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More about Thoughts

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Dramatis Personae

Thesis 1:	A thought is isomorphic with the sentence whose sense it is.
Thesis 3:	Every unambiguous sentence has a unique function-argument analysis.
Sentence A1:	a is parallel to b.
Sentence A2:	The direction of a = the direction of b.
Sentence B1:	For every a, $f(a) = g(a)$.
Sentence B2:	The value-range of f = the value-range of g.
Sentence C1:	There are just as many F's as G's.
Sentence C2:	The number of F's is the same as the number of G's.
Sentence D1:	There exist unicorns.
Sentence D2:	The number of unicorns is not nought.
Sentence E1:	Jupiter has four moons.
Sentence E2:	The number of Jupiter's moons is four.
Thesis T:	The two sentences in each of the pairs A to E express the same sense or have the same content.
Criterion R:	Anyone who grasps the thought expressed by each of a given pair of sentences must immediately recognize one as true if he recognizes the other as true.
Principle K:	If one sentence involves a concept that another sentence does not involve, the two sentences cannot express the same thought or have the same content.

Two fundamental theses In his exceedingly illuminating article "Thoughts" (see [1]), Dr. David Bell sets out a number of requirements that he believes that any successful philosophical theory of thoughts must meet. He believes that Frege's theory of thoughts comes closest, of any yet propounded, to meeting them; and he argues that, among the theses essential to that theory, are those he labels 'Thesis 1' and 'Thesis 3' (see Dramatis Personae above). These theses,

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he contends, are indispensable for the ability of Frege's theory to handle those of the requirements on an acceptable theory of thoughts which he lists under the head of 'rationality', in particular that it yield an account of the logical relations between thoughts.

Despite his admiration for Frege's theory, Dr. Bell claims to detect within it two distinct incoherences. These, he thinks, are difficulties not peculiar to the theory maintained by Frege, which contains certain features that might call for modification on other grounds: rather, they affect any theory of thoughts, and no resolution of them is generally available, or, at any rate, generally accepted. We have, then, first to decide whether the incoherences Bell perceives are genuine; if they are, we have to see what modifications in the theory would be needed to correct them. The first alleged incoherence bears on Theses 1 and 3; so we may start by considering whether Bell is right in maintaining that these two theses are essential to Frege's theory, or whether one or other could be jettisoned without fatal damage to it.

Not only was Thesis 1 held by Frege: he regarded it as a key component of his theory. It is implicit in his doctrine that the sense of part of a sentence is part of the sense of the whole, and explicit in remarks like "This would not be possible if we could not distinguish in the thought parts to which the parts of the sentence correspond, so that the construction of the sentence could serve as a picture of the construction of the thought" ([8], original page 36). Now structure can be ascribed to thoughts only metaphorically, as Frege acknowledged; so the tenability of the thesis depends upon the defensibility of the metaphor. A natural response—virtually a conditioned reflex for professional philosophers—is that Thesis 1 is meaningless in the absence of independent notions of structure for thoughts and sentences. That is a misunderstanding: the two notions are interdependent, and must be explained together. For different purposes, structure may be ascribed to sentences according to different schemes of analysis: the type of structure with which Thesis 1 is concerned is that revealed by what, in his formulation of Thesis 3, Bell calls 'function-argument analysis'. Since functions, for Frege, belong to the realm of reference, Bell clearly means to speak of the semantic theory that constitutes Frege's theory of reference: the structure of a sentence is that which underlies Frege's account of the determination of its truth-value in accordance with its composition. Our understanding of the sentence depends upon our grasp of what determines it as true or as false: so the structure in question is that relevant to our grasp of the thought it expresses. But, conversely, our access to the thought is through that, or some other, sentence expressing it: we arrive at a notion of the structure of the thought via that of the semantic structure of the sentence.

It may appear that, if our conception of the structure of the thought is correlative to that of the (semantic) structure of the sentence, Thesis 1 is vacuous. It is not vacuous, however: rather, it vindicates the attribution of structure to thoughts. Sentences do not encode thoughts, but *express* them: it is only because we can conceive of the thought as having parts corresponding to the parts of the sentence that we can distinguish expressing the thought from a systematic means of identifying it. A thought may be identified, for instance as the weakest additional premise that would make Smith's argument valid, otherwise than by expressing it: you can understand the means of identifying it without yet know-

ing what thought it is. But your grasp of the thought depends upon your grasp of the senses of constituent expressions within a sentence expressing it: that is why Frege was entitled to speak of the sense of such a constituent as consisting in its contribution to the sense of the whole, and as a part of that sense (that thought). You cannot have the thought that planets travel in elliptical orbits, for example, without having the concept of an ellipse: and you cannot have the concept of an ellipse unless you are able to think, of some other closed curve, that it is an ellipse.

It might at first sight be doubted whether Frege would have endorsed Thesis 3: but, on reflection, we can recognize that it is integral to the very idea of a semantic theory, such as he advanced, that the construction of each unambiguous sentence be unique. Different sentences of natural language can of course express the same thought: a sentence may be transformed, without altering the thought expressed, so that its verb is changed from active to passive and its object becomes the new subject. Such distinctions are irrelevant to logic, and hence the two sentences may be represented by the same symbolic formula. It is to this formula that Frege's theory of reference directly applies; and the theory treats each such formula as constructed in a unique way from atomic sentences by logical operations. One might therefore regard a Fregean analysis of a sentence of natural language as taking place in two stages: its representation by a formula of his logical notation, and the syntactic analysis of the formula into parts to which references may significantly be ascribed. Frege no doubt believed that his theory of reference constituted the unique correct semantic theory, but I do not think that Bell needed or meant to endorse such a claim. The uniqueness of the analysis of the sentence is relative to the semantic theory adopted; within that theory, the construction of each sentence must be unique, on pain of ambiguity. For example, a salient feature of Frege's theory is that, although a sentential connective like "if . . . then . . ." does not always stand between complete sentences, the manner in which complex sentences are constructed from atomic ones makes a truth-functional explanation of the connective (as standing between complete sentences) adequate for all occurrences. It follows that, given a sentence representable by a formula of the form "If A then B", where "A" and "B" *are* complete sentences, the first step in analyzing the formula must necessarily be to break it up into the connective and those two subsentences; any other strategy of analysis will destroy the adequacy of the truth-functional explanation.

