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Objects and Existence: Reflections on Free Logic

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Abstract It is usual in free logic to regard free variables and constants as alike, and to distinguish them semantically from bound variables. In the present treatment, by contrast, variables are handled uniformly, while individual constants are regarded as surface artifacts to be fleshed out for deep-structure predicate constructions. Philosophical reasons are presented to support this interpretation; and the logical construction is described informally. The main idea involves a threefold refinement of Quine's Russellian treatment of proper names. First, the existence clause is eliminated. Second, uniqueness is made a formal aspect of the symbolization of singular predicates. Third, no scope distinction is marked in the surface structure, as with Russell's iota-notation, so that, although a sentence involving a singular term might be read in different ways, corresponding to a difference in the way scope is reckoned, the sentence is treated in the logic as if that scope is unknown.

Singular existence claims pose a significant problem for the view that 'exists' is a first-order predicate. It is widely believed that if 'exists' is a first-order predicate, then a sentence like 'Neptune exists', if meaningful at all, must express a necessary truth, or perhaps a trivial truth, but it certainly is not a substantial astronomical discovery. Frege thought so.[1] Russell did too. This belief, however, is incorrect, and Russell's strategy of analyzing an apparent subject expression as a deep structure predicate construction is sufficient to demonstrate this. As long as we deny that the surface 'grammatical' subject of an informative singular existence claim is also its deep 'logical' subject—as with Russell's construction for definite descriptions and Quine's modification for proper names—we can maintain the view that 'exists' is a legitimate first-order predicate.

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Free logic, however, appears to offer more, for it purports to treat an informative singular existence claim as a genuine subject/predicate structure of the form *Fa*, so that the surface 'grammatical' subject just is its 'logical' subject. How is this possible? The answer, of course, is that the subject/predicate structure is an illusion. The Russell/Quine construction was motivated by the desire to have the logical syntax of English singular terms reflect their true semantic role. Free logic, however, seeks to save the appearance of subject/predicate form: the coincidence of logical form with semantic role is dropped, and logical syntax is identified with surface structure. To be sure, Russell also felt the tug of preserving appearances, and so he introduced the iota-operator to make the underlying predicate construction look like a surface subject. Now the iota-operator, being an eliminable surface construction, is only an afterthought to Russell's main analysis; by the very same token, however, it is a paradigm of the sort of enterprise that engages free logic. Below, we show how the constants of free logic can be regarded as surface artifacts masking an underlying predicational (i.e., Russellian) treatment of singular terms. Viewed this way, free logic is not a competitor to the Russell/Quine construction, but a refinement of it.

1 Recall Quine's well-known opening paragraph in "On what there is":

> A curious thing about the ontological problem is its simplicity. It can be put in three Anglo-Saxon monosyllables: 'What is there?' It can be answered, moreover, in a word—'Everything'—and everyone will accept this answer as true. However, this is merely to say that there is what there is. There remains room for disagreement over cases; and so the issue has stayed alive down the centuries. ([19], p. 1)

This passage is puzzling because Quine provides for no intelligible utterance of the incontestable

(1) Everything exists.

There is not the slightest hint that 'exists' is a first-order predicate (unlike [18], p. 151, where Quine admits the possibility). To the contrary, in this passage Quine gives the appearance of embracing Russell's view that *existence* is expressed by the quantifiers: to say that *F*'s exist is to say that *there are F*'s. And, since the quantifier translates the indefinite pronoun 'everything' in (1), there is no way to capture 'exists'.

The intelligibility of (1) demands that 'exists' be treated as a first-order predicate. Let us do so by introducing the existence predicate into first-order logic along with an axiom assuring us that everything exists:

(2) $(x)\Sigma x$.

'Σx' is the universal predicate, true of each and every element of the domain, so that its extension is the domain itself. (2) is our symbolic translation of (1), and the vacuity of (1) is reproduced in (2)'s being a logical truth. (Note that it is true even in the empty domain.)

The disagreement over cases is handled conservatively. First, general existence claims. The standard way of saying that Achaeans exist is to say that *there are such things as* Achaeans. So, 'Achaeans exist' is usually symbolized as

'$(\exists x)Ax$'.[2] However, we can symbolize it equivalently as '$(\exists x)(Ax \ \& \ \Sigma x)$'. 'Achaeans do not exist' usually comes out as '$\sim(\exists x)Ax$', but we symbolize it as '$\sim(\exists x)(Ax \ \& \ \Sigma x)$'.[3] Singular existence claims should be handled in an analogous way. To say that Odysseus exists is to say that *there is such a thing as* Odysseus. So,

(3) Odysseus exists

becomes '$(\exists x)ox$', or, employing the existence predicate, '$(\exists x)(ox \ \& \ \Sigma x)$'. For the slightly more problematic

(4) Odysseus does *not* exist,

we invoke a scope ambiguity. With the name given *large* scope, we have the (logically) false

(4a) Something is Odysseus and does *not* exist,

i.e., '$(\exists x)(ox \ \& \ {\sim}\Sigma x)$'; with the name given *small* scope, however, we have the (as a matter of fact) true,

(4b) *It is not the case that* something is Odysseus and exists,

i.e., '$\sim(\exists x)(ox \ \& \ \Sigma x)$'. To be sure, we have the resources to handle these assertions without the existence predicate. But the important point to note here is that admitting the existence predicate does not lead to any logical incoherence, as has been widely supposed.

There is a small drawback to this analysis in the singular case: when treating the proper name 'Odysseus' as a predicate, the uniqueness of the denotation is lost. We can restore uniqueness, however, by adopting Quine's trick of analyzing a proper name as a Russellian description. Russell, recall, treated a statement of the form *The* ϕ *is* ψ as a conjunction: (a) there exists something that is ϕ and (b) there exists at most one thing that is ϕ and (c) whatever is ϕ is ψ. Existence claims, however, are handled differently. Clause (c) is dropped, so *The* ϕ *exists* becomes: (a) There exists something that is ϕ and (b) there exists at most one thing that is ϕ. 'The present King of France exists', for example, becomes 'One and only one thing is presently King of France'. By analogy, Quine treats (3) as 'One and only one thing is Odysseus', i.e.,

(3a) $(\exists x)(ox \ \& \ (y)(oy \supset x = y))$.

But clause (c) does not have to be dropped for existence claims. With the existence predicate 'Σx' in hand, we can reinstate the clause to get

(3b) $(\exists x)((ox \ \& \ (y)(oy \supset x = y)) \ \& \ \Sigma x)$.

