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Remarks on Frege's Conception of Inference

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1 Whatever the ultimate fate of Frege's logicist thesis, his contribution to the theory of logical inference is indisputably great. He showed how we may represent sentences of arbitrary relational and quantificational complexity in a system where rules of inference are treated as instructions for the manipulation of formulas. In his *Begriffsschrift* he showed how the rules enable us to establish results that might otherwise be thought to depend upon an intuition of serial order.[1] Trouble set in later, with the introduction of an axiom governing the transition from a function to its course of values. But this indicates no deficiency of inference; impeccably correct inferences may proceed from false premises. Validity is one thing, truth another. Deductive inference is truth preserving, and must be so. But in another sense inference has nothing especially to do with truth. We can infer from premises independently of their truth values; a proposition does not have to be true in order for us to be able to derive its consequences. The point about the truth preservingness of validity can be put equivalently in terms of falsehood; an inference is valid if the falsity of the conclusion guarantees the falsity of the conjunction of the premises.

We may express this by saying that while an important use to which we put the machinery of logical inference is *proof*—the derivation of conclusions from premises known to be true—inference and proof are not the same. To conflate them would be as bad as to conflate pure and applied arithmetic—something Frege taught us not to do.

We can make a further appeal to Frege in defending this conception of inference. Logic is concerned with objective matters. Only confusion can result if we allow the validity of an inference to depend upon any psychological conditions. But then it must be a matter of indifference to logic whether the premises of an inference are known to be true or not.

Surprisingly, such an account would provoke a strongly negative reaction from Frege. He expressed himself in ways that seem alien to this conception of

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inference, in ways that suggest the very psychologistic errors he warned us against. I want to clarify both the nature and the source of Frege's doctrine of inference, and to probe its connections with psychologism.

2 The view of Frege's that I am interested in becomes explicit in his writings quite late. This is not, I think, because Frege came late to the view in question. There are indications of it in his earlier works, as I shall show in a moment. Its first clear statement occurs in the second of his long articles on the foundations of geometry and the axiomatic method (1906).[2] He went on repeating the point in published work, notes, and correspondence for the rest of his life. Let us see what Frege says.

> An inference . . . is the passing of a judgment [*Urteilsfällung*] made in accordance with logical laws on the basis of previously passed judgments. Each of the premises is a determinate Thought recognized as true; and in the conclusion too, a determinate Thought is recognized as true. [16], pp. 303–304; [10], p. 318

> From false premises nothing at all can be concluded. A mere Thought, which is not recognized as true, cannot be a premise. Only after a Thought has been recognized by me as true, can it be a premise for me. Mere hypotheses cannot be used as premises. [18], p. 182[3]

> . . . we can infer something only from true sentences. Thus if a group of sentences contains a sentence whose truth is not yet known, or which is certainly false, then this sentence cannot be used for making inferences. . . . When we infer we recognize a truth on the basis of other previously recognized truths according to a logical law. [17] in [29], p. 30 and in [25], pp. 16–17

> Of course we cannot infer anything from a false Thought. [22] in [16], p. 364 and in [10], p. 375

> . . . before recognizing its truth one cannot use a Thought as a premise of an inference, nor can one infer or conclude anything from it. [11] in [16], p.390 and in [10], p. 402

On this view no mere assumption can be a premise. And Frege does reinterpret inference from hypotheses or assumptions as the categorical proof of a conditional (see, e.g., [25], pp. 182–183). That, at any rate, is his official position; but it is contrary to plain common sense and to his own practice. After all, the best way to prove a conditional is often to assume the truth of the antecedent; to deny us this recourse seems merely vexatious. And in the Appendix to [13] Frege deduces, in the ordinary way, various propositions from Axiom 5, which he by that time recognized as logically false.[4] In less guarded moments Frege acknowledges a kind of inference from assumptions. In the letter to Dingler from which I have just quoted [17] he remarks that, from the Thought that 2 is less than 1, and the Thought that anything less than 1 is less than 2, we can derive, "in a purely formal way", that 2 is less than 2. But this would not be inference, but "pseudo-inference," "for the truth of the premises is lacking".[5] It is tempting to say that Frege is reaching for the now familiar distinction between soundness and validity. But Frege's distinction between inference and

pseudo-inference is epistemic: an inference must proceed from premises *known* to be true. And Frege's tone seems to convey the idea that pseudo-inference is mere simulation; as stage assertion is a simulation of assertion.

