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Non-Classical Syllogistic Inference and the Method of Resolution

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Abstract There are non-classical syllogistic inferences where only one of the premises has the form of an ordinary categorical, while the occurrence of the middle term in the other can be embedded in a Boolean term compound or occupy the position of syntactical object of a transitive verb, or both. The aim of this paper is to use the method of resolution, a technique of logical inference widely used in computer science, to show that syllogistic inferences, classical and non-classical alike, share an underlying pattern of cancellation-cum-substitution. We formulate four useful rules of this wider syllogistic and establish them as derived rules of natural deduction for first-order predicate logic.

1 Classical Aristotelian-Medieval syllogistic restricted itself to inferences where premises and conclusion are all categoricals of the familiar A, E, I, O types. Yet non-classical syllogistic patterns of a very similar kind are exemplified by inferences where only one of the premises has the form of an ordinary categorical, while the occurrence of the middle term in the other can be embedded in a Boolean term compound or occupy the position of syntactical object of a transitive verb, or both.[1]

The aim of this paper is to use the method of resolution, a technique of logical inference widely used in computer science, to show that syllogistic inferences, classical as well as non-classical, do indeed share an underlying pattern of cancellation-cum-substitution. In the process, we formulate four useful rules of this wider syllogistic and use the technique of resolution to establish them as derived rules within a familiar system of natural deduction for first-order predicate logic. We hope to demonstrate that these wider syllogistic patterns are common enough and powerful enough to deserve closer study.

2 Valid classical syllogisms can be reduced by "immediate inference" transformations of their premises and/or conclusion to one of two canonical types:

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Type I:	Every M is P	Type II:	Every M is P
(AAA)	Every S is M	(AII)	Some S is M
	Every S is P.		Some S is P.

(Where 'S' stands for the subject term of the conclusion, 'P' for its predicate term, and 'M' for the middle term that does not appear in the conclusion.)

Let us call the universal premise 'Every M is P' the *categorical premise*, and the other premise the *matrix premise*. Then a first approximation to a more comprehensive pattern exemplified by both Types I and II would be this: the conclusion is identical with the matrix premise except that the middle term 'M' has been replaced by the predicate term 'P' of the categorical premise.

The same pattern holds in a wide variety of cases, where the middle term 'M' in the matrix premise is embedded in a Boolean term compound or occupies the position of syntactical object of a transitive verb, or both:

(iii) Every M is P
Some (Every) S is Q and M
―――
Some (Every) S is Q and P .

(iv) Every M is P
Some (Every) S is R to some M
―――
Some (Every) S is R to some P .

(v) Every M is P
Some (Every) S is R to every non-M
―――
Some (Every) S is R to every non-P .

(vi) Every M is P
Some (Every) S is R to some Q and M
―――
Some (Every) S is R to some Q and P .

The pattern holds no matter how deeply embedded 'M' is in the matrix premise. Yet this inference is invalid:

(vii) Every M is P
Some (Every) S is R to some non-M
―――
Some (Every) S is R to some non-P .

It would be useful to have a rule that demarcates occurrences of 'M' in the matrix premise that do anchor valid syllogistic inferences from those that do not.

3 Reviewing Type II syllogisms, we see an alternative reading, with the particular premise as the categorical premise and the universal one in the matrix position:

Type II′:	Some M is P
(IAI)	Every M is S
	Some P is S .

The conclusion is the same as the matrix except that the middle term '*M*' has been replaced by '*P*', the predicate term of the categorical premise, with a *switch* of the matrix quantifier from 'Every' to 'Some'.

This pattern is equally discernible in the following cases:

(viii) Some *M* is *P*
Some (Every) *S* is *R* to every *M*
———
Some (Every) *S* is *R* to some *P* .

(ix) Some *M* is *P*
Some (Every) *S* is *R* to something that is *F* to every *M*
———
Some (Every) *S* is *R* to something that is *F* to some *P* .

But the following case is not a valid inference:

(x) Some *M* is *P*
Some (Every) *S* is *R* to everything that is *F* to every *M*
———
Some (Every) *S* is *R* to everything that is *F* to some *P* .

Here, too, it would be useful to have a rule.

4 Patterns closely analogous to the ones discerned in Sections 2 and 3 are exhibited in sentential logic by the *resolution principle*, a powerful inference rule that anchors a comprehensive method of generating and testing inferences widely used in computer science, the *method of resolution*.[2]

The resolution principle operates on a pair of disjunctive clauses to yield a third disjunctive clause, called their resolvent:

$$\frac{\neg p \vee q \qquad r \vee p}{r \vee q}$$

(where p, q, r can be any sentences, atomic or compound).

