p. 64) to suggest the following interpretation of the categorical propositions:

"All $S$ are $P$" is to be read as
$$(S = \Lambda \cdot P = \Lambda) \vee (S = V \cdot P = V) \vee (S \neq \Lambda \cdot P \neq V \cdot S \subset P)$$
"No $S$ are $P$" is to be read as
$$(S = \Lambda \cdot P = V) \vee (S = V \cdot P = \Lambda) \vee (S \neq \Lambda \cdot P \neq \Lambda \cdot S \cap P = \Lambda)$$
"Some $S$ are $P$" is to be read as
$$(S = \Lambda \cdot P \neq V) \vee (S \neq V \cdot P = \Lambda) \vee (S \cap P \neq \Lambda)$$
"Some $S$ are not $P$" is to be read as
$$(S = \Lambda \cdot P \neq \Lambda) \vee (S \neq V \cdot P = V) \vee \sim(S \subset P).$$

Upon this interpretation, all of the traditional square of opposition, including subalternation, is valid; so are all the traditional syllogisms, including those with weakened conclusions. All conversions and obversions permitted by the distribution rule (RD) are valid on this interpretation. The two basic forms are "All $S$ are $S$" and "Some $S$ are $S$" are tautological on this interpretation. It is assumed, as usual, that the universe is nonempty; but the terms $S$ and $P$ need not be nonempty and need not be nonuniversal.

Now, on this interpretation, the inversion of $A$ is valid. Geach now argues ([2], p. 64) this way.

> . . . To exclude Keynes's defence, no added premiss is needed, and the great unnaturalness of the interpretation is irrelevant. For the question is whether the doctrine of distribution affords a formal test of validity. By constructing a system in which the usual relations of categoricals are maintained, but nevertheless an inference condemned by the doctrine of distribution is valid, I have shown that the doctrine of distribution is useless even as a mechanical test of validity.

But let us see. A term $T$ will be distributed in a disjunctive proposition just in case it is distributed in each disjunct; otherwise there will be a truth-possibility in which we will have information about only part of $T$ and it will be distributed in a conjunction just in case it is distributed in at least one conjunct. Consider the $A$ proposition. In the first disjunct we have

$$P = \Lambda \equiv (x) \sim Px \equiv (x)(y)(Py \supset x \neq y),$$

so $P$ is distributed in that disjunct. In the second disjunct we have

$$P = V \equiv (x)Px \equiv (x)(y)(Py \supset x = y),$$

so $P$ is distributed in that disjunct. In the third disjunct we have as a conjunct

$$P \neq V \equiv (\exists x) \sim Px \equiv (\exists x)(y)(Py \supset x \neq y),$$

so $P$ is distributed in that disjunct also. Thus, in the $A$ proposition on this "unnatural" interpretation the predicate $P$ *is* distributed. As for the conclusion "Some non-$S$ are not $P$" of the inversion, on this "unnatural" interpretation we have in the first disjunct

$$P \neq \Lambda \equiv (\exists x)Px \equiv (\exists x)(\exists y)(Py \cdot x = y),$$

so $P$ is undistributed in that disjunct. Hence it is undistributed in the proposition. Thus, the inversion of $A$, on this "unnatural" interpretation, yields a conclusion in which the predicate $P$ is undistributed. Thus, Geach notwithstanding, even on this "unnatural" interpretation of the categorical forms, the inversion of $A$ is valid but does not constitute a violation of the distribution rule (RD). In short, Geach has still not found a counterexample on which the doctrine of distribution fails.

I conclude that—properly stated—the traditional doctrine of distribution is correct, that Keynes properly stated that doctrine, and that recent alternative criteria for the distribution of terms not only do not improve upon, but are logically equivalent to, Keynes's criterion.

## NOTES

1. [5], pp. 126–127. It is important to remember that each area in the Euler diagrams is assumed to have members (including the area outside the circle constituting the remainder of the universe of discourse).
2. This terminology ($AA_1$) is from [5], p. 127.
3. [9], Sec. 6.11, Sec. 12–26. Compare [10], p. 110.
4. This idea is worked out in detail, but from a slightly different perspective, in [8], Sec. 2. The point is also made briefly in [11], where it is attributed to Peirce, *Collected Papers*, 2.458.

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