I suggest that Frege's motivating idea is this: logic is essentially concerned with justification. In the letter to Dingler he says, of his "purely formal" derivation, that "the truth of the conclusion is no better grounded by means of this pseudo-inference than without it. And this procedure would be useless for the recognition of any truths". This explains why Frege is not content to say just that inference must be from true premises; he goes on in the more extended quotations to say that inference must be from premises that we recognize (*anerkennen*) as true. It cannot be from premises the truth of which is not yet known or from premises known to be false, for such a procedure would not justify or ground its conclusion. That justification is the key to understanding Frege's theory of inference will be the central theme of this essay.

In the unpublished paper called "Logic" [20], which his editors date no more precisely than between 1879 and 1891, Frege tells us a little more about how he conceives the connection between logic and justification:

> Now the grounds which justify the recognition of a truth often reside in other truths which have already been recognized. But if there are any truths recognized by us at all, this cannot be the only form that justification takes. There must be judgments whose justification rests on something else, if they stand in need of justification at all.
>
> And here lies the task of epistemology. Logic has to do only with those grounds of judgments which are truths. To make a judgment because we are conscious of other truths as providing a justification is known as *inferring*. There are laws governing this kind of justification, and to set up these laws of correct inference is the goal of logic. [20] in [21], p. 3 and in [26], p.3, italics in the original

Summarizing, we may say: epistemology and logic are both concerned with justification. Epistemology tells us how to ground our first premises in a noninferential way. Logic tells us how to enlarge our knowledge by specifying the conditions under which a proposition counts as deductively justified—justified by being inferred by truth preserving steps from known propositions. Inference from mere conjecture is not inference proper, for the machinery is put to some etiolated use, as is the machinery of assertion in a theatrical setting.[6]

It is sometimes said that what Frege was driving at is the view that inference must proceed from *asserted* premises.[7] And an asserted premise need not be true. On this interpretation Frege's contrast between inference and pseudo-inference is not a contrast between premises that are known and premises that are not (because they are false, or merely supposed, or believed without sufficient reason) but between premises that the inferrer commits himself to believing, and premises that are mere suppositions. But to interpret Frege in this (some would say charitable) way is doubly to mistake his intention. First of all, the quotations at the beginning of this section make it clear that the premises of an inference must be true and known to be true. Secondly, I do not think that Frege saw any essential connection between inference and assertion, the outward manifestation of a judgment, which is in turn the recognition of the truth of a

Thought. What is essential is that the premises be so recognized, and that may take place "in the mind". In some late remarks on "logical generality" he says of general laws:

> We profit from our knowledge of a law by gathering from it a wealth of particular pieces of information, using the inference from the general to the particular, for which of course *a mental act* [*geistige Arbeit*]—that of inferring—is still always required. [21], p. 278; [26], p. 258, my italics[8]

Of course, if an inference is given external expression, its premises must be accompanied by the sign of assertion, and this is what Frege does in all the derivations in his work that count for him as inference proper. But to insist on this is merely to insist that, if an inference is made explicit, everything logically relevant to it must be made explicit as well, including the fact that the truth of the premises is acknowledged. That is what the assertion sign does for us.[9]

We see here something characteristic of certain exigetical disagreements concerning Frege's philosophy. Readers of Frege who interpret him primarily as a philosopher of language are naturally led to give prominence to the idea of *assertion* in explaining Frege's doctrine of inference. I, on the other hand, seeing much in Frege that is concerned with epistemological questions, incline to the view that the doctrine of inference is explicable in terms of *justification.* These different perspectives must be judged in terms of their consonance with Frege's texts, and I think I have shown already how deficient is the first interpretation in this respect. We shall see how the idea of justification gives further illumination to Frege's doctrine as we try to grapple with the problems to which the doctrine gives rise.

3 Frege's view seems to be that to infer in the proper sense is to proceed by truth-preserving steps from premises known to be true to conclusions which, while they may not have been known prior to the making of the inference, are known in virtue of that inference having been made. Inference is not just truth-preserving; it is justification-conferring.

But what has logic to do with mental acts of judgment? After all, psychologism is the application of mentalistic notions to logic, and Frege tried, surely, to rid logic of such notions: "In logic we must reject all distinctions that are made from a purely psychological point of view" ([21], p. 154; [26], p. 142). It looks as if Frege's view falls prey to the criticism he so often makes of his opponents. To see whether this is so we must look carefully at the kinds of procedures and doctrines that Frege rejects as psychologistic.