The method of resolution, in its purely sentential application, works by reducing each sentence (premise) to conjunctional normal form (CNF), i.e., an n-place conjunction of disjunctive clauses of literals, i.e., atomic sentences or their negations. Then, the resolution principle is repeatedly called upon to "resolve" pairs of distinct disjunctive clauses containing opposed (positive/negative) literals (like 'p' and '$\neg p$'). The effect is to "cancel" such pairs of opposed literals, so the method of resolution could be called, perhaps more aptly, the method of cancellation.

Notice that the premise '$\neg p \vee q$' plays the role of the categorical premise, '$r \vee p$' plays the role of the matrix premise, and the conclusion '$r \vee q$' is the same as the matrix, except that 'q' has been substituted for 'p', in complete analogy with the pattern discerned in Section 2.

The resolution principle includes as special cases modus ponens ($\neg p \vee q$, p/q) and the transitivity of the conditional ($\neg p \vee q$, $\neg q \vee r/\neg p \vee r$), and thus has the required flexibility to anchor all the different kinds of sentential derivations that come up during natural deduction instantiation of the premises of classical syllogisms translated into standard notation (using the instantial variable 'a'):

Type I Syllogism	Type II Syllogism
$(\forall x)\quad (Mx \to Px)$	$(\forall x)\quad (Mx \to Px)$
$(\forall x)\quad (Sx \to Mx)$	$(\exists x)\quad (Sx \wedge Mx)$
$(\forall x)\quad (Sx \to Px)$.	$(\exists x)\quad (Sx \wedge Px)$.
Instantiated Type I	**Instantiated Type II**
$Ma \to Pa$	$Ma \to Pa$
$Sa \to Ma$	$Sa \wedge Ma$
$Sa \to Pa.$	$Sa \wedge Pa.$
Eliminate $\to$:	**Eliminate $\to$:**
$\neg Ma \vee Pa$	$\neg Ma \vee Pa$
$\neg Sa \vee Ma$	$Sa \wedge Ma$
$\neg Sa \vee Pa.$	$Sa \wedge \quad Pa.$
(Straightforward instance of the resolution principle)	(Application of the method of resolution, using Modus Ponens, an instance of the resolution principle)

Notice that standard translations of classical syllogisms with non-compound S, P, M terms are just two steps away from (sentential) CNF, since all that is required to get them into CNF is instantiation and elimination of the conditional in favor of negation and disjunction.

We will exploit the close correspondence between syllogistic patterns of inference and patterns of resolution/cancellation to provide an analysis of the former and establish useful general rules of inference, sentential versions first and then, on their basis, quantified, syllogistic versions.

5 Our formulation of these rules of inference will require a distinction between an *affirmative* as opposed to a *negative* occurrence of a component sentence, closed or open, within a given sentence.

Let us adopt a standard notation for sentential logic, using the connectives '$\neg$', '$\wedge$', '$\vee$', '$\to$', and '$\leftrightarrow$', and follow the standard convention of negating an atomic sentence by directly prefixing the negation sign to it, while negating a compound sentence by first enclosing it in parentheses, and prefixing the negation sign to the left parenthesis. We proceed to eliminate occurrences of '$\to$' and '$\leftrightarrow$' by replacing '$p \to q$' by '$\neg p \vee q$' and '$p \leftrightarrow q$' by '$(\neg p \vee q) \wedge (\neg q \vee p)$'.

The *scope* of a negation sign '$\neg$' is given by the following rules: (i) If a '$\neg$' sign is immediately followed by a '(', then its scope is all of the material between that '(' and the corresponding ')'. (ii) Otherwise, its scope is the atomic sentence that immediately follows it.

We now define an occurrence of a component sentence p (atomic or compound) within a sentence $\Phi(p)$ as *positive* if and only if it lies in the scope of zero or an even number of '$\neg$' signs, and we write $\Phi(\overset{+}{p})$. If p lies within the scope of an odd number of '$\neg$' signs in $\Phi(p)$, then the occurrence is *negative*, and we write $\Phi(\bar{p})$.[3]

For example, to find whether the occurrence of p in '$(p \wedge r) \rightarrow s$' is negative or positive, we replace it by '$\neg(p \wedge r) \vee s$' and note that p lies in the scope of just one '$\neg$' sign; thus the occurrence of p is negative. But the occurrence of p in '$[(p \vee q) \rightarrow r] \rightarrow s$' is positive, because p is in the scope of two '$\neg$' signs when we move to '$\neg[\neg(p \vee q) \vee r] \vee s$'. The sentence '$(p \vee q)$' also has a positive occurrence, while r has a negative occurrence in the same sentence.