There is a persistent belief that clause (c) is dropped here because 'exists' is not a first-order predicate. This is an error. It is dropped because it is *redundant*: (3a) and (3b) are logically equivalent.

The scope ambiguity of the problematic (4) is apparent on this analysis. The large scope interpretation,

(4aa) One and only one thing is Odysseus and does *not* exist,

is logically false; the small scope interpretation, (4bb) *It is not the case both that* one and only one thing is Odysseus and exists,

is contingently true. This latter provides for our ability to formulate meaningful singular denials of existence.

So, with only a minor modification in Quine's analysis, we can treat 'exists' as a first-order predicate without encountering the problems usually thought to stand in the way. (Substantially the same point is made in [4], pp. 278ff.) We find that the existence predicate is redundant in all but *indefinite* existence claims—'Everything exists', 'Something exists', 'Nothing exists'—i.e., claims that require a quantifier to translate the indefinite pronoun.

2 Why have philosophers been so blind to the obvious construction we have just given? Perhaps the most serious philosophical obstacle to the view that existence is a property of objects is a problem we have alluded to already, namely, the ancient *Paradox of Nonbeing* or, as it is sometimes known, *Plato's Beard.* The puzzle goes like this. We can only refer to things that exist; so, if we take singular existential claims to be of subject/predicate form—where the subject stands for something and the predicate says something about that for which the subject stands—then all positive claims of existence turn out to be trivially true (if meaningful) and all negative claims of existence turn out to be trivially false (if meaningful). (For a good discussion of this paradox, see [3].) The modern deflationist[4] response, which we associate most closely with the ideas of Russell and Frege, is to treat the argument as a *reductio* of the premise that existential claims are of subject/predicate form: there is nothing that fails to exist, and so there is nothing to refer to in so claiming.

The deflationist *reductio* entitles us to conclude (taking (3) above as our example) that it cannot be the case both that 'Odysseus' is the subject of the assertion and that 'exists' is the predicate. The received view, however, infers something stronger, namely that 'Odysseus' is not the subject *and* 'exists' is not the predicate. This conclusion is unwarranted: it is an egregious error to infer '$\sim P$ & $\sim Q$' from '$\sim (P$ & $Q)$'. Let us look at the four possible analyses of (3):

(a) 'Odysseus' is subject; 'exists' is predicate.
(b) 'Odysseus' is subject; 'exists' is *not* predicate.
(c) 'Odysseus' is *not* subject; 'exists' is predicate.
(d) 'Odysseus' is *not* subject; 'exists' is *not* predicate.

The *reductio* only eliminates (a). (b) has no plausibility. This leaves (c) and (d). So, 'Odysseus' cannot be the subject of (3), *whether it is true or false that 'exists' is the predicate.* If there is some reason for rejecting 'exists' as a predicate, it must be found elsewhere.

The Paradox of Nonbeing therefore does not speak directly to the issue of whether 'exists' is a predicate, and to that extent it actually gives us little guidance on this controversial issue. Unfortunately, philosophers have constructed a confusing synergy between the Paradox of Nonbeing and the *Ontological Argument* for God's existence. There is a widely entrenched belief that, to avoid the result of the Ontological Argument, one must deny that existence is a property of objects. The Paradox of Nonbeing has been viewed as confirming the logical correctness of this view and taking it one step further. Indeed, it has become standard to argue that 'Odysseus' cannot be the subject of (3) *because* 'exists' is not the predicate. Here is Ryle in "Systematically misleading expressions" making precisely this connection:

> Since Kant, we have, most of us, paid lip service to the doctrine that 'existence is not a quality' and so we have rejected the pseudo-implication of the ontological argument: 'God is perfect, being perfect entails being existent, God exists.' For if existence is not a quality, it is not the sort of thing that can be entailed by a quality.
>
> But until fairly recently it was not noticed that if in 'God exists' 'exists' is not a predicate (save in grammar), then in the same statement 'God' cannot be (save in grammar) the subject of predication. The realization of this came from examining negative existential propositions like 'Satan does not exist' or 'Unicorns are non-existent'. If there is no Satan, then the statement 'Satan does not exist' cannot be about Satan in the way in which 'I am sleepy' is about me. Despite appearances the word 'Satan' cannot be signifying a subject of attributes. ([20], p. 42)

Accordingly, as the view that 'exists' is not a predicate was slowly eroded (see especially [17]), and since this was perceived as being *the* reason for denying that the ostensible subject of an existential claim is the real or logical subject of the claim, so it appeared to some that no philosophical reason stood in the way of holding that (3), although false, could be conceived of as a straightforward subject/predicate statement. This is explicitly Jaakko Hintikka's motivation for the development of free logic:

> It may be objected that any such formalization [of '*a* exists'] will involve the illicit assumption that 'existence is not a predicate'. Fortunately, in a recent note by Salmon and Nakhnikian the standard prima facie objections to treating 'existence as a predicate' have been effectively disposed of. Whether deeper interpretational objections are forthcoming or not, none have been put forward so far; and I doubt very much whether they would at all affect the substance of what we are saying here.
>
> . . . Thus there can be no objection to an attempt to find a formal counterpart to the phrase '*a* exists'. . . . ([9], p. 29)

The shakiness of Hintikka's philosophical underpinnings for free logic are evident. He has focused on the wrong philosophical problem: the classical argument itself requires that 'Odysseus' not be the subject of (3), and this must be so independent of the standard deflationist view that 'exists' is not the predicate.

There is a second way in which the deflationists have muddied the water. Again, I quote from Ryle:

> Take now an apparently singular subject as in 'God exists' or 'Satan does not exist'. If the former analysis was right, then here too 'God' and 'Satan' are in fact, despite grammatical appearances, predicative expressions. That is to say, they are that element in the assertion that something has or lacks a specified character or set of characters by which the subject is being asserted to be characterized. 'God exists' must mean what is meant by 'something, and one thing only, is omniscient, omnipotent and infinitely good' (or whatever else are the characters summed in the compound character of being a god and the only god). And 'Satan does not exist' must mean what is meant by

> 'nothing is both devilish and alone in being devilish', or perhaps 'nothing is both devilish and called "Satan",' or even '"Satan" is not the proper name of anything'. To put it roughly, 'x exists' and 'x does not exist' do not assert or deny that a given subject of attributes x has the attribute of existing, but assert or deny the attribute of being x-ish or being an x of something not named in the statement. ([20], pp. 43–44)

Ryle gives expression to a deep-rooted prejudice that stems from some pronouncements of Frege and Russell to the effect that proper names—*genuine* proper names—cannot be predicates. (We discuss Frege's reasons in detail in [16].) For Ryle, such ostensible subjects as 'God' and 'Satan' are disguised descriptions, explained away *via* paraphrase. It is to this particular analysis of existential claims that Hintikka poses free logic as an alternative:

> Hence such sentences as 'Homer does not exist' can be translated into our symbolism without any questionable interpretation of the proper name 'Homer' as a hidden description. If anybody should set up a chain of arguments in order to show the nonexistence of Homer, we could hope to translate it into our symbolism without too many clumsy circumlocutions. In this sense, the use of an expression for existence is not only possible but serves a purpose. ([9], p. 34)

Note the last sentence of the quotation which reflects, once again, Hintikka's confusion about the reasons for including 'exists' as a predicate.