Bell's first accusation Bell observes that Theses 1 and 3 together imply the corollary that every thought has a determinate structure, corresponding to that of a sentence expressing it. This corollary is then threatened by any example either of a single unambiguous sentence whose structure is not unique, or of two sentences with different structures that express the same thought; and Bell contends that it is from this that the first of the two incoherences he perceives in Frege's theory arises. For he maintains, on pp. 45–46, that Frege was compelled to admit that certain pairs of sentences with "radically different function-argument structures" can express the same thought, and, further, that there are general grounds for this admission. The Fregean examples he offers are the pairs

A, B, and D (see Dramatis Personae). To these we can add pair C, which is of course that pair with which Frege is really concerned in Sections 63–69 of [6], expressed without his technical jargon; he uses pair A merely as a convenient analogy. About pair A, he indeed said in Section 64 that we obtain A2 from A1 by "carving up the content in a way different from the original way", adding that "this yields us a new concept", a passage that Bell quotes. About pair B, he said on p. 11 of [7] that B1 "expresses the same sense" as B2, "but in a different way". As for pair D, Bell is relying on the remark in Section 53 of [6] that "affirmation of existence is nothing but denial of the number nought"; but we can avoid irrelevant side-issues by fastening instead on pair E, of which Frege says in Section 57 that E1 "can be converted into" E2. It is thus Thesis T (see Dramatis Personae) which Bell regards as intrinsically difficult to reject, as having been held by Frege, but yet as incompatible with the fundamental Theses 1 and 3 also held by him.

Concerning Thesis T Frege had, I believe, one of his tacit changes of mind; for it is noteworthy that in [5] he did *not* anywhere claim that the two sides of Axiom V (the two members of pair B) express the same sense. This was questioned by Hans Sluga in [12] on the ground that Frege says that he uses B2 as '*gleichbedeutend*' with B1 ([5], Vol. I, Section 3), and that this word signified 'having the same sense', and not just 'having the same *Bedeutung*'. In view, not merely of Frege's scrupulous terminological precision in [5], but of the fact that the observation follows immediately upon his preliminary explanation in Section 2 of the distinction between sense and *Bedeutung* (reference), this seems somewhat unlikely. If he had still believed the senses to coincide, he could have entertained no doubts about the Axiom, for he held that, if two sentences express the same thought, this will be immediately apparent to anyone who grasps the thought each expresses; yet he said on p. VII of [5], Vol. I, that a dispute might break out over Axiom V, and, in his Appendix to Vol. II, that he had never concealed the fact that it was less evident than the other axioms, and than a logical law ought to be. More likely, by the time he wrote Vol. I of [5], he had come to hold that the claims made in this regard, in [6] and in [7], were too strong. Even so, two questions remain, of which the first is:

Why did Frege ever assert Thesis T? The answer is not far to seek. Let us concentrate on pairs A, B, and C, which exhibit a common pattern, strongly emphasized by Frege in [6]. In each case, the first member states that a certain equivalence relation holds: between certain objects, in pair A; between functions, in pair B; and between concepts, in pair C. The second member states an identity between objects characterized by reference to the two terms of the equivalence relation. The relation between the first and second members of each of these three pairs is that between "This stick and that stick are equally long" and "The length of this stick is the same as the length of that stick". Now Frege insists that in each case we can arrive at an understanding of the second member only if we already have an understanding of the first; our understanding of the second member essentially involves our recognition that it is logically equivalent to the first. How do we make this transition? Individually, almost certainly by being taught to make it: but how could it be made in the first place? Only by reflec-

tion: we come to see that we can express that the equivalence relation holds by stating the identity of certain (abstract) objects. Our understanding of the new form of sentence is mediated by our recognition of its logical equivalence with the old: the meaning of A2, as we ordinarily understand it, is *given* to us by the requirement that it have the same truth value as A1. Since this is so, what more natural than to say that the new sentence has the same content, or expresses the same thought, as the old one? Pairs D and E exhibit a somewhat different pattern; but a similar line of thought seems equally compelling in their case.

Criterion R (see Dramatis Personae) was that usually, though not unwaveringly, employed by Frege from 1891 onwards for two sentences' expressing the same thought. The conclusion that the members of pairs A to E have a common content or sense is reinforced by the fact that they all satisfy Criterion R. No one can be said to grasp the content of A2, for example, unless he is aware that it is true or false according as A1 is true or false; so the content of each must be the same.

Attaining new concepts Now the natural objection to Thesis T is that the second member of each pair involves a concept—that of a direction, a value-range or a number—that the first does not, and hence, by Principle K (see Dramatis Personae), cannot express the same thought. We shall see that the objection is in fact decisive; but our present concern is still with Frege's motivation. He was not in a position to repudiate Principle K; for it is implicit in his frequently repeated thesis that the sense of a part of a sentence is a part of the thought expressed by the sentence as a whole: if that does not mean that a grasp of the sense expressed by the constituent part is a necessary condition for grasping the thought, it means nothing at all. Nor could he deny that Principle K applies to pairs like A and C: for he expressly contends in Section 64 of [6] that by means of the transition from the first member of the pair to the second "we attain a new concept"—that of a direction or of a number. Why, then, did he still think that the content of A2 could be arrived at by analyzing that of A1?