The first thing we must understand is that Frege's opposition to psychologism is not a general condemnation of the role of anything psychological in philosophy, though his more extreme slogans suggest that it is. The attack on psychologism is really an attack on a theory about the nature of cognitive acts; a theory that Frege saw as likely to undermine our respect for reason and argument. He ascribes the theory to Locke and the empiricist tradition, but his active opposition to it was engendered by its apparent influence on the mathematicians, logicians and philosophers of his own day.[1] Frege is never very precise in his characterization of the theory, nor in his delineation of its consequences. But to some extent at least this is understandable. First, he regards it as so fun-

damentally wrong that we have no need to distinguish its variants. And in opposing the theory he is really protesting against an irrationalist *Weltanschauung* which the theory encourages rather than entails. Frege saw himself as grappling with a piece of dangerous ideology; untrue, immoral, and an enemy of clear thinking. And if we examine the prevailing scientific naturalism of Frege's time (to which he makes some reference in his *Foundations of Arithmetic*) it is easy to sympathize with his view.

In what follows I shall try to present Frege's view succinctly, but without possibly contentious clarification, for I want to discover whether there is something in those views that may fairly be called Frege's own, which should lead him to abandon the theory of inference. The present paper is an investigation of the internal stresses to which Frege's system may be subject, rather than of its relation to the wider world of philosophy.

At the core of the theory is the identification of thinking with the having of ideas or mental images. Cognition is thus assimilated to sensation. Frege is not skeptical about the existence of mental images, but he insists that they are an accompaniment to thought, not constitutive of it (cf. [20] in [21], p. 154 and in [26], p. 142). If they were, our thoughts would not be communicable, for there can be no comparing the mental images of different subjects.[11] The picture theory—as we may call it—makes it impossible for one to dispute or criticize the thought of another, and that is relativism:

> If we could not grasp anything but what was within our own selves, then a conflict of opinions [based on] a mutual understanding would be impossible, because a common ground would be lacking, and no idea in the psychological sense can afford us such a ground. [13], Preface, p. xix; [8], p. 17

Frege also saw in the model a tendency to take ideas as the referential objects of thought and experience, particularly in the case of problematic entities like numbers and concepts. Again, we are forced towards relativism: "If the number two were an idea, it would have straight away to be private to me only" ([14], section 27). My numbers may have quite different properties from yours, yet we should not be able to call this a disagreement, for there is nothing common for us to disagree about. And the process of absorbing outer things into the mind does not stop at abstract entities; the model has a general tendency to encourage us to suppose that what we think about and perceive is our own ideas. And that is subjectivism:

> In the end everything is drawn into the sphere of psychology; the boundary between objective and subjective fades away more and more, and even real [*wirchliche*] objects themselves are treated psychologically, as ideas. . . . Thus everything drifts into idealism and with perfect consistency into solipsism. [13], Preface, p. xix; [8], p. 17

Frege's anti-psychologism is an attempt to sweep away relativism and subjectivism by establishing a right view of the mind and its relation to thought and the world. Cognitions are relational states in which the mind grasps an objective Thought (*Gedanke*); and many subjects can grasp and communicate the same Thoughts.[12] Only by recognizing the relational character of thought can we hope to secure an objective world. The relational theory of mind does not

entail realism, for the physical world is not the world of Thoughts; Thoughts are in the realm of sense, and physical things in the realm of reference, as Frege sometimes put it (see, e.g., [21], p. 275; [26], p. 255). The step from Thoughts to things is an extra step, and one that we cannot take with absolute certainty ([16], p. 358; [10], p. 367). But if there is an outer world, we can think and speak of it only via our grasp of senses.[13]

Further evils of the same kind follow if we fail to recognize the relational character of thought. For we will find no objects to attach the notions of truth and validity to but our ideas. In that case there can be nothing true that is not an idea, nothing valid that is not a transition from one idea to another. But ideas are subject only to the associational laws of psychology. So truth and validity must be nothing but psychological notions. In the attack on Erdmann's *Logic* with which he begins [13] he says:

> For me, [what is true] is something objective and independent of the judging subject; for psychological logicians it is not. What Herr B. Erdmann calls "objective certainty" is merely a general acknowledgment on the part of the subjects who judge, which is thus not independent of them, but can change with the constitution of their minds. [13], pp. xvii–xviii; [8], p. 15

I suggest, then that the opposition, as Frege understands it, between psychologism and anti-psychologism is the opposition between the picture and the relational theories of cognition. The picture theory leaves us trapped inside our own heads. Substituting the relational theory gives us: (a) an explanation for the commonality of thought and speech; (b) a bridgehead into the external world; and (c) a regulative, mind-transcendent conception of truth and validity.