Now, Hintikka is certainly correct in questioning the procedure Russell had advocated, i.e., of regarding a proper name like 'Odysseus' as a disguised or truncated description, perhaps 'the most cunning of all Achaeans'. His skepticism about this treatment of names anticipates recent attacks on the so-called "Frege/Russell Description Theory of Names". But this criticism of Russell does not touch Quine, for whom 'x is Odysseus', like 'x is an Achaean', is a predicate in the classic Fregean sense of an expression with a hole in it that can be filled by a proper name to form a sentence. Russell could not be satisfied to leave the issue like this. He thought that the name, since it is predicative, must connote a set of properties that picks out the individual uniquely; accordingly, the name must be regarded as a complex, definable predicate. Quine has no such commitment.[5]

In any event, there is no philosophical or logical problem in taking 'exists' to be a first-order predicate in classical logic. And, adopting Quine's analysis of proper names, we can readily "deny the existence of individuals". So, the philosophical virtues of free logic are somewhat more circumscribed than Hintikka would have us believe.

3 The technical virtues of free logic, however, have not been fully appreciated. Whereas in classical first-order logic individual constants all denote and Existential Generalization permits us to infer

$$Fa$$

from

$$(\exists x)Fx,$$

in free logic this inference is permitted only if the following condition is also satisfied:

a exists.[6]

Free logic therefore enables us to reproduce the subject/predicate structure of a statement like

(5) Odysseus was set ashore at Ithaca while sound asleep,

in first-order notation without engaging the problematic inferences normally associated with that structure. Whereas in classical logic (5) logically implies

(6) There *exists* something that was set ashore at Ithaca while sound asleep,

as well as (3), in free logic it doesn't: (5) can be true or false whatever truth value is assigned to (6) or to (3). And even (3) can be assigned a subject/predicate structure without its turning out to be either a logical truth or a logical falsehood. Consistency and completeness results have been obtained for free logic (e.g., see [5]), so it would seem that individual constants can be set free from the classical constraint that they denote, and that singular existential claims can be treated as subject/predicate statements.

There remain, of course, philosophical questions about the subject/predicate form free logic assigns to (3) and to (5). Consider (5): Are we to suppose that 'Odysseus' is referring to something and the rest of the sentence is saying about *him* that *he* was set ashore at Ithaca while sound asleep? Or consider (3): Are we to suppose that 'Odysseus' is referring to *something*, that the rest of the sentence is saying about *him* that *he* has the property of existence, and finally that (3) is neither logically true nor logically false? The classical answer has been "No" in each case. What does free logic say? van Fraassen and Lambert side with the classical view:

> In our development [of free logic], *talk* about non-existent objects is just that—"talk" is what is stressed. "Non-existent" object, for us, is just a picturesque way of speaking devoid of any ontological commitment. In this regard our own development is motivated by what Russell called "a robust sense of reality". ([13], p. 200)

Hintikka, however, tantalizes us with the more exciting prospect of referring to a nonexistent individual and saying of that individual that it does not exist:

> Existence can be a predicate in the sense that it is possible to use a formal expression containing the free individual symbol *a* as a translation of the phrase '*a* exists', without running into any logical difficulties.
>
> . . . we can now meaningfully deny the existence of individuals; formulas of the form $\sim(Ex)(x = a)$ are not all disprovable any more. ([9], p. 34)

But instead of a philosophical follow-up, Hintikka retreats to surface syntax. He does the same in defending Quine's well-known ontological slogan, *To be is to be the value of a variable*:

> No matter how Quine himself originally conceived of the meaning of the dictum, it seems to me that by far the most important way of interpreting it is

> to take it to say that formulas of the form $(Ex)(x = a)$ serve as a formalization of the common-sense phrase 'a exists'. For what $(Ex)(x = a)$ says is that the individual referred to by a is identical with one of the values of the bound variable x; and being identical with one of its values is obviously the same as simply being one of the values. ([9], pp. 40–41)

Once again, Hintikka dangles the idea of referring to nonexistent individuals. But, surely, $(\exists x)(x = a)$ cannot *say* that "the individual referred to by a is identical with one of the values of the bound variable x" if a is a nondenoting singular term. For it doesn't refer to any individual. And, by the same token, $\sim(\exists x)(x = a)$ cannot truthfully *say* that the individual referred to by a is *not* identical with one of the values of the bound variable x. What could it possibly mean to say that *a given individual* is not the value of a bound variable when there is *nothing* (on this view) that is not a value of a bound variable?

Free logic stands with Russell rather than with Meinong in holding that there is nothing that fails to exist: the domain of the quantifiers *is* the domain of existents.[7] The classical semantic account of an atomic statement Fa is that a denotes something, and Fa is true if the item a denotes has the property F denotes. Free logic ascribes the form Fa to statements like (3) and (5), but rejects the classical account: we cannot suppose that 'Odysseus' denotes an individual and that the rest of the sentence says something about him, for the term denotes nothing. Why, then, impose the form Fa on (5)? The answer is clear: to create an *illusion*. (The same is true for the subject/predicate assignment to (3).)

Russell had argued that descriptions occurred in deep structure only in predicate constructions. Accordingly, any sentence of the form *The* ϕ *is* ψ is symbolized in its Russellian form without a subject expression *The* ϕ: 'ϕx' is the underlying predicate Russell invokes. Russell could have completed his analysis at this point. But he chose not to. For he also sought to maintain the surface illusion of subject/predicate form in symbolic notation, and to this end he introduced his iota-operator (this aspect of his theory is stressed in [10]). Russell could thus engender the illusion of referring to things that do not exist without committing himself to the Meinongian ontology that results from taking descriptions to be genuine subject expressions. Some of those who developed free logic confused illusion with reality, and thought that it represented more of a philosophical advance, more of a challenge to the heart of Russell's treatment of descriptions (and especially to Quine's extension to proper names) than it actually is. Our suggestion is that we construe free logic as attempting to engender an illusion similar to the one Russell sought to create with his iota-notation, i.e., of a surface subject masking a deep-structure predicate. Only—and here is the *real* advance—free logic does it better. To develop the connection with Russell, however, we need a Russellian-type predicative analysis of proper names that avoids the existential commitment Russell's analysis carries with it. We now turn to this.