This, too, is readily explained: he adopted Thesis T by false analogy with the type of sentence concerning which he repeatedly insisted that distinct analyses can be given of one and the same sentence, and that it is essential to logic that they can. The process of analysis is described both in [4] and in '*Booles rechnende Logik*', the latter paper being one submitted for publication but rejected. In the simplest case, we pick out one or more occurrences of one singular term within the sentence, and conceive of it as replaceable, in all the occurrences we have picked out, by any other singular term; we may then view the sentence as composed out of that singular term and the remaining (constant) part (called in [4] the 'function'), which expresses a concept. In the light of this analysis, the content of the sentence may be viewed as saying of the thing denoted by the singular term that it falls under the concept. Since the sentence may contain more than one singular term, and each of them may have more than one occurrence, this operation can be executed in a number of different ways. We may also pick out each of two singular terms, in one or more occurrences, conceiving of each as replaceable, in those occurrences, by another singular term; we are then viewing the content as being to the effect that a relation, expressed

by the constant part, obtains between the things denoted by the two terms. Yet more sophisticated forms of analysis allow us to pick out expressions for concepts or relations, again to be regarded as replaceable by other expressions of the same type. The choice of an analysis for any one given sentence is thus very wide.

Frege regarded this conception of analysis as one of his great discoveries, and characterized it by saying that "instead of putting together the judgment from an individual thing as subject and an already formed concept as predicate, we conversely analyze the judgeable content and so attain the concept" ('*Booles rechnende Logik*' in [10], p. 18). By thus representing a complex predicate, to which a quantifier can be attached to form a sentence, as not being formed out of its parts, but as arrived at by omitting its argument from a complete sentence, he was able to treat all complex sentences as built up, ultimately, from closed atomic sentences. He did not need, like Tarski, to resort to so semantically problematic, if syntactically simple, a device as taking *open* atomic sentences as the basis; and this lends an especial purity to his semantics.

More importantly, Frege characterized the process of analysis as a means of concept-formation, more powerful than the Boolean operations and the process of abstraction generally admitted; his favorite example was our forming the concept of the continuity of a function. The analysis of a sentence enables us to discern a pattern in the sentence, a pattern it shares with every other sentence resulting from inserting an argument into the argument-place of the 'function'. We may equally well say that we discern a pattern within the content; but the recognition of this pattern was not needed for a grasp of the content, but occurs when that content has already been grasped. It is for this reason that Frege says in [4] that "it has nothing to do with the conceptual content, but is solely a matter of the way we view it". The pattern is not imposed, since it was there to be discerned; but it is selected from among many different equally discernible patterns, which is why each sentence or thought is capable of being analyzed in different ways. Thus it would in principle be possible to grasp the thought expressed by "Brutus killed Brutus" without noticing that it exhibited a pattern shared by "Cato killed Cato", but not by "Brutus killed Caesar": it is by noticing that common pattern that we attain the concept of suicide.

For Frege, deductive reasoning was not a mechanical process of eliciting new ways of stating what was already known: rather, analytic judgments could extend our knowledge. What explained how this could be was precisely the fact that analysis, and with it the formation of concepts, was an essential constituent of logical inference; a step indispensable to constructing a quantified sentence, and equally to recognizing the validity of an argument in which it figured. Since the form of the analysis was not uniquely determined by the content of the sentence analyzed, deductive reasoning must therefore be, in some part, a creative intellectual operation. All this put Frege, almost alone amongst philosophical logicians, in possession of an account of how deductive reasoning could be simultaneously certain and fruitful.

Two of Frege's central doctrines were that one and the same content was capable of distinct analyses, and that different sentences of natural language could have the same content. Often the difference reflected a variation in the focus of attention; and hence one sentence might better accord with one anal-

ysis of the content, another with another. What made analysis a process of concept-*formation* was the fact that a grasp of the concept revealed by the analysis was not integral to an apprehension of the content being analyzed. These views together render it entirely comprehensible how Frege came to claim that, by analyzing the content of the first member of any of our pairs, one could extract a concept which obtained explicit expression in the second member, but that the content itself remained unaltered.

The relation thus claimed to obtain between the first and second member of a pair is in fact precisely that between a sentence and its definitional replacement. We may analyze the sentence "13 is greater than 1 and, for all n, if n divides 13, $n = 13$ or $n = 1$" into "13" and the concept-expression obtained by omitting all three occurrences of "13" from the sentence. By this means, we attain the concept of primality, which that concept-expression expresses. We may therefore use that concept-expression to define the predicate "_ is prime", and then replace the original complex sentence by "13 is prime". This last sentence obviously involves the concept of primality, whereas the whole point of regarding the analysis of the original sentence as a means of attaining that concept was that a grasp of the content of that sentence did not require us to see it as splitting up in that way, nor, therefore, to have the concept of a prime number. But, since the latter sentence has been introduced by stipulating that it have the same truth value as the former, it seems that they must have the same content.

The untenability of Thesis T For all that, Thesis T must be pronounced indefensible. It cannot be justified by Criterion R, which Frege indeed often used to good effect in order to show that two sentences did *not* have the same sense, in other words as a necessary condition for expressing the same thought. But, although he never acknowledged the fact, it is far less plausible as a sufficient condition. Its inadequacy when so used is shown when it conflicts with Principle K. The principle is compelling: no one can be said to grasp the thought that a certain politician is dishonest, for example, if he lacks the concept of dishonesty. Since possession of the concept is essential to a grasp of the thought, the thought cannot be identified with one that can be grasped by someone who does not possess the concept. It follows that Principle K rules out an identity of sense between the members of each of the pairs A to E. In doing so, it contradicts Criterion R for identity of sense. Possibly Frege never noticed that Principle K conflicts with Criterion R, to both of which he was committed; but it is plain that, when they conflict, it is Principle K to which we should hold fast.