4 We may now ask whether Frege's conception of inference represents a concession to psychologism. In conformity with the methodological decision of Section 3, we shall treat this as the question whether the conception falls victim to the kinds of objections that Frege brought against the picture theory.

Suppose, as Frege says, that the premises of an inference must be recognized truths. Now what is recognized by one person may not be by another; so what counts as an inference for one person may not count as such for another. But if logic is to be an objective science—as Frege insists it must—its results should be valid for everybody: not in the sense that everyone recognizes the results of logic as valid, but in the sense that logical results should be stateable without reference to the possibly idiosyncratic perspectives of particular subjects. The objectivity of truth, so Frege tells us, consists in the fact that what is true contains no reference to the subject who judges. While we may think that 'I am cold' is true for me but not for you, we do so only because we confuse the indexical sentence with the nonindexical Thought it expresses, to which truth properly attaches. The Thought I express when I say 'I am cold' is just as true for you as it is for me ([20] in [21], p. 146 and in (26], p. 134–135). We can put the point another way by saying that if there can be a definition of what it is for a Thought to be true—as Frege thinks there cannot ([28] in [16], p. 344 and in [10], p. 353)—that definition would involve no parameter specifying a subject. But Frege seems to embrace exactly this kind of subject-relativity for inference.

In the second of the quotations with which we began Section 2 he says "Only after a Thought has been recognized *by me* as true can it be a premise *for me*".

Frege says nothing in his own defense; indeed he does not seem to realize that there is a problem here, vigorous though he is in searching out indications of psychologistic errors in others. What defense can we construct for him?

Given what we have said about Frege's objections to psychologism we should expect him to be able to show that his conception of inference is neither relativistic nor subjectivistic: to show that (i) if a conclusion is inferred by one subject, a contrary conclusion can never be inferred by another, and (ii) if a conclusion is inferred by a subject, that inference must be equally available to all rational agents.

For anyone to infer a proposition, certain conditions must prevail. The premises and conclusion must be true; the conclusion must be obtained from the premises in accordance with the rules of inference; the inferrer must recognize the truth of the premises. The first condition ensures that inference is not relativistic, since p and $\sim p$ are never both true (assuming an absolutist conception of truth). The question of subjectivity arises in respect of the two other conditions. Since an inference, properly displayed, involves the application of elementary rules like the rule of modus ponens we may say that the inferential steps are capable of being carried out by any rational agent.

The question that remains concerns the recognition of truth. Called upon to defend himself, Frege could make a distinction between objective and subjective senses of recognition, this distinction arising naturally out of his attack on psychologism. In the objective sense, to recognize a Thought as true is to be in possession of a justification for it; it is that justification another agent must possess in order to make the inference his own. And the test of a justification is its availability to all rational agents. Here Frege is close to Kant. Frege's distinction between "the grounds that justify a conviction and the causes that actually produce it" ([21], p. 159; [26], p. 147), echoes Kant's remark that "The holding of a thing to be true is an occurrence in our understanding which, though it may rest on objective grounds, also requires subjective causes in the mind of the individual who makes the judgment" [31], [A820, B848]. This is a distinction that the picture theory cannot make, for there is no purely internal, psychological difference between justification and mere persuasion. Kant goes on: "The touchstone whereby we decide whether our holding a thing to be true is conviction or mere persuasion is therefore external, namely the possibility of communicating it and of finding it to be valid for all human reason". And Frege makes the same point when he insists that what is objective is that which is "exactly the same for all rational beings" ([21], p. 7; [26], p. 7). In particular, arithmetic is objective because it is "given directly to our reason" and so "utterly transparent to it" ([14], Section 105). Here at least we can expect to find justification accessible to all rational agents. If Frege's mathematical inferences are genuine, it will be a test of our rationality that we should make them our own.