Moving to first-order quantification theory in a standard notation that includes the quantifiers '$\exists$' and '$\forall$', variables 'x', 'y', 'z', etc., and atomic and compound open sentences 'Sx', 'Px', 'Mx', 'Ay', 'By', 'Rxy', 'Fxy', '$Gxyz$', '$Ay \wedge Rxy$', etc., we can assign a positive or negative occurrence to every *closed sentence*, atomic or compound, and every *open sentence*, atomic or compound; in particular, we can assign a + or − value to every open sentence in one free variable (this is the widest possible generalization of a traditional *monadic term*, and it is monadic terms that play the central role in syllogistic reasoning).[4]

For example, in the sentence '$(\forall x)[(\forall y)(Ay \rightarrow Rxy) \rightarrow Bx]$', '$Bx$' has a positive occurrence, 'Rxy' has a negative occurrence, 'Ay' has a positive occurrence, '$(\forall y)(Ay \rightarrow Rxy)$' has a negative occurrence, etc., as we see once we move to '$(\forall x)[\neg[(\forall y)(\neg Ay \vee Rxy)] \vee Bx]$'.

6 The premises and the conclusion of the resolution principle

$$\begin{array}{c} \neg p \vee q \\ \underline{p \vee r} \\ q \vee r \end{array}$$

are all in CNF *relative to p, q, r*, i.e., treating p, q, r as literals (even if they are not atomic). We will express the result of reducing a sentence Φ to CNF *relative* to a sentence p contained in Φ as CNF $\Phi(p)$.

The inferential mechanism of the resolution principle can be seen to operate through a mutual cancellation of the pair of affirmative-versus-negative occurrences of the recurrent component p (the "middle term" of classical syllogistic) and an insertion/substitution of the partner of the negative occurrence of p in the place of the positive occurrence of p.

But what if the matrix premise $\Phi(p)$ is not in CNF relative to the "middle term" p? We will show that when $\Phi(p)$ is reduced to CNF $\Phi(p)$, all the occurrences of p in CNF $\Phi(p)$ that were generated from the original occurrence of p in question will have the same + or − value as the original. This means that the value of p remains invariant through all the equivalence transformations that are required to get us from $\Phi(p)$ to CNF $\Phi(p)$, and, therefore, that value in $\Phi(p)$ *predicts* one or more corresponding affirmative or negative occurrences of p in CNF $\Phi(p)$; hence, it predicts the possibility of cancellation against the negative occurrence of p in the categorical premise in accordance with a *direct* application of the resolution principle. Thus, + or − values function as indicators of forthcoming cancellation/substitution.

The steps that lead from $\Phi(p)$ to CNF $\Phi(p)$ are the following[5]:

Step 1: Eliminate $\rightarrow$ and $\leftrightarrow$ in favor of $\wedge$, $\vee$, and $\neg$.
Step 2: Move every $\neg$ sign as far inward as it can go, by:
(2.1) eliminating double negations
(2.2) using De Morgan's Laws.
Step 3: Use the distributive laws for $\wedge$ and $\vee$.

Of these steps, (1) has been built into the method of assigning + or − values, so it has been already performed.

Step (2.1) obviously leaves overall + or − values unaffected because it removes two $\neg$ signs at a time, leaving the overall number of $\neg$ signs even or odd just as before.

Step (2.2) proceeds in accordance with DeMorgan's laws to distribute a $\neg$ sign whose scope is an entire conjunction or disjunction to the components of that conjunction or disjunction which were within the scope of the original $\neg$ sign anyway, so it leaves the overall value unaffected by neither increasing nor diminishing the number of $\neg$ signs that each component is in the scope of.

Step (3) obviously leaves + or − values unaffected. It may multiply the number of occurrences of a given sentence, but each occurrence will retain the positive or negative value of the original.

Therefore, we conclude that reduction to CNF $\Phi(p)$ leaves the overall + or − value of p in Φ unaffected. (The sequence of steps is not relevant either.)