Since Frege was himself committed to Principle K, he ought never to have asserted Thesis T; and his other doctrines in no way compelled him to do so. In [6] Frege eventually decided that A2 cannot actually serve to define A1, nor C2 to define C1; this by itself should have warned him that there was something amiss in holding them to have the same content. In fact, Principle K disallows identity of content even for definitionally equivalent predicates, and sentences containing them, at least given Frege's doctrine of analysis. The sentence "13 is prime" and its definitional expansion are, of course, intimately connected: the sole difference between them is that what, in the terminology of [4], was not integral to the content of the expanded sentence, but was merely one out of dif-

ferent possible ways of regarding it, has become integral to the content of its definitional abbreviation "13 is prime". The predicate "_ is prime" has indeed the same sense as the complex predicate serving as its definiens; but this sense is not a constituent of the thought expressed by the expanded sentence.

Gregory Currie in [3] tries to vindicate Frege by attributing to him two distinct theories (or concepts) of sense, a weak and a strong one: the weak one is that concerned with the information conveyed by a sentence; the strong one allows Frege to justify the analytic definitions of arithmetical notions that he offers in [6], involving the replacement of an intuitive notion by a precise one. This is not very plausible. It is hard to see how there could be any acceptable concept of sense according to which an imprecise expression could bear the very same sense as a precise one: precision and imprecision are features of the *sense* itself, not of the manner of expressing it. Currie does not attempt to delineate the alleged strong concept of sense; he merely asserts that Frege had such a concept, which of course explains nothing. But, in any case, the replacement of an intuitive notion by a precise one, though exemplified by Frege's explicit definition of "the number of *F*'s", is irrelevant to the logical equivalence of C1 and C2. It *does* indeed bear on the definition of "There are just as many *F*'s as *G*'s" (C1) to mean "There is a one-one map of the *F*'s onto the *G*'s"; but this definition should be kept separate from the requirement that C1 and C2 have the same truth value, since it would be quite possible to accept the definition of cardinal equivalence in terms of one-one mapping while refusing to take numerical terms at face value as genuine singular terms denoting objects, or, indeed, to do the converse. Frege runs the two together in Section 63 of [6] because his contention that C2 is to be explained in terms of C1 would be idle unless C1 were independently definable; but, when he discusses the analogous pair A, he does not enquire how A1 (the concept of parallelism) is to be defined, and considers the logical equivalence of A2, not with some definiens of A1, but with A1 itself. Once the distinction is clearly made between (i) the definition of cardinal equivalence (equinumerosity) by means of one-one mapping, and (ii) the equation of cardinal equivalence with numerical identity (pair C), the temptation to think that Thesis T has anything to do with replacing vague everyday notions with mathematically precise ones is dispelled, and, with it, Currie's account of Frege's alleged strong concept of sense.

Frege's idea that we attain a concept by analyzing a content already given is, on his own principles, wrongly applied to the pairs A to E. Since, by means of analysis, we discern a pattern that was there to be discerned, the possibility of discerning it must be apparent from a complete representation of the structure of the sentence; but A1 cannot be displayed as dissectable into two terms for directions and the sign of identity. It would be perfectly in order to observe that A2 could be analyzed, not only as saying of the direction of *a* and the direction of *b* that they are identical, but also as saying of the lines *a* and *b* that they stand in the relation that the direction of one is the same as that of the other: in this way, given the content of A2, we could arrive at the concept of being parallel. It would therefore be possible to maintain that A1 is a disguised form of A2, and can be understood only by recognizing it as conventional shorthand for A2; its apparent form would not then be its true form, but would conceal a tacit reference to directions. But this is precisely the view Frege is opposing: he argues

that "this is to stand the true state of affairs on its head", since the concept of parallelism is prior to that of a direction. We may express this by saying that the direction of explanation runs from A1 to A2: A1 is explained independently of A2, but must be invoked in any explanation of A2. It is true that Frege appears to be proposing to analyze E1 as shorthand for E2, for E2 is interpretable by means of his definitions, whereas he gives no independent explanation of E1, but merely remarks that "it can be transformed into" E2. Against this, however, the same objection might be brought, namely that the adjectival use of number-words is prior to their substantival use and to the concept of a number; and so his procedure at this point requires a justification he does not offer. Certainly, it would be contrary to all Frege's beliefs to propose that to say that *F* and *G* have the same value for every argument is shorthand for saying that their value-ranges coincide: the former statement involves only identity and universal quantification, in explaining neither of which is any allusion made, however tacit, to value-ranges.

My own view is that Frege is right about the direction of explanation as between the members of pairs A and C, and that it is a minor flaw in his presentation in [6] that he represents statements of number like E1 as explicable in terms of statements of identity between numbers like E2. This, however, is irrelevant to the present issue. If the direction of explanation runs from the second member of any of these pairs to the first—if, for example, in the pair A, "is parallel to" is definable by "has the same direction as"—there is no problem: the first member can then be arrived at by analysis of the second, and is a definitional replacement of it. By what was said earlier about definition, the two sentences would still not have precisely the same content; but their contents would be bound in that intimate connection that results from definition. If, conversely, the direction of explanation runs from the first member of a pair to the second, the second cannot be obtained by analysis from the first. Frege was simply mistaken in thinking that it could; in my opinion, he came to realize his mistake, although without ever clearly perceiving that his criterion for expressing the same thought did not constitute a sufficient condition for doing so. We indeed have, in the transition from the first member of such a pair to the second, a mode of concept-formation, and one that follows a pattern common to many distinct instances; but it is a different mode from that which Frege discovered and so strongly emphasized. The first of Bell's two crises for Frege's theory of thoughts, and for theories of thoughts in general, is thus resolved.