It is helpful to consider what Frege would say about the relations between the predicates "is true", "is beautiful" and "is justified". "Is true" is an objective predicate; we do not introduce it by saying what it is for something to be true *for someone*. "Is beautiful", on the other hand, is thoroughly subjective; we can only ever say that something is beautiful for a certain subject. One sub-

ject may find a thing beautiful, another not; yet there would be here no disagreement, for ". . . the proper work of art is a structure of ideas within us and the external thing—the painting, the statue—is only a means for producing the proper work of art within us" ([21], p. 144; [26], p. 132). Now the application of "is justified" is to be explained in terms of what it is for something to be justified for a subject. But conflicting propositions can never be justified for different subjects, and whether something is justified for a subject does not at all depend upon the subject's ideas. Justification thus occupies a position between truth and beauty, but in crucial respects it is closer to truth. It is, we may say, subject-involving, but it is not subjective, as beauty is. Psychologism is the error of treating nonsubjective concepts as if they were subjective. The present conception of justification avoids any such error. So goes the answer that Frege might give us.

5 We can hardly be satisfied with this answer unless Frege can provide us with a concrete demonstration of the kind of justification he has in mind. For propositions that are or can be deductively justified there is no particular problem. To justify such a proposition we have merely to cite the already known propositions from which it follows. But what is the objective justification for those propositions that are treated as axioms? In a paper written at the end of his life Frege acknowledges three nondeductive sources of knowledge: the empirical, the logical and the geometric-temporal (cf. [27] in [21], pp. 286–294; and in [26], pp. 267–274). He emphasizes the fallibility of the empirical source—sense experience—and the extent of its dependence on the other two sources, but he does not say whether the empirical source depends essentially upon the having of subjective ideas. Since Frege's own concern was the mathematical sciences, we would expect from him a fuller account of their sources. (In what follows I shall ignore the very substantial problem of empirical knowledge.) In this late essay Frege emphasizes also the fallibility of the logical source of knowledge—without saying exactly what that source is. He says that it is "wholly inside us", but also that logical errors arise because of the dependence of the source on language. In particular, our ability to form proper names to which no objects correspond leads to trouble. We form the expression "extension of the concept *F*", and assume that there must be something that this expression refers to. But there is no such thing. This, apparently, is the lesson of Russell's paradox.

The geometrical source of knowledge is the least liable to error. It does not depend upon subjective ideas; that we cannot visualize an infinite totality of geometric points is no barrier to our recognition of the result that every interval contains infinitely many such points. Knowledge of the infinite comes from the geometrical rather than from the empirical source of knowledge. These remarks are preparatory to the attempt at a new foundation for arithmetic based on geometry that Frege was contemplating at this time.[14]

From our point of view, this discussion is thin and unilluminating. It gives us little idea about what the logical and geometrical sources of knowledge are. Not enough, certainly, for us to decide whether these sources may count as objective in the required sense. Philip Kitcher has suggested a reason why Frege has so little to say on the subject: he simply assumes that the nature of these sources has been identified and analyzed by Kant.[15] A rough Kantian taxonomy

tells us that the source of logical knowledge is conceptual analysis; the source of geometric knowledge is the pure intuition of space. At the time when Frege thought that arithmetic could be founded on logic, he differed from Kant only in thinking that the logical source of knowledge can provide us with knowledge of particular objects—the logical objects or extensions of concepts. Given what we have already seen of Frege's apparent indebtedness to Kant, Kitcher's suggestion is a plausible one. But to complete the defense we are constructing of Frege's theory of inference we need to ask, specifically, what does Frege take to be the justification—the objective justification—for the axioms of his logical system? What does he offer his readers by way of conceptual analysis that will enable them to see his axioms as certain truths and thus to make his inferences their own? How exactly, in the present case, are they to exercise that capacity for analysis which is the logical source of knowledge? Notoriously, Frege is all but silent on this matter, particularly concerning the problematic Axiom 5, the ultimate source of Russell's paradox.[16] (Let us concentrate our attention on this axiom.)

