Notre Dame Journal of Formal Logic
Volume 34, Number 2, Spring 1993

Apodeictic Ecthesis

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Abstract A formal interpretation is constructed for Aristotle's apodeictic syllogistic, including the proofs by ecthesis. Ecthesis is here taken to involve an appeal to singular propositions. A system of singular ecthesis is constructed, and the whole system of apodeictic syllogisms is based on it. On the basis of this reduction, it is argued that (1) the 'necessarily' in Aristotle's apodeictic propositions is, in the first instance, a *de re* predicate-modifier; (2) Aristotle's system is not a modal logic in the modern sense; (3) an extensionalist reading can be given of Aristotle's apodeictic syllogistic; (4) the modal syllogistic formalizes some, but not all, aspects of Aristotelian metaphysics.

0 Introduction The present paper extends the results of Thom [8] by showing that Aristotle's apodeictic syllogistic can be based on a system of singular syllogisms and rules of ecthesis. The claim is not that Aristotle intended his apodeictic syllogistic to be so based: the case against such an interpretation is documented in Smith [5] (§5). Rather, the claim is simply that Aristotle's system can be so based, just as his assertoric syllogistic can be based—in the manner of Thom [6]—on a system of 'expository' assertoric syllogisms, and that this reduction to the logic of singular apodeictic propositions casts light on a number of questions of philosophical interpretation. These are: (1) the question—posed in Becker [2]—whether the 'necessarily' in Aristotle's apodeictic propositions is a *de re* predicate-modifier or a *de dicto* proposition-forming operator on propositions, and the question—posed in Patterson [4]—whether it is a modifier of the copula; (2) the question—made acute by Wieland's [11] suggestion that apodeictics do not imply their corresponding assertorics—whether Aristotle's system is indeed a modal logic in the modern sense; (3) the question whether an extensionalist reading can be given of Aristotle's apodeictic syllogistic; (4) the question—which motivates much of [4] and Van Rijen [10]—of the relation between the modal syllogistic and Aristotelian metaphysics.

Received January 20, 1992; revised April 27, 1992

There are four sections. In Section 1 it is argued that if Aristotle's system is understood to include his remarks on the *rejection* of formulas then it is inconsistent; accordingly, a revised, consistent system is formulated. Section 2 presents an interpretation of Aristotle's apodeictic ecthetic proofs, which is axiomatised in a system properly including the revised Aristotelian system. In Section 3, semantics are developed for this system. The system is shown to be sound but incomplete relative to these semantics. Section 4 deals with the four questions of philosophical interpretation mentioned above.

1 A revised Aristotelian system Aristotle's presentation of apodeictic syllogistic is based on some unspecified systematization of non-modal syllogistic. In addition, it uses the following modal axioms:

Le conv $\dfrac{Lba^e}{Lab^e}$ **Li conv** $\dfrac{Lba^i}{Lab^i}$ **La conv** $\dfrac{Lba^a}{Lab^i}$

Barbara LLL $\dfrac{Lcb^a Lba^a}{Lca^a}$ **Celarent LLL** $\dfrac{Lcb^e Lba^a}{Lca^e}$

Darii LLL $\dfrac{Lcb^a Lba^i}{Lca^i}$ **Ferio LLL** $\dfrac{Lcb^e Lba^i}{Lca^o}$

Barbara LXL $\dfrac{Lcb^a ba^a}{Lca^a}$ **Celarent LXL** $\dfrac{Lcb^e ba^a}{Lca^e}$

Darii LXL $\dfrac{Lcb^a ba^i}{Lca^i}$ **Ferio LXL** $\dfrac{Lcb^e ba^i}{Lca^o}$

with 'Lba^a' for 'b belongs necessarily to all a' (or 'all a must be b'), 'Lba^e' for 'b belongs necessarily to no a' (or 'no a can be b'), 'Lba^i' for 'b belongs necessarily to some a' (or 'some a must be b'), and 'Lba^o' for 'b necessarily does not belong to some a' (or 'some a cannot be b').

The system assumes rules for the Direct Reduction of other theses to these axioms. As shown in Thom [7] (§§5–7), these can be a rule of Substitution, a rule of Permutation, and a rule of Cut. Since no Indirect Reductions are given, the system includes no rule warranting such a procedure. But it does make use of some (unstated) rules of Ecthesis to prove:

Baroco LLL $\dfrac{Lbc^a Lba^o}{Lca^o}$ **Bocardo LLL** $\dfrac{Lca^o Lba^a}{Lcb^o}$.

