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Czeżowski on Wild Quantity

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In [2] Czeżowski argued for a view similar to one held by Leibniz (see [4]) and, most recently, extensively defended in one version by Sommers (for example: [5]-[10]; something like it is also suggested in [1], pp. 239-241). It is the view that singular propositions are logically formulable either as universals or as particulars. Sommers calls this "wild" quantity. Czeżowski intended to substitute this view for the traditional, scholastic view that singular propositions are logically universal. They are so considered because their subject terms are distributed. Czeżowski presented reasons for thinking that singulars might also be construed particularly. He wrote:

> Thus singular propositions may be said to be a kind of hybrid; they have in common with universal propositions that they are subalternant to particular propositions and belong to the field of the relation of contrariety; while by their subalternation to universal propositions and by belonging to the field of the relation of subcontrariety they are akin to particular propositions. . . . A singular proposition may be interpreted either as universal, in view of its being subalternant to the particular, or as particular, in view of its subalternation to the universal. If one of the two contradictory singular propositions is interpreted in the first manner, then the other must be interpreted in the second. [2], p. 394

To summarize, singulars can be viewed as universals because: (i) they are subalternant to particulars, and (ii) they have contraries. Singulars can be viewed as particulars because: (a) they are subalterns of universals, and (b) they have subcontraries. Moreover, given any contradictory pair of singulars, it must be the case that one is universal and the other is particular.

Both the scholastics and moderns like Czeżowski and Sommers want the same thing—a categorical formulation for singular propositions. Why else would one seek a tacit quantity for singular subjects? Czeżowski summarized the logic of singulars in his "hexagon of opposition".

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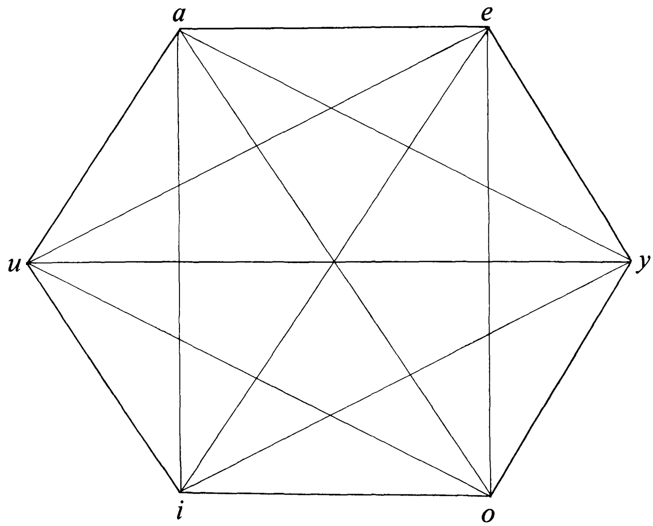
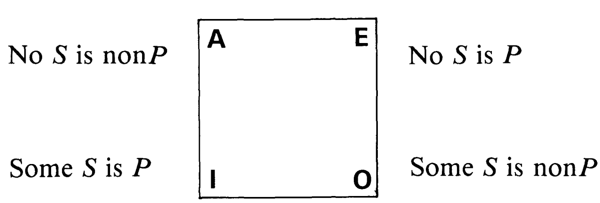


Here *u* is the form of a positive singular and *y* is its contradictory. The hexagon suggests that wild quantity is like a third kind of quantity along with universal and particular. Thus there are six, rather than four, categorical forms. That this is a mere illusion however requires a new look at universal quantity.

Sommers has provided arguments (especially in [10], Chap. 14) to show that universally quantified propositions may be defined by the negation of particulars. But under certain conditions they remain undefined. Thus, particulars are logically primitive in a way that universals are not. For example, 'Every *S* is *P*' is defined as 'No *S* is non*P*' (the negation, or contradictory, of 'Some *S* is non*P*') as long as either 'Some *S* is *P*' or its subcontrary 'Some *S* is non*P*' is true. When these both fail to be true, then their corresponding universals are undefined. Both particulars will fail to be true when the subject fails to refer (e.g., 'Some even prime greater than 7 is less than 40'), or when both particulars presuppose a false proposition (e.g., 'Some Trojans belonged to the Maoist party'), or when the subject is undetermined with respect to the predicate term (e.g., 'Some man will walk on Mars in 2009'), or when the predicate term is semantically inapplicable to the subject (e.g., 'Some numbers are green'). Cases in which a proposition fails to be true in either of its particular forms are "vacuous".

Let us construct a square for 'Some Spaniard is a philosopher' ('Some *S* is *P*').



Notice that a form like 'No *S* is *P*' is simply shorthand for 'Not: some *S* is *P*' (thus in English we say: 'No man is an island' = 'Not a man is an island' = 'It is not the case that a man is an island' = 'It is not the case that some man is an island' = 'Not: some man is an island'). So this square is constructed in terms of particular quantity, term negation ('non . . . '), and sentence negation, contradiction ('Not:'). Universal forms are definable as long as the sentences are nonvacuous. Given our sample reading they are nonvacuous; so we have

$$\text{Every } S \text{ is } P =_{df} \text{No } S \text{ is non}P$$
$$\text{Every } S \text{ is non}P =_{df} \text{No } S \text{ is } P \ .$$

Let us call these the *a* and *e* forms respectively (cf. [3]). Normally for nonvacuous cases, **A** = *a* and **E** = *e*. For vacuous cases, *a* and *e* are undefined. In effect, then, we have taken the term/sentence negation distinction to be formally more primitive than the universal/particular distinction. Modern logic recognizes only the latter distinction. Traditional logic recognized both distinctions, but took neither as more primitive than the other. By recognizing both, and taking one as primitive, we can make the formal vacuous/nonvacuous distinction. Moreover, an approach which makes quantity distinctions secondary would naturally hold more promise for one seeking to formulate singulars.

Consider now a square with singular propositions, for example, 'Socrates is wise' ('*S* is *W*'), etc.

Not: *S* is non*W*	Not: *S* is *W*
S is *W*	*S* is non*W*

For now we leave these forms unlabeled, though the law of excluded middle guarantees the contradictoriness of diagonally opposite pairs. Now the thesis of wild quantity says that a singular proposition is implicitly particular but entails its universal. If a singular proposition is vacuous, however (e.g., 'Socrates is axiomatic'), then its universal forms are undefined. So, for vacuous cases (and given singular '*S*') we have

Not: some *S* is non*X*	Not: some *S* is *X*
Some *S* is *X*	Some *S* is non*X*

But for nonvacuous cases (e.g., 'Socrates is wise') we have

1. **A** = *a* (nonvacuousity)
2. **E** = *e* (nonvacuousity)
3. *a* = **I** (singularity)
4. *e* = **O** (singularity).

Thus:

Socrates is wise ____________________ Not: Socrates is wise
A, *a*, **I** **E**, *e*, **O**

By labeling **A**, *a*, **I** as *u* and **E**, *e*, **O** as *y* we get the middle horizontal of Czeżowski's hexagon, and thus expose the illusion of wild quantity as any kind of third quantity.

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