4 Proper names are usually treated as individual constants in first-order logic, but there are alternatives to this treatment. Quine, as we have seen, suggests that we analyze them as Russellian descriptions. So,

(8) John is a man,

becomes 'One and only one thing is John and is a man', which expands to the conjunction of the three claims:

(8a) There is at least one thing that is John
(8b) There is at most one thing that is John
(8c) Whatever is John is a man.

That is, the analysis includes an explicit *existence* claim (8a) and an explicit *uniqueness* claim (8b). (Note that, because of the first two clauses, (8c) can be replaced by 'Something is John and is a man'.)

Let us modify this analysis. First, the uniqueness clause. Unlike individual constants, which are assured by their form alone a unique denotation, predicates can denote an indefinite number of things; (8b) is required to ensure that the predicate 'x is John' denotes uniquely. We prefer to make uniqueness of denotation a *formal* feature of the notation, as it is with individual constants. Let us, therefore, expand the usual list of predicate letters in first-order logic to include lower case 'f', 'g', 'h', etc., which will be governed by the axiom

$$(x)(y)(fx \supset (fy \supset x = y))$$

to ensure that each is true of at most one object. With these *singular predicates*[8] in hand, we can safely drop the uniqueness clause (8b). Then, since our primary intention is to drop the existence requirement Russell explicitly imposed on the logic of singular terms, we will also drop the existence clause (8a).[9] With the existence and uniqueness clauses having been attended to, we turn to (8c).

If we follow the pattern of (8c), then, given the modifications we have just made, a sentence like 'Odysseus is an Achaean' would be symbolized as '$(x)(\mathrm{o}x \supset \mathrm{A}x)$'. The general principle for the symbolization is to take a sentence containing the proper name a to be about whoever or whatever is denoted by a. So, 'Odysseus is shrewd' becomes '(x)(x is Odysseus $\supset$ x is shrewd)', and 'Everybody fears Odysseus' becomes '$(x)(y)$(x is Odysseus $\supset$ y fears x)'. This analysis soon leads to difficulties. The point is immediate with existence claims. It is as incorrect to take the singular claim (3) to be the universal affirmative

(9) $(x)(\mathrm{o}x \supset \Sigma x)$

as it is to take the general claim

(10) Unicorns exist

to be the universal affirmative

(11) $(x)(\mathrm{U}x \supset \Sigma x)$.

Both are logically true, because '$(x)\Sigma x$' is a logical truth. Whatever is a unicorn must exist, since everything exists[10]; but, of course, there are no unicorns. So nontrivial assertions of existence (whether singular or general) cannot be treated as universal affirmatives; by the same token, nontrivial denials of existence (whether singular or general) cannot be treated as particular negatives. The interesting positive claim that unicorns exist is that *there is at least one*; the interesting negative claim that unicorns don't exist is that *there are none.* The desired translation for (3) is not (9), but

(12) $(\exists x)(\mathrm{o}x \,\&\, \Sigma x)$,

and the desired translation for (10) is not (11), but

(13) $(\exists x)(\mathrm{U}x \,\&\, \Sigma x)$.

'Odysseus does not exist' and 'Unicorns do not exist' are translated, respectively, as the negation of (12) and the negation of (13).

The situation is the same for ordinary predicates. 'Achaeans are men' is ordinarily translated as '$(x)(\mathrm{A}x \supset \mathrm{M}x)$'. But if 'Achaeans exist' were handled similarly as '$(x)(\mathrm{A}x \supset \Sigma x)$', it would turn into a logical truth, since everything exists. 'Achaeans are not men' goes in as '$\sim (x)(\mathrm{A}x \supset \mathrm{M}x)$'. But, if we handled 'Achaeans do not exist' in the same way, it would become '$\sim (x)(\mathrm{A}x \supset \Sigma x)$', which is a logical falsehood. (This is especially vivid when phrased in the logically equivalent form '$(\exists x)(\mathrm{A}x \,\&\, {\sim}\Sigma x)$'. Everything exists, so there can be nothing that is an Achaean and yet fails to exist.)

In general, there are two distinct symbolizations for statements that are classically of the form *Fa*: $(x)(ax \supset Fx)$ and $(\exists x)(ax \,\&\, Fx)$. Now, if a unique *a* exists, the symbolizations are logically equivalent, i.e.,

$$(\exists x)ax \supset [(x)(ax \supset Fx) \equiv (\exists x)(ax \,\&\, Fx)],$$

so there is no logical point in favoring one translation over the other.[11] But if *a* does not exist, a difference emerges. So, *both* ways of treating proper names are needed. Consider, for example, the two translations of

(14) Odysseus is a unicorn,

namely:

(15) $(x)(\mathrm{o}x \supset \mathrm{U}x)$
(16) $(\exists x)(\mathrm{o}x \,\&\, \mathrm{U}x)$.

If Odysseus exists, (15) and (16) have the same truth value, one that depends entirely on whether the predicate is true or false *of him*. If Odysseus does not exist, however, the situation is different: there is no "objective reality" to hold on to, and (15) turns out to be trivially true while (16) turns out to be trivially false. Insofar as we are inclined to assign any truth value to (14) in this case, it will be the one which is (arbitrarily) determined by the standard story. Odysseus is supposed to have been a man, not a unicorn. So (14) is false. And this would seem to dictate that we arbitrarily symbolize (14) as (16)[12] and 'Odysseus is not a unicorn' as '$\sim (\exists x)(\mathrm{o}x \,\&\, \mathrm{U}x)$'. But then we must have already determined the truth value of the statement before we can symbolize it correctly, which is a severe drawback. We prefer a notation that enables us to fudge the issue of truth value for statements containing nondenoting singular terms.

5 A complex English construction containing a definite description must be symbolized in Russell's iota-notation either with large scope or with small scope. English, however, provides a notation which is inherently ambiguous ('The present King of France is not bald'), allowing us to speak without forcing us to identify the scope of the description. The surface-structure singular term 'The present King of France' is ultimately cashed in (if Russell is correct) for either a large

scope or a small scope predicate construction, but we are able to use it without committing ourselves one way or the other. If we construe the two translations, (15) and (16), as, respectively, underlying small scope and large scope predicate constructions for the surface atomic structure (14), we shall have solved our problems of the last section and, in effect, provided a satisfying rationale for the syntax of free logic.