Two kinds of analysis The doctrine of alternative possible analyses appears to contradict Bell's Thesis 3 with which we started. Bell resolves the matter by holding that all but one of the alternative analyses recognized by Frege are always *partial* analyses, whereas it is only a complete analysis that should be unique. The example he uses is the sentence "Brutus killed Caesar", which can be correctly analyzed into the proper name "Brutus" and the concept-expression "__ killed Caesar", and equally correctly into the proper name "Caesar" and the concept-expression "Brutus killed __". This enables him to dismiss as spurious certain apparent counterexamples to unique analyzability. But even if the contrast between partial and complete analyses suffices to explain the "Brutus killed

Caesar" case, it does not serve for all. The fact is, rather, that we have to do with two quite different senses of "analysis". The analysis spoken of in Thesis 3 uncovers what Frege refers to as the structure of the thought, as a whole composed of parts: it shows how the thought is to be represented as constructed in stages from thoughts expressible by atomic sentences, and how those thoughts in turn have as constituents the senses of simple names, predicates, and functional expressions. This conception is indeed fundamental to Frege's semantics; but it is not what he is referring to when he uses words translatable as "analysis". If we use the term "analysis" for the former process, we should use some other for what Frege means when he says that one sentence or thought admits of different analyses: I have suggested "decomposition". The latter process does *not* purport to display the structure of the thought, in that sense of "structure" in which a grasp of the thought depends upon an apprehension of its structure. Rather, it picks out a pattern common to that thought and a range of others, a perception of which is *not* required in order to grasp the thought. A recognition of this pattern yields a new concept which is not, in general, a constituent of the thought, though it is attained by regarding the thought as exemplifying that pattern; but, as Frege makes clear in [4], it will be a constituent of other thoughts, in particular one attributing a property to that concept (a thought expressed by a quantified sentence).

When, by omitting all three occurrences of the numeral "13", we subject to the process Frege referred to as 'analysis' the arithmetical sentence cited earlier, "13 is greater than 1 and, for all n, if n divides 13, $n = 13$ or $n = 1$", we have not taken a first step towards an analysis in Bell's sense: on the contrary, the concept-expression so obtained nowhere appears in the process of displaying the structure of the sentence as built up from atomic sentences, for this must begin by breaking it up into two subsentences conjoined by "and". The reconciliation between Thesis 3 and the doctrine of alternative analyses is, therefore, to be effected, not merely by demanding uniqueness only for complete analyses rather than partial ones, but by distinguishing two quite different notions of analysis.

Do contents have structure? An interpretation of Frege due originally to Hans Sluga has found a few adherents, among them Steven Wagner. The interpretation rests on construing quite literally Frege's apparently comprehensive claim quoted above, that "instead of putting together the judgment from an individual thing as subject and an already formed concept as predicate, we conversely analyze the judgeable content and so attain the concept". If we make no distinction between the two senses of "analysis", this appears to imply that we initially grasp the content as unstructured. That conclusion is presented by Wagner in [13] as Frege's doctrine, understood, as Wagner emphasizes, by analogy with Kant's epistemology of space and time. "Contents," he tells us (p. 8), "are *unstructured* bearers of truth-values, and propositions (as I am now using the word) are structures which the mind formulates by way of representing content to itself"; hence "the content . . . is itself an entity prior to these propositions and more objective". Contents exist independently of us, but we can grasp them only by imposing a propositional structure on them; grasping a content

is thus itself "a process of analysis". The propositional structure is "the mind's contribution"; but "a single content may admit radically different analyses", and hence be "represented by us to ourselves by distinct propositions". It therefore makes no sense to ask "whether a particular concept or term appears" in a given content, "although this question makes sense for any of the propositions that express" it.

A little reflection should show that it is unnecessary to attribute to Frege so radically Kantian a view of judgeable contents in order to guarantee that concepts are never among their building blocks, but are always extracted from them. It could still be allowed that the contents of complex sentences have a structure depending on how they are built up from the contents of atomic sentences by operations corresponding to negation, application of the sentential connectives and quantification; all that is needed is that the contents of the atomic sentences be initially apprehended as devoid of structure. But Frege does not appear at any stage to have believed even this. Thus he says, in '*Booles rechnende Logik*', that "in order to be capable of being so analyzed, the expression of the judgeable content must already be articulated. From this one may conclude that at least the properties and relations that are not further analyzable must have their own simple designations" ([10], pp. 18–19): in other words, even atomic sentences are complex and contain simple expressions for the primitive properties and relations. He goes on to touch upon a difficulty not peculiar to his theory, saying, "It does not follow from this that the ideas of these properties and relations can be formed unconnected with things" (that is, with objects); "rather, they arise simultaneously with the first judgments in which they are ascribed to things". The thought, 'That leaf is green', cannot be unstructured, since, for anyone to have it, he must be capable (as Evans's 'generality constraint' requires) of thinking of other objects that they are green, and of thinking other things concerning the leaf; moreover, he must be aware that he is capable of this. Likewise at the linguistic level: the sentence "That leaf is green" might be new to him, but he can understand and use it because he knows what "that leaf" means and what "is green" means. On the other hand, to know what "is green" means consists in knowing what is meant by saying of any one thing that it is green; one can understand the phrase only as a component of sentences. Correspondingly, one can grasp the property of being green only by knowing what it is to think, of something, that it is green. Frege offers no deep account of the matter; but, by speaking of the primitive properties and relations as "arising simultaneously" with the judgments involving them, he rules out the idea that we *first* grasp the judgeable contents and only subsequently discern the properties and relations within them.

Wagner will reply that this is not the view he takes Frege to hold: the properties and relations arise simultaneously because *we* cannot grasp the unstructured contents without imposing a structure on them. Such a reply, however, reveals how gratuitous his contrast between judgeable contents and 'propositions' is; for, in that case, it can only be from the *propositions* that we can extract concepts, and Frege's dictum about this cannot after all be used to support the thesis that *contents* are unstructured. There can be, for Frege, no more and less objective: there are only the wholly objective and the wholly subjective. Moreover, there is no true analogy between Wagner's doctrine of propositions and Kant's

doctrine of space. Different individuals may have qualitatively very different intuitions of space; but Wagner requires that there may be two distinct propositions which we all grasp and which impose different structures on the same content-in-itself, and there is no analogue of this in Kant's account of our apprehension of objects as in space. Wagner speaks of the way *he* uses the word "proposition"; but we need to know to which Fregean term or concept that use corresponds. Wagner ascribes his quasi-Kantian theory simply to Frege, without differentiation of period, citing both [6] and [5]. In Frege's post-1890 doctrine, contents were split into thoughts and truth values. Thoughts—even those expressed by atomic sentences like "Etna is higher than Vesuvius"—have parts, including the sense of the name "Etna", and hence also the sense of "is higher than"; truth values have no structure, and there are only two of them. Thoughts presumably correspond to Wagner's propositions; but there is no place in which to locate his unstructured contents. For the early period, conversely, it is impossible to say which term corresponds to Wagner's "proposition".