7 Now it becomes easy to show that the following two rules hold for sentential logic (SR for "Syllogistic Rule"):

$$\text{SRI: } \begin{array}{ll} (1) & \neg p \vee q \\ (2) & \Phi(\overset{+}{p}) \\ \hline (3) & \Phi(q/\overset{+}{p}). \end{array} \qquad \text{SRII: } \begin{array}{ll} (1) & \neg p \vee q \\ (2) & \Phi(\bar{q}) \\ \hline (3) & \Phi(p/\bar{q}). \end{array}$$

(Where $\Phi(q/\overset{\pm}{p})$ represents the result of replacing one or more positive/negative occurrences of p in $\Phi(\overset{\pm}{p})$ by q. Note that the resolution principle is a special case of SRI.)

Proof outline of SRI: (There may be more than one occurrence of p in Φ but each one can be treated separately.)

Let the sequence of steps that got us from $\Phi(\overset{+}{p})$ to CNF $\Phi(p)$ be $S_1, S_2, \ldots, S_n$. CNF $\Phi(p)$ may contain more than one occurrence of p generated from the original affirmative occurrence, but they will all be affirmative as well. Now pair the first premise '$\neg p \vee q$' with each disjunctive clause in CNF $\Phi(p)$ that contains one of these affirmative occurrences, apply the principle of resolution, and replace the clause which functioned as the second premise in resolution by the conclusion of the resolution argument. The cumulative result can be represented as CNF $\Phi(p)(q/\overset{+}{p})$. Now apply a sequence of corresponding steps in exact reverse order, i.e.: $S'_n, S'_{n-1}, \ldots, S'_2, S'_1$, to get from CNF $\Phi(p)(q/\overset{+}{p})$ to $\Phi(q/\overset{+}{p})$.[6]

Proof outline of SRII: This is a special case of SRI:

$\neg p \vee q$ is equivalent to

(1′) $\neg(\neg q) \vee \neg p$

$\Phi(\bar{q})$ is equivalent to

(2′) $\Phi(\neg(\neg q)/\bar{q})$.

Since q is in the scope of an odd number of $\neg$ signs in Φ, $\neg q$ is in the scope of an even number of $\neg$ signs in Φ, hence we can reinterpret (2′) as:

(2″) $\Phi((\overset{+}{\neg} q))$.

Applying SRI to (1′) and (2″) we get:

(3′) $\Phi((\neg p)/(\overset{+}{\neg} q))$.

But substituting (affirmative) occurrences of $\neg p$ for affirmative occurrences of $\neg q$ in Φ can be reinterpreted as substituting (negative) occurrences of p for negative occurrences of q in Φ. Hence we get our desired conclusion:

(3) $\Phi(p/\bar{q})$.

8 We are now in a position to state and prove the syllogistic, quantified versions of the sentential principles SRI and SRII.

SRI*: Categorical Premise: $(\forall x)\ (Fx \to Gx)$ or $(\forall x)\ (\neg Fx \vee Gx)$
Matrix Premise: $\Phi(\overset{+}{Fy})$
Conclusion: $\Phi(Gy/\overset{+}{Fy})$.

where Fx, Gx are open sentences in one free variable x, and Fy, Gy are open sentences in one free variable y, not necessarily distinct from x.

In our proof we will use standard techniques of natural deduction.[7] The result will be to establish SRI* and SRII* as derived rules of natural deduction.

Proof outline of SRI:* We begin by working on the Matrix Premise. We have already eliminated $\to$ and $\leftrightarrow$ in favor of $\wedge$, $\vee$, and $\neg$. We now drive negation signs in until no quantifier is in the scope of a negation sign. Let the sequence of equivalence transformations that accomplishes this be: $S_1, S_2, \ldots, S_{n-1}, S_n$. Let the result be represented as $\Phi'(\overset{+}{Fy})$.

We instantiate each bound variable in $\Phi'(Fy)$, moving from left to right and using US (Universal Specification) or ES (Existential Specification) according to the case. Let the steps of instantiation be $i_1, i_2, \ldots, i_{K-1}, i_K$.

Step i_j uses US or ES to eliminate the j-th quantifier in $\Phi'(\overset{+}{Fy})$, say $(Q_j z_j)$, and substitute the instantial variable a_j for the bound variable z_j. Let $y = z_m$. Then the cumulative result of this sequence of instantiations is a complex truth-function of atomic sentences of the form $\Phi''(\overset{+}{Fa_m})$.

We now use the same instantial variable a_m to instantiate the bound variable x in the categorical premise, thus ending up with the following pair of statements:

$$\neg Fa_m \vee Ga_m$$
$$\Phi''(\overset{+}{Fa_m}).$$

By the sentential version of SRI, which we have already proved, these two yield:

$$\Phi''(Ga_m/\overset{+}{Fa_m}).$$

We now use generalization in reverse. We use a sequence of steps $g_k, g_{k-1}, \ldots,$ g_2, g_1, such that g_j corresponds to the step i_j of instantiation. If i_j used US (ES), g_j will use UG (EG) to restore quantifier $(Q_j z_j)$ and bound variable z_j by eliminating the instantial variable a_j. In the case of g_m, it will restore the quantifier $(Q_m y)$ and the bound variable y in the open sentence Gy. The result will clearly be $\Phi'(Gy/\overset{+}{F}y)$.