Let us introduce some notation. We will take

$$[\theta a]Fa$$

to be short for

$$(x)(ax \supset Fx).$$

We will take

$$F[\theta a]a$$

to be short for

$$(\exists x)(ax \,\&\, Fx).$$

The first, $[\theta a]Fa$, the *small scope* reading, carries no existential commitment; the second, $F[\theta a]a$, the *large scope* reading, does.

We can see the impact of this scope distinction in the Ontological Argument, a version of which goes like this.

God has all perfections, and existence is a perfection, so God exists.

Let us phrase the argument this way:

God is perfect
Whatever is perfect has existence
———————————————
God exists.

Let '$\mathrm{g}x$' be the singular predicate 'x is God' and let '$\mathrm{P}x$' be the predicate 'x is perfect'. Now, the second premise is symbolized as:

$$(x)(\mathrm{P}x \supset \Sigma x).$$

But there are two ways of symbolizing the first premise, and also two ways of symbolizing the conclusion. With the first premise assigned the small scope reading, '$[\theta \mathrm{g}]\mathrm{Pg}$' (i.e., '$(x)(\mathrm{g}x \supset \mathrm{P}x)$'), the conclusion that validly follows is the small scope reading, '$[\theta \mathrm{g}]\Sigma \mathrm{g}$' (i.e., '$(x)(\mathrm{g}x \supset \Sigma x)$'). This conclusion is trivially true because everything exists. The interesting claim, the one with bite, that is thought to be inferred is the large scope reading, '$\Sigma[\theta \mathrm{g}]\mathrm{g}$' (i.e., '$(\exists x)(\mathrm{g}x \,\&\, \Sigma x)$'); and this requires the large scope reading of the premise '$\mathrm{P}[\theta \mathrm{g}]\mathrm{g}$' (i.e., '$(\exists x)(\mathrm{g}x \,\&\, \mathrm{P}x)$'), thereby begging the question of the argument. There is, then, no reason to fear that in taking existence to be a first-order property the Ontological Argument will go through.

Let us extend our notation. We will introduce "individual constants" that will enable us to create an illusion: 'Fa', although it looks atomic, is actually either '$[\theta a]Fa$' or '$F[\theta a]a$', but there is nothing about the sentence that tells us which it is. Let us, for example, introduce the individual constant 'o' for Odysseus. (We trust that the context will forestall confusing this symbol with the one introduced earlier, viz. '$\mathrm{o}x$' for the predicate 'x is Odysseus'.) Then we can express (3) as 'Σo'. The small scope reading, '$[\theta o]\Sigma o$', expands to '$(x)(ox \supset \Sigma x)$': this is a logical truth, the gutless claim that is true even if there is nothing that is Odysseus. The large scope reading, '$\Sigma[\theta o]o$', expands to '$(\exists x)(ox \,\&\, \Sigma x)$': this is the important claim, true if something is Odysseus, and false if nothing is. So, the surface-structure "atomic statement" containing the "individual constant" 'o' is expanded into one of two possible deep-structure constructions involving the singular predicate 'ox'. There is nothing about (3) to tell us which of the two readings is assigned.

Let us use this new notation to symbolize the Ontological Argument. Let us use 'g' as our constant for 'God', so the argument becomes:

$$\frac{\begin{array}{l}\mathrm{Pg} \\ (x)(\mathrm{P}x \supset \Sigma x)\end{array}}{\Sigma \mathrm{g}}.$$

Is this argument valid? On the classical understanding of the constants, the argument is surely valid. But with the current understanding of the constants, it is invalid, for we have no information about the scopes of the underlying predicates in the first premise 'Pg' and in the conclusion 'Σg'. If each "atomic" statement were expanded to expose an underlying predicate construction that had small scope, the argument would be valid. If each "atomic" statement were expanded to expose an underlying large scope predicate construction, the argument would be valid. But the possibility remains of expanding the premise to a small scope construction and the conclusion to a large scope construction, and that would make the argument invalid. So, the argument above is invalid because there are readings on which the premises come out true and the conclusion comes out false.

We have a way, then, of understanding proper names, which makes them quintessentially syncategorematic expressions. They have no meaning in isolation, but only in context. Their use masks a scope distinction that must be taken account of at the most elementary level, namely, that of an ostensible atomic statement. The constant is eliminable either for a large scope or a small scope introduction of a singular predicate, but it is not part of the logical form of the statement to say which. This is, in effect, the role of individual constants in free logic.

6 The *atomic-looking* sentences of free logic mask an underlying predicate construction. Each such sentence is replaceable by a constant-free sentence, one that contains a predicate construction corresponding to the original's constant. A *storybook* function will pair sentences containing constants with constant-free sentences, in essence, by assigning a scope to the atomic-looking sentence. The constant-free expansion has a truth value in a given model under an interpretation; the atomic-looking sentence paired with it will share the same truth value. If 'c' is a nondenoting proper name then in some cases 'Fc' will turn out true (being paired by a storybook with the small scope reading), and in some cases 'Fc' will turn out false (being paired by a storybook with the large scope reading). But, if 'c' designates an element of the domain, 'Fc' always has the same truth value since scope is irrelevant.[13] A sentence is *valid* if it comes out true for every storybook under every interpretation in every model.

There are two ways in which this characterization of free logic differs from many familiar in the literature. First, we assume a sharp distinction between free variables, on the one hand, and constants on the other.[14] Our second, and related difference, is much more controversial: we take

(17) $(x)(x = x)$

to be valid in free logic, but not

(18) $a = a$.

Our reasoning is straightforward, given the notion of validity we are working with. There are four possible ways of expanding (18) to constant-free sentences:

(18a) $(y)(ay \supset (x)(ax \supset x = y))$
(18b) $(\exists y)(ay \ \&\ (x)(ax \supset x = y))$
(18c) $(y)(ay \supset (\exists x)(ax \ \&\ x = y))$
(18d) $(\exists y)(ay \ \&\ (\exists x)(ax \ \&\ x = y))$.

