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Reply to Burgess and to Read

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1 Introduction Either John is foaming at the mouth or John is biting the carpet. John is not foaming at the mouth. Therefore, John is biting the carpet. Such an instance of Disjunctive Syllogism (*DS*) is undoubtedly intuitive, but a form of inference which is intuitive is not thereby valid. There are (at least) three positions which can be taken concerning the validity of *DS*. *First: DS* is valid, and the "or" in it is the two-valued extensional "or". Thus, the argument form Extensional Disjunctive Syllogism (*EDS*), *i.e.*, $A \vee B$, $\sim A / \therefore B$, is valid. *Second: EDS* is invalid. There is a valid argument form, Intensional Disjunctive Syllogism (*IDS*), namely $A + B$, $\sim A / \therefore B$, where "+" is intensional disjunction. Whenever you have a valid example of *DS*, it is because it is an instance of *IDS*. *Third:* The examples of *DS* which seem intuitive are often instances of *EDS*; but this does not make *EDS* valid, and it is not. Whenever it seems intuitive to infer using *EDS*, it is because there is an extra assumption, that things are "normal", which ensures the truth of the conclusion and which explains the apparent intuitiveness of *EDS*.

Recently (in [8]), I defended the third of these. Read (in [9]) defended the second. In the course of my argument, I made the further claim that there are precise sufficient conditions for when the truth of the premises of *EDS* would ensure the truth of the conclusion and that these conditions obtained whenever there was an intuitive example of *EDS*. Both Read and Burgess ([4], see also his [5] and [6]) understood me to be trying to prove my claim by appeal to the validity of *EDS* in the metatheory, an appeal which they took to be circular. In Section 2 of this note, I will argue that there is no circularity in my position. In Section 3, I will argue that my position is a stable one, in that no collapse into a generally valid *EDS* follows from it. In Section 4, I will briefly respond to some of Burgess's other points from [4].

2 The appeal to normality We need some definitions. A theory for a logic L is a set of sentences closed under the consequence relation $\vdash_L$. It is useful to consider the situation we find ourselves in when deducing according to "natu-

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ral" logic, as a theory closed under the natural consequence relation $\vdash$. This has the virtue that actual deductive behavior in the course of theory construction in the sciences can be seen as data about how propositions are related by $\vdash$. Closely connected with the question of whether $A \vdash B$, is the question of whether B belongs to all natural theories to which A belongs. Read objects to my too-quick identification of these two questions; but I do not rely on it in what follows.

A theory will be said to be consistent iff for every sentence A, not both A is in the theory and $\sim A$ is in the theory (or—what is equivalent given metalinguistic laws of De Morgan, Double Negation and Commutation hereafter assumed—either A is not in the theory or (extensionally) $\sim A$ is not in the theory). A theory will be said to be prime iff for every extensional disjunction $A \vee B$ in the theory, either A is in the theory or (extensionally) B is in the theory.

I claimed that theories which are *intuitively well behaved or normal* are closed under *EDS*, and that counterexamples to *EDS* are to be found in abnormal theories only, though that should hardly daunt the fearless logician. I then claimed that a sufficient condition for a theory to be closed under *EDS* is that it be consistent and prime. In proving this, I appealed to something looking like *EDS* in the metatheory. Both Read and Burgess objected that I had no right to such an appeal.

But this is not so. Let me make clear what my contention is. I claim that (given a normal metatheory which we should be able to ensure), for any consistent prime theory *Th* and for any propositions A, B, from $A \vee B \in Th$ and $\sim A \in Th$ it is deducible that $B \in Th$. My argument is in two stages. The first stage is in the metametatheory.[1]

> From the premises that a theory is normal and that $A \vee B$ and $\sim A$ are in the theory, it is deducible that B is in the theory.
>
> The metatheory (of this paper) is normal.
>
> $\therefore$ The metatheory is such that it is deducible that B is in it from the premise that $A \vee B$ and $\sim A$ are in it.

The premises of this argument were not justified by any appeal to *EDS*, but to the pretheoretic data available to us. There do seem to be intuitive examples of *EDS*, and the metatheory needed to put through the argument to follow is minimal: first-order logic with a single binary relation $\in$ with quite weak properties and a relation $\vdash$ which is also quite weak. No reason for suspicion of abnormality or paradox here. That is, I don't claim to *prove* the truth of the above two premises. Proof will have to stop somewhere, especially in the epistemology of logic. I offer support for them, of a reasonable kind.