Wagner's interpretation enables him to ascribe to the Frege of [5] and [6] the view expressed in '*Booles rechnende Logik*' ([10], p. 38), that numbers "are first created by thinking", or, at least, that this is true of the extensions of concepts with which he identifies them. But, quite rightly faithful to Frege's conclusion that we cannot introduce numerical terms by simply stipulating that C2 is to be equivalent to C1, Wagner finds himself forced to concede that, if such a stipulation were made, C2 would merely express an unstructured relation between the concepts F and G, not a genuine identity between objects (p. 9): it thus only shows how numerical terms, if we had them, could be used to restructure the content of C1. It is plain to Wagner that Frege thought it essential to be able to treat the numerical terms in C2 as genuine terms standing for objects in the domain of the individual variables. We obtain such terms by the explicit definition Frege gives of the numerical operator: but, since many alternative definitions of it would yield the logical equivalence of C2 and C1, the understanding of C2 as containing genuine terms for numbers involves more than is given by that equivalence, and it follows (p. 10) that we cannot obtain a concept of number by analyzing C1. Hence, in the end, Wagner's interpretation proves impotent to justify assigning the same content to C1 and C2 (or, presumably, any of the other pairs).

Bell's second accusation The second incoherence Bell claims to detect in Frege's theory concerns the status of thoughts (and senses generally) as objects; we may say, although Bell does not express it in this way, that it has to do with Frege's mythology of the third realm. For Frege, thoughts are objects existing independently of our grasping them. Bell claims, however ([1], p. 46), that "it is impossible for an object to fulfill the role of a sense", and quotes Frege's possibly unguarded remark that "a truth-value cannot be part of a thought, . . . because it is not a sense but an object" ([11], p. 35) in support. He justifies his claim by arguing that, for Frege, "to have an object in mind is to have grasped the sense of some expression which has that object as its reference", but that, when applied to the case in which the object is itself a sense, this leads to a vicious infinite regress.

For all objects other than senses, Frege adhered faithfully to Kant's dictum that every object must be given to us in a particular way; the sense of a singular term consists in a way in which its referent is given to us, and we can think about or be aware of an object only by grasping such a sense. When we grasp a sense, however, *it* cannot be given to us in any one way rather than another; our mode of awareness of it is not constituted by our grasp of any *other* sense. That is because everything that contributes to determining the reference of an expression is part of its sense: we do not go to the reference via the sense, as one goes from Oxford to Leeds via Birmingham; the sense is the route, and not an intermediate station. It thus appears that, if senses are objects, they are the only objects which we can apprehend immediately, without conceiving of them in any particular way. We might say that, in grasping a thought or a constituent sense, we apprehend it *as it is*, or in its entirety: not, of course, that we know everything there is to know about it, but that we identify it by its constitutive properties, that is, by what makes it the thought or thought-constituent that it is.

It might be objected that Bell's phrase "to have an object in mind" properly applies to thinking *about* an object, or possibly also to apprehending it in perception or to comparable modes of awareness of it, whereas grasping a thought is not thinking *about* it, but thinking it: in grasping the thought, we are thinking about whatever the thought is about. But the objection, as it stands, merely serves to underline, not invalidate, the contrast between grasping a thought and apprehending other objects. For if, as Frege supposed, thoughts are independently existing objects, grasping a thought can only be a mode of awareness of an object, however unlike our awareness of an object of any other kind it may be. If thoughts are objects, they are objects standing to us in an epistemic relation utterly different from all others; and this suggests that Frege's theory should be amended by denying that thoughts are objects at all.

Bell's response to the difficulty concerns thoughts as contents of the activity of thinking rather than as the senses of sentences. His solution is to hold that thinking and judging should be construed as intransitive rather than transitive verbs: not on the model of agent-act-object, but on that of agent-act; thoughts, in other words, are the referents of mere cognate accusatives. And there his essay abruptly ends. It is true that he has published a sequel, "The art of judgement", [2], in which, taking off from a remarkable anticipation of Wittgenstein's observations about rules that he discovered in the *Critique of Pure Reason*, he seeks help from Kant's *Critique of Judgment*: but I shall not discuss this. Instead, I want to consider where Bell leaves us at the end of "Thoughts".

We have to ask whether the crisis is as grave as Bell believes it to be; and then to ask what will happen to the Fregean theory if Bell's resolution of the crisis is adopted. The end of "Thoughts" is abrupt, because, as with Frege's attempted solution of Russell's paradox, Bell has proposed an emendation of Frege's theory without reviewing the effect this would have on the merits of the theory already canvassed. As he remarks, Frege advanced the thesis that thoughts are not merely objects, but objects that exist timelessly and independently of us, because he thought it necessary in order to safeguard their objectivity. That is, since he allowed no intermediate status between the radically subjective and the wholly objective, he thought it followed from the fact that thoughts are accessible to all that they are independent of any. The objectivity

of thoughts forms for Bell another class of requirements upon an adequate theory of thoughts, requirements that he recognizes Frege's theory as meeting; above all, the requirement that different individuals can grasp the same thought, and that one individual can communicate a thought without residue to another. What Bell believes himself to have discovered is that Frege's theory contrives to satisfy the requirements of objectivity only at the price of violating other requirements that come under the head of interiority, namely that thoughts can be grasped and judged to be true by individual subjects.