If nothing is a, (18a) and (18c) turn out to be true. But, because of the existential quantifier out front, (18b) and (18d) will be *false* if nothing is a. So, since there are constant-free expansions on which it comes out false, (18) cannot be valid. This apparently violates deep intuitions on the part of some theorizers.[15] But we find no compelling reason to believe that (18) is a logical truth. Since Cicero exists, the object Cicero must be identical with himself. Pegasus, however, does not exist, and so the parallel argument, viz. that the object Pegasus must be identical with himself, does not get off the ground: there is no individual Pegasus. It is plausible, then, to hold that 'Cicero = Cicero' is true, and also that 'Pegasus = Pegasus' is false. The falsity of (18) does not lead to inconsistency; it simply implies that a does not exist.

Let us now look, informally, at those formulas which represent the salient difference between classical and free logic. We can easily see that the classically valid

(19) $(x)Fx \supset Fa$

fails in free logic, because one of its instances, '$(x)\Sigma x \supset \Sigma a$', is invalid: '$(x)\Sigma x$' is a logical truth while 'Σa' is not. We are unable to disprove

(20) $Fa \supset (\exists x)Fx$

in the same way because '$\Sigma a \supset (\exists x)\Sigma x$' is valid since '$(\exists x)\Sigma x$' is. But the fact that '$\sim\Sigma a \supset (\exists x)\sim\Sigma x$' is a logical falsehood does the job.[16] The claim 'a exists' has frequently been expressed in free logic as '$(\exists x)(x = a)$'. This must be understood to be the large scope existence claim '$\Sigma[\theta a]a$', which we will symbolize as '$\Sigma_1 a$'.[17] That is,

$$[Fa \ \&\ \Sigma_1 a \ \&\ (x)(Fx \supset Gx)] \supset Ga$$

is a logical truth, but

$$[Fa \ \&\ (x)(Fx \supset Gx)] \supset Ga$$

is *not*.[18] The inference goes through if we are assured that *a exists*, but not otherwise. Because

(21) $\Sigma_1 a \supset [F[\theta a]a \equiv [\theta a]Fa]$

is valid in free logic, so that movement between constant and variable is no longer problematic, the valid free logic correlates to (19) and (20) are, respectively,

(19a) $[(x)Fx \ \& \ \Sigma_1 a] \supset Fa$
(20a) $[Fa \ \& \ \Sigma_1 a] \supset (\exists x)Fx.$

(20) is not valid in free logic because it is not true on every reading of the constant, i.e., on both the large scope and small scope readings. The two scope readings for (20) are

(22a) $[\theta a]Fa \supset (\exists x)Fx$

and

(22b) $F[\theta a]a \supset (\exists x)Fx;$

and these get expanded, respectively, to

(22aa) $(x)(ax \supset Fx) \supset (\exists x)Fx$

and

(22bb) $(\exists x)(ax \ \& \ Fx) \supset (\exists x)Fx.$

Now, (22bb) comes out true no matter how we interpret the predicates '*ax*' and '*Fx*'. Not so for (22aa): if nothing is *a* and nothing is *F*, (22aa) comes out false. And so long as there is the possibility of falsehood on an interpretation, (20) cannot be a logical truth.

Now consider (20a). The two readings of '*Fa*' are, once again, '$[\theta a]Fa$' and '$F[\theta a]a$'. The large scope existence claim '$\Sigma_1 a$' (i.e., '$(\exists x)(ax \ \& \ \Sigma x)$') is logically equivalent to the simpler '$(\exists x)ax$'. So, the two possibilities are

(23a) $([\theta a]Fa \ \& \ (\exists x)ax) \supset (\exists x)Fx$

and

(23b) $(F[\theta a]a \ \& \ (\exists x)ax) \supset (\exists x)Fx.$

These are expanded, respectively, to

(23aa) $((x)(ax \supset Fx) \ \& \ (\exists x)ax) \supset (\exists x)Fx$

and

(23bb) $((\exists x)(ax \ \& \ Fx) \ \& \ (\exists x)ax) \supset (\exists x)Fx.$

And, as the reader can readily verify, these are both true no matter how we interpret the predicates *ax* and *Fx*. So far, so good.

Consider, now, (19). We replace '*Fa*' by

(23a) $(x)Fx \supset [\theta a]Fa$

and

(23b) $(x)Fx \supset F[\theta a]a.$

These expand, respectively, to

(23aa) $(x)Fx \supset (x)(ax \supset Fx)$

and

(23bb) $(x)Fx \supset (\exists x)(ax \,\&\, Fx)$.

Although (23aa) comes out true for any '*ax*' and '*Fx*', (23bb) does not: everything might be *F* even though nothing is *a*. So, (19) is not valid.

7 Here is a brief sketch of our formal system of free logic, **FL**. The vocabulary of the language of **FL** is a quintuple ⟨**V**, **P**, **SP**, **L**, **C**⟩. **V** is an infinite set of *variables*, $v_1, \ldots, v_n, \ldots$. **P** is an infinite set of *predicates of degree n*, with P^{ij} the *i*-th predicate of degree *j*. **SP** is an infinite set of *singular predicates of degree 1*, with p^i being the *i*-th member. **L** is the set of *logical signs*, including:), (, ⊃, and ~. All others are assumed to be defined in the usual way. **C** is an infinite set of *constants*, the *i*-th constant being the numeral *i*. The sets **V**, **P**, **SP**, and **L** are pairwise disjoint. The set **F** of *generic predicates* is the union of **P** and **SP**. The set **T** of *terms* is the union of **V** and **C**. We will use the letters t, t', etc., to represent terms. We will use the usual notation *F*, *G*, *H* for predicates, reserving the special notation and the term 'singular predicate' when we need to speak precisely of them. We assume the usual notation available to us for our informal discussion of the language.

The set **A** of *atomic sentences of* **FL** will contain anything of the form

$F^{in}v_1 \ldots v_n$

i.e., a predicate of degree *n* followed by *n* variables. (For the special case of the identity predicate, instead of $F^{=2}v_1v_2$ we will use the more usual $v_1 = v_2$.) The set $\mathbf{S}_{\mathrm{CF}}$ of *constant-free sentences* of **FL** is defined inductively:

(a) Every atomic sentence is a sentence
(b) If A and B are sentences, so are $(A \supset B)$ and $\sim A$
(c) If x is a variable and A is a sentence, then $(x)A$ is a sentence
(d) The only sentences are those obtained from (a)–(c).