Now for the second stage of my argument, "drop down a level" to the metatheory. For suppose that *Th* is consistent and prime; I claim that from this fact together with $A \vee B \in Th$ and $\sim A \in Th$ it is deducible that $B \in Th$. For from $\sim A \in Th$ (*i.e.*, not—not—$\sim A \in Th$) together with the consistency of *Th* (either not $\sim A \in Th$ or not $A \in Th$) it is deducible that not $A \in Th$ (by appeal to the conclusion of the Stage I argument). Then from $A \vee B \in Th$ by primeness, either $A \in Th$ or $B \in Th$; hence it is deducible that $B \in Th$ (by appeal to

Stage I again). It is apparent that no question-begging appeal to the validity of *EDS* has been made here. A version of *EDS* has been used, but *only as a property of a particular theory*, and support has been given for that.

3 Formalizing the argument It is evident that a formal version of the foregoing argument can be written down in a straightforward way, taking a single binary predicate $\in$, the usual extensional connectives, and, wherever "it is deducible that" occurs, $\vdash$. Or, instead of $\vdash$, use a metatheoretic $\rightarrow$. For the conclusion $(Con(Th) \,\&\, Pr(Th) \,\&\, (A \vee B) \in Th \,\&\, \sim A \in Th) \rightarrow (B \in Th)$, the extra properties needed for $\rightarrow$ are substitutivity with respect to the equivalences of De Morgan, Double Negation and Commutation, and the two rules (a) Transitivity for $\rightarrow$, and (b) $A \rightarrow B$, $C \rightarrow D / \therefore A \,\&\, C \rightarrow B \,\&\, D$. A special case is where we take Th = The True, so that "$\in Th$" is a truth predicate. I presume that this constitutes an answer to Burgess' demand ([4], pp. 49, 51) for a theorem formalizing the principle (Consistency & Primality & $(A \vee B)$ & $\sim A) \rightarrow B$. Of course this is not to say that the $\rightarrow$ in question is entailment, since, for example, an enthymematic $\rightarrow$ would do (e.g., [1], p. 259, or [2]). On the other hand, even if we take the $\rightarrow$ of the metatheory to be entailment, *it does not follow that all theories are classical.* To see this, just note that the logical structure of the object language theories has been left unspecified. Nothing prevents them, therefore, from being theories of any of the usual relevant logics. The first stage of the argument delivers the conclusion that B may be deduced from $A \vee B$ and $\sim A$ only for instances of A and *B from the particular metatheory*. Nothing follows about *unrestricted* theoremhood of $((A \vee B) \,\&\, \sim A) \rightarrow B$, so it is open to us to invest the $\rightarrow$ with unrestricted substitution instances corresponding to a weaker logic than classical.

This is far from eclecticism. As defined by Burgess, that is the view that relevant logic is only "appropriate for certain extraordinary abnormal situations . . . no logic provides canons of validity that are necessary and sufficient for all situations . . . logics have to be local, . . . different situations have different logics" ([4], p. 50). If this means that there are no logical truths and no valid arguments,[2] then I am certainly not committed to it. The view advocated here is consistent with the position that there are some universally valid argument forms, and some argument forms which in more restricted circumstances take us from truths to truths. It would be confusion to describe this as the thesis that relevant logic is *only* appropriate in abnormal situations. One might hold instead that relevant logic describes the correct universal validities, while classical logic is a special case, holding only over a restricted domain.

I take it that the fact that nonclassical object-language theories are describable by weak metatheory (and any supertheory) in the fashion of this paper demonstrates the logical stability of my position. So one is led to ask what kinds of epistemic considerations Read and Burgess would severally appeal to in support of their own differing positions. I suspect that Read's view brings him dangerously close to logical skepticism.[3] He seems to think that unless some kind of proof of the unrestricted validity of an argument form is forthcoming, then one would never be justified in moving from its premises to its conclusion in a particular case. But if any argument form is valid, then some inference rules are not justified by being proved from others. Burgess, on the other hand, might

be making a much stronger demand than I attributed to him at the beginning of this section, namely, the demand to produce a fully developed relevant metalogic, truth theory, model theory, set theory, the lot ([4], p. 51). This ploy is sufficiently common to deserve a name, so let us call it *The Fallacy of the Conservative Theorist*: Unless My Opponents Have a Fully Developed Countertheory, All Their Arguments Against Me are Unsound. But, of course, the above result holds in any supertheory of our metatheory, no matter how much extra baggage it gets.