We now perform corresponding steps $S'_n, S'_{n-1}, \ldots, S'_2, S'_1$ to $\Phi'(Gy/\overset{+}{F}y)$ to restore negation signs to positions corresponding to their original positions in $\Phi(\overset{+}{F}y)$. We thus end up with $\Phi(Gy/\overset{+}{F}y)$.

Proof outline of SRII:* Quantified SRII can be formulated as follows:

SRII*:	Categorical Premise:	$(\forall x)(Fx \rightarrow Gx)$ or $(\forall x)(\neg Fx \vee Gx)$
	Matrix Premise:	$\Phi(\bar{G}y)$
	Conclusion:	$\Phi(Fy/\bar{G}y)$.

The proof proceeds exactly as that of SRI*, except that it uses sentential SRII instead of sentential SRI in the relevant part of the proof between the chains of instantiation (specification) and generalization.

9 We must now tackle the issue of the proper formulation of syllogistic rules that cover cases where the categorical premise is a particular, not a universal predication.

It will help if we keep two things apart: (a) The detailed *process of inference*, which proceeds first through reduction to CNF and then cancellation (resolution) of opposed sentences, and (b) *the result* of this process, the conclusion of the inference, which may or may not be easily recoverable from the matrix premise as it stands by simple substitution or some other transformation of comparable simplicity.

The efficacy of SRI* and SRII* as rules of inference is due to the fact that the result of inference by cancellation in these cases can easily be recovered from the matrix premise by simple substitution, so that we are able to "jump to the conclusion" eschewing the actual mechanics of inference. In the cases where the categorical premise is particular, however, such "jumping to the conclusion" is possible only under rather stringent conditions.

Let us initially restrict ourselves to sentential logic for the sake of simplicity.

The simplest possible case exemplifying sentential patterns corresponding to the syllogistic ones discerned in Section 3 above is this:

"Categorical" Premise:	$p \wedge q$
Matrix Premise:	$\neg p \vee r$
Conclusion:	$q \wedge r$.

The conclusion can be recovered from the matrix premise by substituting q for $\neg p$, but also changing $\vee$ to $\wedge$.

But if we introduce a little additional structure in the matrix premise, getting the result involves considerable rearrangement of the matrix.

For example, let us consider the following premises:

(1) $p \wedge q$
(2) $(p \wedge r) \rightarrow s$.

The CNF for (2) is (2′): $\neg p \vee \neg r \vee s$.
From (1) and (2′) we get the conclusion

(3) $q \wedge (\neg r \vee s)$ (or $q \wedge (r \rightarrow s)$)

which departs quite considerably from (2) in its structure.

Even worse is:

(1) $p \wedge q$
(2) $(p \vee r) \rightarrow s$.

The CNF for (2) is (2′) $(\neg p \vee s) \wedge (\neg r \vee s)$ which together with (1) gives us the conclusion

(3) $(q \wedge s) \wedge (\neg r \vee s)$.

In both these cases, the negative value of the occurrence of p in the matrix premise is still a reliable indicator of forthcoming cancellation, but the results of cancellation depart considerably from the structure of the matrix premise and cannot be usefully recovered from it.

It is quite clear that only under stringent conditions can we get a neat, predictable conclusion.

The only rule in the cases we are considering that is useful enough and allows an extension involving quantification seems to be the following—a straightforward generalization of the simplest case we considered above:

Sentential SRIII:

First Premise:	$p \wedge q$
Matrix Premise:	$\Phi((p \overset{+}{\rightarrow} r))$
Conclusion:	$\Phi((q \wedge r)/(p \overset{+}{\rightarrow} r))$.

This rule underlies another one which can be reduced to it:

Sentential SRIV:

First Premise:	$p \wedge q$
Matrix Premise:	$\Phi((p \overset{-}{\wedge} r))$
Conclusion:	$\Phi((q \rightarrow r)/(p \overset{-}{\wedge} r))$.

Principle SRIV applied to the first case we considered above gives a different conclusion:

$$\frac{\begin{array}{l} p \wedge q \\ (p \wedge r) \rightarrow s \end{array}}{(q \rightarrow r) \rightarrow s.}$$

This conclusion is weaker than the one we got before by going through the detailed process of inference, but it can be recovered by an orderly process of substitution from the matrix premise.