The set **AL** of *atomic-looking sentences* will contain anything of the form

$F^{in}t_1 \ldots t_n$

i.e., a predicate of degree *n* followed by *n* terms. (Again, we use $t_1 = t_2$ instead of the unfamiliar $F^{=2}t_1t_2$ to express the identity of terms.) Clearly, every atomic sentence is atomic-looking (i.e., $\mathbf{A} \subseteq \mathbf{AL}$), but not conversely. The set **S** of *sentences* of **FL** is defined inductively:

(a) Every atomic-looking sentence is a sentence
(b) If A and B are sentences, so are $(A \supset B)$ and $\sim A$
(c) If x is a variable and A is a sentence, then $(x)A$ is a sentence
(d) The only sentences are those obtained via (a)–(c).

Every constant-free sentence of **FL** is a sentence of **FL** (i.e., $\mathbf{S}_{\mathrm{CF}} \subseteq \mathbf{S}$). The set of sentences of **FL** looks just like the set of sentences of classical logic (except for the addition of the singular predicates). This is the illusion of the notation.

The axioms for our system **FL** are built up from more familiar systems in a conservative way. There is only one rule of inference, *modus ponens*:

(MP): if $\vdash A$ and $\vdash A \supset B$, then $\vdash B$.

First we introduce:

Truth Functional Logic

A1 $A \supset (B \supset A)$
A2 $(A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))$
A3 $(\sim A \supset \sim B) \supset (B \supset A)$.

A1–A3 along with MP yield a consistent and complete system of truth functional logic, **TF**. Next, we add:

Constant-Free First-order Logic

A4 $(x)(A \supset B) \supset ((x)A \supset (x)B)$
A5 $A \supset (x)A$, when x is not free in A
A6 $(x)\, A \supset (y/x)A$, so long as y is not *captured* in A.

A1–A6 along with MP yield a consistent and complete constant-free first-order logic without identity. Next, we add:

Constant-Free Identity Theory

A7 $(x)(x = x)$
A8 $(x)(y)(x = y \supset (A \supset (y/x)A))$.

A1–A8 along with MP yield a consistent and complete constant-free first-order logic with identity. Next, we add:

Singular Predicate Theory

A9 $(x)(y)(p^a x \supset (p^a y \supset x = y))$.

A1–A9 along with MP yield a consistent and complete *Constant-Free First-order Logic with Singular Predicates and Identity*. Finally, to get **FL**, we add:

Nondenoting Constants

A10 $((x)A \;\&\; (\exists x)f^a x) \supset (a/x)A$, if x is free in A.

A1–A10 along with MP yield **FL**. The notion of a *deduction of* Φ *from* Γ is defined in the usual way.

Next, we define the notion of a model **M** for **FL**, and also of truth in a model. The important point to remember about the construction is that the constants are to be eliminated.

We define σ: $\mathbf{C} \to \mathbf{SP}$, such that for $i \in \mathbf{C}$, $\sigma(i) = p^i \in \mathbf{SP}$. (For ease of interpretation, we use 'a', 'b', 'c' as constants and 'p^a', 'p^b', 'p^c' as the respective singular predicates.) We extend σ to σ': $\mathbf{AL} \to \mathbf{S}_{\mathrm{CF}}$, i.e., to a function that maps any sentence containing a constant into one that does not. We do so inductively on the degree n of the predicate F^{in}:

Degree 1: Then we have something of the form $F^{i1}a$. Either

$$\sigma'\colon F^{i1}a \to [\theta a]F^{i1}a = (x)(p^a x \supset F^{i1}x)$$

or

$$\sigma': F^{i1}a \rightarrow F^{i1}[\theta a]a = (\exists x)(p^a x \;\&\; F^{i1}x).$$

Degree n (assuming that σ' is defined for all degrees $< n$): Then we have something of the form $F^{in}a_1 \ldots a_{n-1}a_n$. Either

$$\sigma': F^{in}t_1 \ldots t_{n-1}a_n \rightarrow [\theta a_n]F^{in}t_1 \ldots t_{n-1}a_n = (x)(p^{an}x \supset F^{in}t_1 \ldots t_{n-1}x)$$

or

$$\sigma': F^{in}t_1 \ldots t_{n-1}a_n \rightarrow F^{in}[\theta a_n]t_1 \ldots t_{n-1}a_n = (\exists x)(p^{an}x \;\&\; F^{in}t_1 \ldots t_{n-1}x).$$

Finally, σ' extends uniquely to a function $\underset{\sim}{\sigma}'$ (the *storybook*) from the set of *sentences* containing constants ($\mathbf{S} - \mathbf{S}_{\mathbf{CF}}$) to those that do not ($\mathbf{S}_{\mathbf{CF}}$).

A *model* **M** for **FL** is a triple $\langle \mathbf{D}, \mathbf{f}, \sigma \rangle$. The *domain* **D** is a *nonempty* set. The *interpretation function* **f** is such that

$$\mathbf{f}: \mathbf{V} \rightarrow \mathbf{D},$$

i.e., it assigns to each variable $v_i \in \mathbf{V}$ an element of the domain $\mathbf{f}(v_i) \in \mathbf{D}$;

$$\mathbf{f}: \mathbf{F}^j \rightarrow \mathbf{D}^j,$$

i.e., it assigns to each predicate $F^{ij} \in \mathbf{F}$ a set of j-tuples $\mathbf{f}(F^{ij}) \subseteq \mathbf{D}^j$. (In the case of a *singular predicate* p^i, $\mathbf{f}(p^i) \subseteq \mathbf{D}$ is a set consisting of at most one element. The function $\sigma': \mathbf{AL} \rightarrow \mathbf{S}$ is defined as above, associating each $s \in \mathbf{AL}$ with an $s' \in \mathbf{S}$, and then extended to a *storybook* $\underset{\sim}{\sigma}'$.

Finally, we turn to the notion *true in model* **M** for **FL**. The basic idea is that each sentence with constants is associated with some sentence with no constants, and the former is true in **M** iff the latter is. We use the notation $\vDash_{\mathbf{M}} s$ to express that $s \in S$ is true in model **M**. Then $s \in S$ is *true in* **M** *relative to an interpretation* **f** *and storybook* $\underset{\sim}{\sigma}'$ if

Atomic Sentences

1. $\vDash_{\mathbf{M}} F^{in}v_1 \ldots v_n[\mathbf{f}]$	iff $\langle \mathbf{f}(v_1), \ldots, \mathbf{f}(v_n) \rangle \in \mathbf{f}(F^{in})$
[2. $\vDash_{\mathbf{M}} v_1 = v_2[\mathbf{f}]$	iff $\langle \mathbf{f}(v_1), \mathbf{f}(v_2) \rangle \in \mathbf{f}(F^{=2})$]
3. $\vDash_{\mathbf{M}} p^i v_1[\mathbf{f}]$	iff if $\mathbf{f}(v_2) \in \mathbf{f}(p^i)$ then $\langle \mathbf{f}(v_1), \mathbf{f}(v_2) \rangle \in \mathbf{f}(F^{=2})$