4 Sundry loose ends This brings me to the question of who has misrepresented whom. I have already argued that Burgess and Read have misunderstood me. Burgess claims that I misrepresented him, and that his intent was all innocence itself: only to show against Anderson and Belnap that common sense employs *EDS*. As I said in [8], Burgess's first paper is best understood as an attack on Anderson and Belnap, but some remarks suggest that his aims are more general. I do not think that anyone could read his paper and not get that impression. Here are just a few points. The aim of the card game example was not just to show that common sense goes his way, but also that "the relevantist" would do "badly" and "in social life, diplomacy, and other areas". I deny this. Notice, too, the inference from "not common sense" to "bad". Again, his arithmetical example insinuates the less-than-innocent conclusion that "the honor of priority goes to Wyberg", the implication being that Wyberg's argument was valid ([6], p. 102). I claim that if arithmetic is inconsistent, then Wyberg's argument is invalid, so the "commonsense" presumption that Wyberg's argument is valid masks the presupposition of consistency. I take it that in disputing the implication of validity, I was meeting Burgess's "challenge" "to explain away some apparent examples of commonsense instances of *DS*" ([4], p. 45). Needless to say, to fail to take up such a challenge is to lose some presumed competition by default. The debate might at this point degenerate into semantic trivialities about how narrow in application were Burgess's phrases like "the relevantist" and "the Anderson-Belnap systems E and R", and he hastens to tell us how big is the gulf between Anderson and Belnap, and Routley ([4], p. 45). I think, in fact, that Burgess's first paper was written largely in ignorance of what had been published *about E and R* by others, an impression his second paper certainly hastens to counteract. Again, his assurance ([6], p. 104) that he was concerned "solely with the original Anderson-Belnap account of 'relevant' logic and with their claim that their systems E, R, etc., are in better agreement with common sense than is classical logic", would have helped him better had he not *contrasted* it with "the discovery of serendipitous applications" such as logics of ambiguity; instead of a contrast with, say, Meyer's work.

Burgess uses so many rhetorical devices that his papers read like a list of textbook examples of informal fallacies. I do not propose to catalog all of these, but let me caution readers against fallaciously reasoning on the basis of Burgess' second paper according to the *Fallacy of Divide-and-Conquer*: The Opponents are in Disagreement about Some Issues, Therefore All Their Theses are False. Certainly there is disagreement on some issues, but it is simply distortion to say that "Routleyism and Andersonianobelnapism are so dissimilar that it is misleading to apply a single label 'relevantism' to both" ([4], p. 45). Routley, who

describes himself as a relevantist, ([10], pp. 21–23), is hardly a *mere* paraconsistentist: it is not possible to read Routley's published work since [13] without grasping the quite central role played in it by relevance (e.g., [12]). Far from there being huge dissimilarity, the point of the Fine-Meyer-Plumwood-Routley-Urquhart semantics was that it offered an *explanatory* account of the Anderson-Belnap systems, and particularly the prized property of relevance (e.g., [13], or [11], p. 394). For that matter, the generality of that semantics, particularly the move to inconsistent and nonprime or incomplete theories, offered an explanatory account of the intuitions, and the limitations of those intuitions, behind relevantism, classicalism, paraconsistentism, intuitionism, connectivism, and modal logic. The particular application here, that the semantics made it clearer what were the options in dealing with *DS* and that one might propose a semantically based explanation of the illusory intuitiveness of *EDS*, seems to me to represent considerable progress over the original Anderson-Belnap account of *DS*.

The simple point against both Burgess and Read is this. Logic does not operate in a vacuum, but on deductive theories. While all the theories of a logic need to be closed under the deducibility relation of the logic, it is possible for some theories of the logic to be closed under additional rules as well, for instance *EDS*; and it would be surprising if we could not sometimes know this and exploit it. My further point against Burgess still stands: that the conditions under which *EDS* holds might be so normal that there is produced the illusion, even in intelligent and expert deducers, that it is valid. This is not to be disposed of by the methods of medieval Christianity invoked by Burgess in the opening quotation of [4].

NOTES

1. I do not rely on a rigid distinction between object language and metalanguage, which is one of the less satisfactory aspects of the classical paradigm. The distinction is used here only for expository purposes.

2. In point of fact this is a view with which I have recently become more sympathetic [7]. But Burgess was in no position to conclude this on the basis of my paper.

3. Cf. also Belnap and Dunn [3]. In the spirit of Belnap and Dunn, we might object: "But what if your metatheory is abnormal?" But what if? That does not count against the claim that *if* a theory *is* normal, *EDS* holds for it. And I take it that it is not so hard to believe that the present metatheory is normal.

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