The province of syllogistic reasoning can now be seen to consist of those cases where the combinatorial possibilities for cancellation/resolution of opposed pairs of "middle terms" are tractable enough to permit the reaching of a conclusion from the matrix premise through simple substitutions.

10 *Proof outline of Sentential SRIII:* Our premises are:

First Premise: (1) $p \wedge q$
Matrix Premise: (2) $\Phi((p \overset{+}{\rightarrow} r))$.

We use steps $S_1, S_2, \ldots, S_{n-1}, S_n$ to reduce (2) to its CNF, say (2′). In the process we get one or more disjunctive clauses with positive occurrences of $p \rightarrow r$. Each one of these clauses has the form $(p \rightarrow r) \vee \Psi$, or, equivalently, $\neg p \vee r \vee \Psi$. Using resolution on (1) and each such clause, we obtain in each case:

$q \wedge (r \vee \Psi)$ or equivalently $(q \wedge r) \vee (q \wedge \Psi)$ which implies $(q \wedge r) \vee \Psi$.

The cumulative result of substituting in (2′) the results of such resolution for the clauses containing the positive occurrences of $p \rightarrow r$ will clearly be (3′): CNF $\Phi(p \rightarrow r)((q \wedge r)/(p \overset{+}{\rightarrow} r))$.

Applying to (3)′ a sequence of corresponding steps $S'_n, S'_{n-1}, \ldots, S'_2, S'_1$ we get $\Phi((q \wedge r)/(p \overset{+}{\rightarrow} r))$.

Proof outline of Sentential SRIV: We start with the premises:

First Premise: (1) $p \wedge q$
Matrix Premise: (2) $\Phi((p \overset{-}{\wedge} r))$.

Since $(p \wedge r)$ has an overall negative value in (2), it is in the scope of an odd number of negation signs.

(2) is equivalent to

(2′) $\Phi(\neg(\neg(p \wedge r))/(p \overset{-}{\wedge} r))$.

Clearly, $\neg(p \wedge r)$ has an overall positive value in (2′) and (2′) can be reinterpreted as:

(2″) $\Phi((\neg(p \overset{+}{\wedge} r)))$.

Applying SRIII to (1) and (2″) we get

(3′) $\Phi(\neg(q \rightarrow r)/(\neg(p \overset{+}{\wedge} r)))$.

But substituting (affirmative) occurrences of $\neg(q \rightarrow r)$ for affirmative occurrences of $\neg(p \wedge r)$ in Φ can be reinterpreted as substituting (negative) occurrences of $(q \rightarrow r)$ for negative occurrences of $(p \wedge r)$. Hence we get our desired conclusion:

(3) $\Phi((q \rightarrow r)/(p \overset{-}{\wedge} r))$.

11 We can now state and prove the quantified versions of SRIII and SRIV. The expressions used in stating these principles appear complicated, but the substitutions involved are simple and clearly indicated.

	SRIII*	**SRIV***
Categorical Premise: (1)	$(\exists x)\ \overset{+}{(Fx \wedge Gx)}$	$(\exists x)\ \overset{+}{(Fx \wedge Gx)}$
Matrix Premise: (2)	$\Phi\big(\overbrace{(\forall y)\Psi(\overbrace{(Fy \rightarrow Wy)}^{+})}^{}\big)$	$\Phi\big(\overbrace{(\forall y)\Psi(\overbrace{(Fy \wedge Wy)}^{-})}^{}\big)$
Conclusion: (3)	$\Phi\left\{\dfrac{(\exists y)\Psi\big((Gy \wedge Wy)/\overbrace{(Fy \rightarrow Wy)}^{+}\big)}{\underbrace{(\forall y)\Psi\big((Fy \rightarrow Wy)\big)}_{+}}\right\}$	$\Phi\left\{\dfrac{(\exists y)\Psi\big((Gy \rightarrow Wy)/\overbrace{(Fy \wedge Wy)}^{-}\big)}{\underbrace{(\forall y)\Psi\big((Fy \wedge Wy)\big)}_{+}}\right\}$.