We may well doubt that Frege needed to interpret the objectivity of thoughts so strongly as he did in order to meet the requirements that fall under that head, and yet feel uncertain whether the theory will continue to meet them if it is amended as Bell proposes; especially so since the emendation jettisons the Fregean principle, anticipated by Bolzano, that thoughts are not contents of the mind. Acts of thinking may be differentiated in many ways: the same thought may be grasped as the senses of different sentences, in one or more languages. But we have seen that there cannot be distinct ways of grasping it. Now when thinking and judging are construed as mental acts directed towards extramental objects which form the contents of the thoughts or judgments, there is no difficulty in seeing how thoughts and judgments made by others at other times *can* have the same contents, though it is not ruled out that some thoughts of mine may be such that nobody else's could have their content. But to say that "think" and "judge" should be construed as intransitive verbs is to say that which thought is grasped or judged to be true is a matter of the way in which one thinks or judges; and then it is much more doubtful whether an adequate sense can be given to saying that two people think or judge in the same way, and so may be said to have the same thought or make the same judgment. Such a sense depends upon conceptually separating the content from the act of thinking and providing it with a criterion of identity distinguishable from anything binding it to an event in the subject's mental life. Now is not that to say that we must construe that event on the model of agent-act-object? And does that not make Frege right, after all?

Reflexivity Others of Bell's requirements on a successful theory of thoughts stand in danger from his emendation. Among these are those coming under the head of 'reflexivity'. As Bell puts it ([1], p. 38), we can think and judge about thoughts and judgments; hence thoughts and judgments can figure as the objects of other thoughts and judgments as well as the contents of acts of thinking and judging. Bell rightly contends that Frege's own principles demand that the reference of such a phrase as "the sense of the expression '*A*' "—let alone, we may add, of one like "Boyle's law"—cannot but be an object. In thinking about thoughts, we may refer to them without expressing them, for instance by a name like "the fundamental theorem of algebra" or a description like "the weakest additional premise that would make the argument valid"; and the thought is then given to us in a particular way, as other objects are. Frege's theory indeed meets the requirements of reflexivity admirably: it will surely no longer be able to meet them if thoughts and judgments are not recognized as objects at all.

Reflexivity requires, then, that thoughts and their constituent senses must

be objects of some kind, which, like other objects, can be given to us in one way or another. That, Bell might reply, holds good when we are referring to them or thinking *about* them; but we have another way of apprehending them, namely grasping them, to which other objects furnish no analogy. It is, he might say, when we grasp them that we know them directly, rather than in some particular way. This might be objected to as a confusion. To think about something is to have it as an object of reference. The way we think about it is the sense: but, for that very reason, the sense is not a second object of our thinking, but simply the way in which we are thinking about the referent; it would therefore be better not to call grasping a thought a means of apprehending it, if by this is meant being aware of it as we are aware of what we perceive and what we think about. But Bell might retort that it is this very confusion which he is aiming to expose, since it is engendered precisely by Frege's mythology of the third realm, by his treating senses as objects. More exactly, senses are objects inasmuch as they are possible objects of reference and possible subjects of thought: but Frege's mythology makes them objects of the mental act of grasping; and the error lies in conceiving that act as having an object at all.

This solution cannot be adequate. It is not merely that senses *can* be objects of reference, things we think about as well as contents of our thinking, but that one of several ways of referring to a sense goes via our grasp of it: Frege is therefore justified in speaking of 'oblique' reference. It does not matter here whether he is right to invoke this notion in respect of the *oratio obliqua* clause that follows a verb like "believe": we are concerned, rather, with phrases of the form, "the thought that . . ." and "the sense of ' . . . '". As Bell correctly observes, it is because we grasp the direct sense of the sentences in the "that"-clause or of the expression in quotation marks that the phrase identifies a thought or a sense for us, namely the sense of that very sentence or expression: the prefix "the thought that" or "the sense of" requires us to turn from the referent to the way it is referred to. Grasping a sense, although primarily a way of directing our attention upon the referent, must also involve awareness of that sense itself: otherwise we could not exploit a grasp of the sense as one way of directing attention upon it, when it is the sense that we are thinking about. It thus seems, after all, that grasping a sense must be classed as a mode of apprehending it; if so, it constitutes a direct apprehension of it.

But what is it to apprehend something directly? The difference is that between referring to a thought and expressing it. We can, in general, understand a phrase that refers to a thought without knowing which thought it is that is referred to; but we cannot understand one that expresses a thought without thereby knowing which thought that is—that is, what it is to *express* a thought. However, in view of the obscurity of the notion of knowing which object satisfies a given condition—of knowing who committed the murder, for example—this intuitively natural reply does not furnish a sufficient explanation. It will not do, either, merely to say that to apprehend an object directly is to know its essential or constitutive properties. A game, for example, is constituted by its rules; so if a game is specified to me by a statement of its rules, I can identify it by its constitutive properties, even though I may not know anything else about it, such as whether it is enjoyable, when it was invented or where it is played. But the analogy breaks down. We can distinguish, with other objects, between their

constitutive properties and the way in which those properties are expressed, for instance between the rules of the game and the manner in which the rules are stated: by contrast, when a thought is identified as that expressed by a sentence, any variation in the way the sentence determines the thought, that is, in the sense of the sentence, will induce a variation in the thought.