The most that one can glean from the slender indications that Frege gives us, together with their assumed Kantian background, is that the truth of the axiom is constitutive of the concept of a concept; it is something we can recognize to be true by an analysis of that concept. In the Preface to [13] Frege says that the axiom "is what people have in mind when they speak of the extension of a concept". In an article written in 1906 [23] Frege discusses the idea which is really the problematic part of Axiom 5: that to every concept there corresponds an extension. Surprisingly—for this is several years after Russell's discovery—Frege treats the idea with no great skepticism: "an actual proof [of the proposition] can scarcely be furnished. We will have to assume an unprovable law here. Of course it isn't as self evident as one would wish for a law of logic. And if it was possible for there to be doubts previously, these doubts were reinforced by the shock from Russell's paradox". In the same place, Frege says of extensions: "By means of our logical faculties we lay hold upon the extension of a concept, by starting out from the concept". A little later: "The extension of a concept simply has its being in the concept". Forgetting for the moment that Frege is here trying to justify something that he really knows to be false, his words suggest the idea that to think of a concept is thereby to think that the concept has an extension, if only an empty one. A concept without an extension is no concept at all. So if we understand the notion of a concept we are guaranteed the truth of Axiom 5 and hence the existence of logical objects—extensions. Thus a justification for the acknowledgment of Axiom 5 is ensured for anyone who grasps the concept of a concept. And that is a basic condition on a rational intelligence.[17]

But if this is Frege's argument, he was far from sure about it. Many other things he says suggest that the axiom simply does not have the epistemic status required of an axiom proper.[18] He admits in the Preface to [13] that "a dispute can arise" as to its truth. The further remark that "in any event the place is pointed out where the decision must be made" suggests that the axiom is put forward tentatively, as a hypothesis to be tested by its consequences (as it was, so destructively, by Russell). And in the Appendix to the second volume he says of the axiom, "I have never concealed from myself its lack of the self evidence

which the other [axioms] possess, and which must properly be demanded of a law of logic. . . . I should gladly have relinquished this foundation if I had known of any substitute for it". Writing to Russell in 1902 he says: "I myself was long reluctant to recognize courses of values and hence classes; but I saw no other possibility for placing arithmetic on a logical foundation. . . . I have always been aware that there are difficulties connected with this . . ." ([29], p. 223, [25], p. 141).

We see, I think, that Frege's characterization of inference, while it is not susceptible to the more obvious objections against psychologism, does not reflect Frege's own logical practice, or even his own more reflective representation of that practice. Frege was prepared, at the time of writing [13], to present as genuine inferences deductions from premises, at least one of which he could not honestly regard as an established truth, and concerning the justification of which he could provide no more than the vaguest hints. The theory of inference embodies his preference for a certain epistemological ideal: knowledge as a deductive system grounded in self-evident axioms. This ideal permeates his whole philosophy and is, I believe, the motivating force behind his logicist program. It shows itself also in an extraordinarily uncritical attitude towards Euclidean geometry which he regards as a paradigm of a deductive science and which he falsely claims has "exercised unquestioned sway for 2000 years" ([21], p. 184; [26], p. 169). Russell's paradox ought to have brought him to a reassessment of that ideal and to a recognition that inference may be carried out in a more relaxed epistemic setting. Instead, it was only after Russell's discovery that Frege began explicitly to insist that inference must be from known truths. I am left with the impression of one in the position of those unfortunate believers investigated by Festinger: one whose reaction to the failure of his system is a heightened affirmation of faith (cf. [7]).

NOTES

1. See [9], Section 3. For comment see [1].

2. The debate began with an exchange of letters between Frege and Hilbert concerning the nature of the axiomatic method, and was fueled by Frege's reading of Hilbert's *Foundations of Geometry* which first appeared in 1900. Frege published a two-part essay on the subject in 1903, entitled "On the Foundations of Geometry". He was replied to in the same year by Alwin Korselt, and published an extended response and development by the same title in 1906. See [29], pp. 58–80 and 147–151, and [25], pp. 31–52 and 92–94. Also see [32], which contains the papers by Frege and Korselt. For a useful discussion see [34], chapter 3. I have made some alterations to the translation of passages that appear in this essay. I use 'Thought' as a translation of Frege's *'Gedanke'*, distinguishing it from 'thought' in the psychological sense. For our purposes we may take 'Thought' to be roughly synonymous with 'proposition'.

3. Enclosing these notes to Jourdain, Frege says "You may use them as you see fit" [29], p. 115; [25], p. 76).

4. [13], II. In the main body of the work all axioms and theorems are prefixed with the judgment stroke "⊢", which indicates that the proposition following is asserted (see volume I, Section 5). Of the derivations in the Appendix Frege says ". . . in consideration of the doubtful truth of it all I shall omit the judgment stroke" ([13], II, p. 256; [8], p. 130).

5. For another place where Frege allows the drawing of conclusions from a supposition, see e.g. [12] in [16], p. 136 and in [10], p. 149.