Quantified SRIII*

Proof outline: We begin as in the proof of quantified SRI* by working on the Matrix Premise. We have already eliminated $\rightarrow$ and $\leftrightarrow$ in favor of $\wedge$, $\vee$, and $\neg$. We now drive negation signs in until no quantifier is in the scope of a negation sign. Since the expression consisting of the universal quantifier and its scope has a positive occurrence, the quantifier itself remains unchanged. Let the sequence of equivalence transformations that accomplishes this be: $S_1, S_2, \ldots, S_{n-1}, S_n$. The result can be represented as:

$$(2')\quad \Phi'\big(\overbrace{(\forall y)\Psi'(\overbrace{(Fy \rightarrow Wy)}^{+})}^{+}\big).$$

We first instantiate the categorical premise by ES using an instantial variable not present in (1) or (2), say 'a_0':

We get:

$$(1')\quad Fa_0 \wedge Ga_0.$$

We now instantiate each bound variable in (2′), moving from left to right and using US or ES according to the case. Let the steps of instantiation be $i_1, i_2, \ldots, i_{k-1}, i_k$.

Step i_j uses US or ES to eliminate the j-th quantifier in (2′), say $(Q_j z_j)$, and substitute the instantial variable a_j for the bound variable z_j. Let $y = z_m$. Then Q_m is $\forall$ and a_m can be chosen to be a_0. The cumulative result of this sequence of instantiations is a complex truth-function of atomic sentences of the form

$$(2'')\quad \Phi''\big(\overbrace{\Psi''(\overbrace{(Fa_0 \rightarrow Wa_0)}^{+})}^{+}\big).$$

Applying sentential SRIII to (1′) and (2″) we get:

$$(3')\quad \Phi''\big(\underbrace{\Psi''((Ga_0 \wedge Wa_0)/\overbrace{(Fa_0 \rightarrow Wa_0)}^{+})}_{+}\big).$$

We now generalize in reverse. We use a sequence of steps $g_k, g_{k-1}, \ldots, g_2$, g_1, such that g_j corresponds to the step i_j of instantiation. If i_j used US(ES), g_j will use UG(EG) to restore quantifier $(Q_j\ z_j)$ and bound variable z_j by eliminating the instantial variable a_j. In the case of g_m, however, it will use EG to put the quantifier $(\exists y)$ in place of the former quantifier $(\forall y)$ because premise (1′) was obtained by ES from (1). The cumulative result will clearly be:

$$(3'')\quad \Phi'\big((\exists y)\Psi'(Gy \wedge Wy)/(\forall y)\Psi'(\overbrace{(Fy \rightarrow Wy)}^{+})\big).$$

Applying corresponding steps $S'_n, S'_{n-1}, \ldots, S'_2, S'_1$ to (3″) to restore negation signs to their original positions, we end up with

$$(3) \quad \Phi((\exists y)\Psi((Gy \wedge Wy)/\overbrace{(Fy \rightarrow Wy)}^{+})/\underbrace{(\forall y)\Psi((Fy \rightarrow Wy))}_{+}).$$

Quantified SRIV: Proof Outline:*

The proof proceeds exactly as that of SRIII*, except that it uses sentential SRIV instead of sentential SRIII in the relevant part of the proof between the chains of instantiation and generalization.

12 The reader can easily verify that cases (iii) to (vi) of Section 2, when translated in a standard way into the language of first-order predicate logic, are instances of SRI*, since the middle term '*M*' yields an open sentence that has a positive occurrence in the matrix premise, so that cancellation can occur against the negative occurrence of '*Mx*' in the categorical premise '$(\forall x)(\neg Mx \vee Px)$' upon instantiation. The contrary is the case with case (vii).

Similarly, cases (viii) and (ix) of Section 3, when translated, are instances of SRIII*, while (x) is not. The matrix premise in case (ix) translates as:

$$(\exists x)/(\forall x)\{Sx \wedge (\exists y)[(\forall z)(Mz \rightarrow Fyz) \wedge Rxy]\}$$

where the expression '$(\forall z)(Mz \rightarrow Fyz)$' has an overall positive occurrence, and the expression '$Mz \rightarrow Fyz$' has an overall positive occurrence within the scope of the quantifier '$\forall z$', so the conditions of SRIII* are met.

On the contrary, the matrix premise of case (x) translates as:

$$(\exists x)/(\forall x)\{Sx \wedge (\forall y)[(\forall z)(Mz \rightarrow Fyz) \rightarrow Rxy]\}$$

where the problem is that when the second conditional is eliminated, it becomes clear that the expression '$(\forall z)(Mz \rightarrow Fyz)$' has an overall negative occurrence, which violates one of the conditions of SRIII* (when the negation sign in '$[\neg(\forall z)(Mz \rightarrow Fyz) \vee Rxy]$' is driven in, '$\forall z$' changes to '$\exists z$' and we have to use different instantial variables to instantiate the existentially quantified particular categorical premise and the expression '$(\exists z)\neg(Mz \rightarrow Fyz)$' so that cancellation of the middle term '*M*' cannot occur).