The point is general, since any means of identifying an object is taken by Frege to be a sense: if not one expressible in the language as we have it, then at least one capable of expression in language. A means of identifying a thought may vary while the thought remains the same, but not if it is identified by being grasped. For grasping a thought is to other means of identifying it as expressing it is to other means of referring to it: any variation in the manner in which the thought is grasped will induce a variation in the thought itself. To grasp it *is* to have hold of a means of expressing it, or, if not that exactly, then of the principle whereby some possible (complex) sign would express it: it is, as it were, what is left of understanding when we take away the expression understood, comparable to the knowledge of the meaning of a word possessed by someone who has temporarily forgotten that word and is searching for it. In all other cases, a distinction obtains between what is identified and the means of identification; but, when a sense is grasped, the means of identification and the thing identified coincide. Although it can also be identified in other ways, a thought, or a sense generally, is something that can be used to identify itself, precisely because it can be expressed; and it can be expressed because it is already a means of identification, although, in the first instance, a means of identifying, not itself, but its referent. It is the use of an expression of the thought or sense to refer to that very thought or sense that prompts us to say that in grasping it, that is, in understanding the expression (actually or potentially), we apprehend it directly. This terminology is harmless provided that we explain the phenomenon in this way, and not as exemplifying a mysterious cognitive faculty for mental contact with objects of one particular kind, objects that mediate our awareness of objects of all other kinds. It would make no sense to speak of grasping other objects, or of apprehending them directly; there is therefore no substance to saying that we apprehend them only indirectly.

Now does this provoke the crisis that Bell discerns? That is far from evident. The theory requires that, when we refer to or think about anything, we refer to or think about it in a particular way, but that, in doing so, we are conscious of the way in which we are referring to or thinking about it: all that is actually essential is that we should be capable of becoming conscious of that. The theory further requires that such modes of identification should have sufficient structure for there to be a clear way of identifying *them*—a criterion of identity for the way in which a referent is given—so that it makes sense to speak of fastening upon any one such mode as an object of our thought. Finally, the theory requires that these modes of identification be capable of embodiment in linguistic expressions, by means of which they can be conveyed by one person to another. The embodiment consists, as Bell states, in an isomorphism: the contribution of features of the mode of identification or sense to the whole should be mirrored by the contribution of the parts of an expression to the manner in which the reference of the whole is determined. Given all this, it remains no mystery how a sense can become itself an object of thought, and how, among other

ways, it can be reflexively identified—that is, identified *by means of itself*. When so reflexively identified, there is then naturally the impression that we are apprehending it directly, in a manner in which no other type of object can be directly apprehended. The theory thus appears to account, without any incoherence, for the facts that prompt Bell to deny that senses are objects.

The third realm But is this a sufficient solution? Is not Bell still right to deny that grasping a sense is to be understood on the act-object model? Was not Frege wrong to conceive of thoughts as eternal, immutable objects, and does not his notion of the third realm remain a piece of misleading philosophical mythology? One way of rejecting it as mythology is to maintain that senses are intrinsically *of* expressions. Frege, again like Bolzano, argues that the existence of a sense does not depend upon its being expressed or even grasped; and this must be allowed, inasmuch as there are surely thoughts that have never been and never will be expressed or grasped. But this does not show that senses are not, metaphysically, *of* expressions, as the powers of a chess piece are *of* the piece. Although there are powers which no piece, in any past or future version of chess, has ever had or will ever have, to conceive of such a power is to conceive of a piece as having that power; and likewise, on the proposed view, to conceive of a sense is to conceive of an expression as having that sense, whether or not there ever has been or will be such an expression. This of course conflicts with Frege's view that there could be beings who grasped the thoughts we grasp, but not as expressed in language or symbolism. It is temerarious to deny this possibility: what we may reasonably deny is that Frege's theory of thoughts gave any explanation of how it may be so.

We need a distinction, among objects, that Frege did not draw, namely between transcendent objects and immanent objects. A dance step, for example, is an immanent object. There appears to be no clear answer to the question whether or not we should construe the statement that a dancer danced a certain step on the model of agent-act-object. It would be unreasonable to deny that the dance step is an object: two dancers can execute the same step, and the step may have been danced many times, and hence have existed a long time, before a particular dancer danced it; a dance step has objective properties, and can be spoken of and thought about. On the other hand, the step did not exist, and could not have existed, antecedently to or independently of anyone's dancing it: it is this which distinguishes it as an immanent object from a transcendent one like a shoe. Thoughts and other senses should not be banished, as Bell wishes to banish them, from the category of objects; but they are immanent, not transcendent, objects, and hence not inhabitants of a realm altogether independent of us and our activities. Since we *can* conceptually separate the thought from the particular act of thinking, it does no harm to construe that act as having an object. To characterize the act as objectless would suggest that we *cannot* conceptually distill its content; the opposite mistake, which was Frege's, is to regard the object as transcendent.

Spontaneity Appealing both to Kant and to Wittgenstein, Bell proposes as a requirement for a successful theory of thoughts what he calls the 'Principle of

Spontaneity', which lays down that the performance of a learned or rule-governed act should not be so explained as to presuppose the prior performance of an act of the same type. He maintains that the second incoherence he claims to detect in Frege's theory entails the violation of this principle. The theory does violate the principle, in my opinion; not, however, for this reason, but because of Frege's realism. That realism requires that we are able to recognize the sense as determining the reference, even when we ourselves have no capacity to identify the referent. On this view, the sense of a proper name constitutes a condition that an object must satisfy to be its referent, and that of a predicate a condition for it to apply to any object: the sense will be definite if it is determinate, for any object, whether or not it is the bearer of the name and whether or not the predicate applies to it. But this determination is impersonal: it is effected, not by us, but by the way things are. Hence, when Frege makes his demand that it be determined, for every object, whether or not it falls under a given concept, he frequently feels constrained to add that *we* may be unable to decide this. He therefore cannot explain a grasp of the sense of a name or predicate as consisting in an ability to recognize, in favorable circumstances, that the name does or does not refer to a given object, or that the predicate does or does not apply to it. Rather, a grasp of the sense must consist simply in knowing what condition an object must satisfy to be the referent of the name or for the predicate to apply. But such knowledge can amount only to a *judgment* that the name denotes, or the predicate applies to, any given object just in case that object satisfies such-and-such a condition; and thus the grasp of a sense is explained in terms of a judgment as to the truth of a certain thought, and Bell's principle of spontaneity is flouted.

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