6. In one place Frege tries to separate justification and inference. In a brief paper that Scholtz dated around 1906 but which could have been written much earlier, Frege tells us that "The task of logic is to set up laws according to which a judgment is justified [*gerechtfertigt*] by others, irrespective of whether these are themselves true" ([21], p. 190; [26], p. 175). It is ironic that while Frege seems to want to say that inference has nothing to do with whether the premises are true or false, he still speaks of "justifying", rather than simply of "inferring".

7. Analysing some of the passages I have quoted, R. H. Stoothoff concludes that "Frege did not mean to deny the possibility of using a false thought as premise for an inference: certainly he admitted the possibility of inference from a thought which is mistakenly asserted, i.e., from a thought whose truth is mistakenly acknowledged. But he maintained that only a thought that is categorically—not just hypothetically—asserted can be used as a premise of an inference" ([35], p. 407). I also took this view in an earlier essay ([5], p. 81).

8. Michael Dummett asks "why does [Frege] express [the doctrine] by saying, 'We can make inferences only from true premises', rather than 'We make inferences only from premises which we take to be true'?" Dummett answers: "taking something to be true is a psychological matter, and Frege had set his face against the importation of psychology into logic. If we make the emendation, 'We make inferences only from premises which we have asserted', and recognize that assertion is not something psychological, the difficulty is overcome" ([6], p. 313). But Dummett is simply wrong about what Frege says: in the quotations given in Section 2 above he says that a premise must be both true and recognized as true. Taking to be true, in the sense in which one can mistakenly take something to be true, is not plausibly what Frege has in mind here. And in none of these quotations is there any mention of assertion.

   There is one place where Frege connects inference with assertion. In a paper assumed to date from 1915 Frege says that ". . . 'true' makes only an abortive attempt to indicate the essence of logic, since what logic is really concerned with is not contained in the word 'true' at all but in the assertoric force with which a sentence is uttered" ([21], p. 272; [26], p. 252). Prima facie, this supports Dummett's interpretation. But I would describe it as a case where Frege is, for other reasons, speaking in the formal rather than the material mode. He is discussing the view that adding the words "is true" to a sentence indicates assertoric force. This is not so: "P is true" is just another sentence that may or may not be asserted. This naturally leads him to speak of assertion rather than judgment as the essence of logic, since assertion rather than judgment goes with the linguistic expression of a Thought. However one construes this passage, taken in conjunction with the evidence I have already presented that inference is connected only with the mental act of judgment, it cannot count for very much.

9. This is further supported by a remark in a draft of a letter to Jourdain, written in 1914: "What is to serve as the premise of an inference must be true. Accordingly, in *presenting* an inference one must utter the premises with assertoric force, for the truth of the premises is essential to the correctness of the inference" ([19] in [29], p. 127, and in [25], p. 79, my italics). The same draft contains the astonishing claim that "Whoever understands a sentence uttered with assertoric force adds to it his recognition of its truth. If a proposition uttered with assertoric force expresses a false Thought, then it is logically useless and cannot strictly speaking be understood". The letter was rewritten before it was sent, and these passages do not recur. Little weight, therefore, can be attached to them.

10. On Locke see [21], p. 115; [26], p. 105. For Frege's criticisms of contemporary views see, e.g., [14], Parts I, II and III. See also [2], pp. 3–38, and [30].

11. See [24] in [16], p. 146 and in [10], p. 160; [28] in [16], p. 353 and in [10], p. 362.

12. [16], p. 148n, [10], p. 162n; [16], p. 361–362, [10], p. 371.

13. See [16], p. 148, [10], pp. 161–162. For more on the character of Frege's anti-psychologism, see my [4].

14. For more on Frege's geometrical construction see my [3].

15. I am indebted to Kitcher's paper [33] for the stimulus it has given to my thinking on this subject.

16. In a somewhat simplified form, the axiom says that two concepts have the same extensions if and only if the same objects fall under both.

17. In his interesting [36] Crispin Wright takes a similar line, attempting to base arithmetic not on the notion of extension, but on the notion of number itself. He takes the principle "$Num(F) = Num(G)$ iff the Fs and the Gs are 1-1 correlated" to be constitutive of the concept number. The Peano Axioms follow in second-order logic.

18. Axioms "are truths for which no proof is given in our system, and for which no proof is needed. It follows from this that there are no false axioms, and that we cannot accept a Thought as an axiom if we are in doubt about its truth . . ." ([21], p. 221; [26], p. 205).

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