13 In some cases, we need to "doctor" one or more premises before applying one of the SR* rules. For example, let us consider the valid argument: Some *M* are *B*, Every *R* to some *M* is *C*/ Every *R* to every *B* is *C*. This goes over as follows:

(1) $(\exists x)(Mx \wedge Bx)$

(2) $(\forall x)\{[\exists y)(My \wedge Rxy)] \rightarrow Cx\}$.

We have to first eliminate $\rightarrow$ before applying any of the SR* rules:

(2′) $(\forall x)[\neg((\exists y)(My \wedge Rxy)) \vee Cx]$.

If we just drive the $\neg$ sign in we get:

(2″) $(\forall x)[(\forall y)(\neg(My \wedge Rxy)) \vee Cx]$.

Now we can apply SRIV*, to get:

(3) $(\forall x)[(\exists y)(\neg(By \rightarrow Rxy)) \vee Cx]$.

(3) is equivalent to

(3′) $(\forall x)[\neg((\forall y)(By \rightarrow Rxy)) \vee Cx]$

and

(3″) $(\forall x)[((\forall y)(By \rightarrow Rxy)) \rightarrow Cx]$.

(3″) is easily translated into: Every *R* to every *B* is *C*.

Another obvious form of "doctoring" is rearranging the order of quantifiers in order to get a categorical premise with the "middle term" in subject position. For example, if we have as premises:

(1) Some *A* is *R* to every *M*, i.e., $(\exists x)\{Ax \wedge (\forall y)(My \rightarrow Rxy)\}$

and

(2) Some *B* is *F* to some *M*, i.e., $(\exists x)\{Bx \wedge (\exists y)(My \wedge Fxy)\}$.

(2) is subject to the following equivalence transformations:

(2′) $(\exists x)(\exists y)\{Bx \wedge (My \wedge Fxy)\}$

(2″) $(\exists y)(\exists x)\{My \wedge (Bx \wedge Fxy)\}$

(2‴) $(\exists y)\{My \wedge (\exists x)(Bx \wedge Fxy)\}$.

We can now apply SRIII* with (2‴) as the categorical premise and (1) as the matrix premise, to get:

(3) $(\exists x)\{Ax \wedge (\exists y)[(\exists z)(Bz \wedge Fzy) \wedge Rxy]\}$.

In English: Some *A* is *R* to something that some *B* is *F* to.

A general strategy suggests itself. We first check to see if two occurrences of a given monadic term in two premises are opposed so that cancellation may be forthcoming, and then try to doctor the premises accordingly (that is, with respect to the given term as the "middle term" of a syllogistic inference) to get instances of one of the SR* rules.

If cases that require some such prior doctoring be included, I believe it is no exaggeration to claim that about 80–90% of the examples of inferences in current logic textbooks are in essence syllogistic.[8] The fact that logicians, when called upon to construct fairly complex textbook examples of inferences involving quantifiers, almost invariably tend to fall back on syllogistic patterns, doctored or undoctored, is adequate testimony to the pervasiveness, naturalness, and reach of syllogistic reasoning in ordinary non-mathematical thinking.

NOTES

1. My study of non-classical syllogistic is heavily indebted to the work of Fred Sommers. (See [8], especially Chapters 7 and 9, and [9]. See also recent studies of his work in

syllogistic in Englebretsen [2].) His method of cancellation and his unfamiliar +− notation are close relatives of the method of resolution and can be studied against its background.

2. The method of resolution was developed by J. A. Robinson ([6] and [7]). A useful account of it can be found in Davis and Weyuker [1] Part 3, Chapters 11 and 12.

3. The preceding definitions of the scope of a negation sign and the positive/negative occurrence of a component are indebted to McIntosh [4] pp. 395-396.

4. For an illuminating discussion of the relationship among terms, predicates, and open sentences, see Quine [5], Chapter 20.

5. See [1], pp. 236-237.

6. In the interest of brevity, we will not give an explicit formulation of the correspondence intended between steps S_n and S_n', since it is quite obvious.

7. See Mates [3], Chapters 6 and 7, for a representative example.

8. The import of Sommers's work is demonstrating how much inference can be accomplished without overt instantiation, i.e., by methods that do not employ the full apparatus of bound variables of standard predicate logic. Syllogistic inference occupies the very core of his algebraic inferential techniques.

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