Molecular Sentences

4. $\vDash_{\mathbf{M}} \sim A[\mathbf{f}]$	iff not $\vDash_{\mathbf{M}} A[\mathbf{f}]$
5. $\vDash_{\mathbf{M}} (A \supset B)[\mathbf{f}]$	iff either not $\vDash_{\mathbf{M}} A[\mathbf{f}]$ or $\vDash_{\mathbf{M}} B[\mathbf{f}]$ (or both)
6. $\vDash_{\mathbf{M}} (x)A[\mathbf{f}]$	iff for every $d \in \mathbf{D}$, $\vDash_{\mathbf{M}} A[\mathbf{f}(d/x)]$

Consider now $\underset{\sim}{s}_1 \in [\mathbf{S} - \mathbf{S}_{\mathbf{CF}}]$:

7. $\vDash_{\mathbf{M}} \underset{\sim}{s}_1[\mathbf{f}]$	iff $\vDash_{\mathbf{M}} \underset{\sim}{\sigma}'(\underset{\sim}{s}_1)[\mathbf{f}]$.

It is clear from the way this definition is constructed that the statements involving constants are all semantically eliminable, in that such a given statement is expandable into a statement containing no constants, and the truth value of the original will just be whatever truth value is associated with the constant free version. Those sentences containing constants, however, are clearly not atomic statements.

Finally, we define validity. Let Γ be a set of sentences, and let Φ be a sentence. Γ *logically implies* Φ, $\Gamma \models \Phi$, iff for every domain **D**, interpretation **f**, and storybook $\underline{\sigma}'$, if $\models_{\mathbf{M}} \Gamma[\mathbf{f}]$ then $\models_{\mathbf{M}} \Phi[\mathbf{f}]$. A sentence Φ is *valid* iff $\models\Phi$, i.e., for every domain **D**, interpretation **f**, and storybook $\underline{\sigma}'$, $\models_{\mathbf{M}} \Phi[\mathbf{f}]$.

NOTES

1. In an early (i.e., antedating *Grundlagen*) unpublished manuscript, "Dialogue with Punjer on existence," ([7], pp. 53–67), Frege toyed with the idea of taking 'exists' to be a first-level predicate, even identifying 'exists' with 'being self-identical', but he ultimately shied away because he could not then account for the informativeness of existential claims. His view did not change even when he clarified the distinction between sense and reference. The decision to treat existence as a second-level predicate had already been deeply integrated into his view that number is a property of concepts, not of objects.

2. Let us adopt the following abbreviations: 'ox' for 'x is Odysseus'; 'Ux' for 'x is a Unicorn'; 'Ax' for 'x is an Achaean'; 'Mx' for 'x is a man'. The use of a lower case 'o' is intentional, and will be explained below.

3. '$\sim(\exists x)(\mathrm{A}x \,\&\, \Sigma x)$' is equivalent to '$(x)(\sim\mathrm{A}x \vee \sim\Sigma x)$'; and since '$(x)\Sigma x$' is logically true, the right disjunct is logically false, so the disjunction reduces to '$(x) \sim \mathrm{A}x$', i.e., '$\sim(\exists x)\mathrm{A}x$'.

4. The term is from [1]. The alternative, Meinongian view, which Berlin calls *inflationist*, is to treat the argument as a *reductio* of the claim that we can refer only to existents.

5. Kripke distinguishes explicitly between the theory of reference proposed by Frege and Russell, on the one hand, and the linguistic reform proposed by Quine. Only the former draws his fire. See [11], p. 343.

6. Universal Instantiation is similarly modified in free logic: if every (existent) object is *F*, then *a* will also be *F*, provided that *a* is an (existent) object. These two constraints on EG and UI constitute a necessary condition for something's counting as a free logic. See [12] for a historical overview and complete references.

7. One semantic structure for free logic takes the values of the bound variables to coincide with existents, but allows the constants to designate elements in a superset, consisting of the existents plus the elements of an "outer domain" (see [14]). Nonetheless, it is still false that some things do not exist.

8. The singular/general distinction is just that, a distinction between how many things a term applies to. The confusion between this distinction and the subject/predicate distinction has led to unfortunate philosophical positions. I discuss these in [16]. The idea of treating proper names as predicates has been argued in [15], and most notably in [21] and [22].

9. This enables us to translate 'There is such a thing as Odysseus' as we had originally tried to do in Section 1, as '$(\exists x)\mathrm{o}x$'.

10. We could say: "We cannot conceive of a unicorn except as an existing unicorn." Is *this* Kant's point?

11. That is, in this case, the difference between *all* and *some* disappears. This is, perhaps, the sense in which, as Sommers puts it, proper names have "wild scope".

12. The general principle goes like this. If we have a truth "about" the nonexistent *a*, take the singular claim as a universal affirmative, so that it is true by false antecedent. On the other hand, if we have a falsehood "about" the nonexistent *a*, take the singular claim to be a particular affirmative, since it will be false because *a* does not exist.

13. Given any model, with a domain and an interpretation, the truth of a claim about an element *in the domain* is fixed. Claims "about" nonexistent objects can go either way. A sentence involving constants (even nondenoting constants) always has a truth value. We could, however, amend this picture to accommodate truth valueless sentences. We need only suppose that, in some cases, a storybook is a *partial function*, for the story does not tell us whether a given atomic-looking sentence is true or false. (This way of dealing with the truth valueless case is rather different from van Fraassen's *supervaluations*, and avoids the difficulties his own construction led him into. See [6].) The 'hole' Frege spoke about in the case of a nondenoting proper name is not the lacking of an argument; the hole is in the story.

14. As does van Fraassen [5]; Hintikka [9] does not. This is not a controversial position: different systems simply handle the matter differently.

15. For example, van Fraassen [5] thanks Karel Lambert for convincing him that (18) is a logical truth. Burge [2] and Grandy [8] express different points of view on the issue.

16. This is van Fraassen's example, [5] p. 219: "If Pegasus does not exist, then there exists something that does not exist."

17. This enables us to use Σa as a noncommittal existence claim. We can therefore take 'Σg' to be 'God exists' without any real ontological commitment (because of the possibility of the small scope reading).

18. For the antecedent could be true while the consequent is false, and in that case it would follow that *a* doesn't exist.

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