Aristotle also discusses rejected formulas. He orders them deductively, reducing the rejection of some formulas to that of others. The unreduced rejections are explained by means of counter-examples. Some adjustments need to be made to Aristotle's account here. (i) He claims at [1] 31a38-31b3 that the rejection of Felapton XLL reduces to that of Ferio XLL, whereas the truth is that the rejection of the latter reduces to that of the former; (ii) he fails to observe that the rejection of Festino XLL and Ferison XLL reduces to that of Ferio XLL, that the rejection of Disamis LXL reduces to that of Darii XLL, and that the rejection of Bocardo XLL reduces to that of Felapton XLL; he rejects all these moods (except the first, which he overlooks) by the method of counter-example (32a1-3, 31b32-33, 31b40-41). If we make the appropriate adjustments, this procedure in effect involves taking the following to be rejected axiomatically:

Lo conv $\dfrac{Lba^o}{Lab^o}$ **Barbara XLL** $\dfrac{cb^a Lba^a}{Lca^a}$ **Celarent XLL** $\dfrac{cb^e Lba^a}{Lca^e}$

Darii XLL $\dfrac{cb^a Lba^i}{Lca^i}$ **Felapton XLL** $\dfrac{ca^e Lba^a}{Lcb^o}$ **Baroco LXL** $\dfrac{Lbc^a\ ba^o}{Lca^o}$.

In addition, he rejects the following pair axiomatically (i.e., by counter-example):

Baroco XLL $\dfrac{bc^a Lba^o}{Lca^o}$ **Bocardo LXL** $\dfrac{Lca^o\ ba^a}{Lcb^o}$.

This, however, leads to trouble. For, if the ecthetic proofs of Baroco LLL and Bocardo LLL are accepted, then they can be adapted to validate Baroco XLL and Bocardo LXL. This can be shown as follows. Aristotle describes the ecthetic proof of Baroco LLL thus:

> . . . it is necessary for us to set out that part to which each term does not belong and produce the deduction about this. For it will be necessary in application to each of these; and if it is necessary of what is set out, then it will be necessary of some part of that former term (for what is set out is just a certain 'that'). Each of these deductions occurs in its own figure. ([1] 30a9-15)

Instead of using the particular negative apodeictic premise 'Some *a* cannot be *b*', the procedure is to consider a premise relating to 'that part' of *a* which cannot be *b*. This part cannot be *c*, says Aristotle, and so some *a* cannot be *c*. But what is the reason why the selected part *cannot* be *c*? The context does not explain. It might be because all *c must* be *b*; or it might be because all *c is b*. But in either case Baroco XLL, as well as Baroco LLL, is validated. For, concerning the selected part of *a*, we can reason as follows: it cannot be *b*, but all *c* either must be or is *b*, so the selected part cannot be *c*. This syllogism is in the second figure, like Baroco itself. (By similar reasoning it can be shown that a simple adaptation of the ecthetic proof of Bocardo LLL validates Bocardo LXL. Formal representations of these proofs are given below.)

It is clear therefore that, if Aristotle's system is taken to include the counter-examples as well as the ecthetic proofs, then it is inconsistent, being committed to the validity as well as the invalidity of Baroco XLL and Bocardo LXL. This fact obliges the interpreter to revise Aristotle's system, either by dropping the ecthetic procedure for apodeictic forms, or by allowing Baroco XLL and Bocardo LXL as valid. It is argued in [8] (pp. 147–148) that Aristotle's counter-examples to these moods are unconvincing. Given this, and since the rejection of nothing else depends on their rejection, it seems preferable to allow Baroco XLL and Bocardo LXL as valid, retaining the procedure of ecthesis. In what follows, I shall ignore Aristotle's rejection of these moods, and I shall refer to the Aristotelian system thus modified as the revised Aristotelian system.

2 An ecthetic system In Aristotle's ecthetic proof of Baroco LLL and Bocardo LLL, there is an ambiguity concerning whether the selected term ('that part' with reference to which the ecthetic proof proceeds) is a general or a singular term. In the present paper, I take it to be a singular term.

We now define an ecthetic system, containing singular term variables:

$$x, y, z, \ldots,$$

general term variables:

$$a, b, c, \ldots,$$

and alphabetic variants of the following forms:

ax (x is a), ax^u (x is not a), Lax (x must be a), Lax^u (x cannot be a),

as well as the forms of proposition recognized in the revised Aristotelian system.

The non-modal base of our ecthetic system consists of the axioms:

$$\text{(A1)}\ \frac{ax}{ax} \qquad \text{(A2)}\ \frac{ba^a}{ba^i} \qquad \text{(A3)}\ \frac{ba^a\ ax}{bx} \qquad \text{(A4)}\ \frac{ba^e\ ax}{bx^u}$$

$$\text{(A5)}\ \frac{bx\ ax}{ba^i} \qquad \text{(A6)}\ \frac{bx^u ax}{ba^o}$$

plus rules of Substitution, Permutation, and Cut, and the four ecthetic rules:

$$\text{(R1)}\quad \text{If}\ \vdash\frac{Q\ ax}{bx}\ \text{then}\ \vdash\frac{Q}{ba^a}$$

(where Q is a possibly null sequence of forms, and x does not occur in Q)

$$\text{(R2)}\quad \text{If}\ \vdash\frac{Q\ ax}{bx^u}\ \text{then}\ \vdash\frac{Q}{ba^e}$$

(where Q is a possibly null sequence of forms, and x does not occur in Q)

$$\text{(R3)}\quad \text{If}\ \vdash\frac{Q\ ax\ bx}{p}\ \text{then}\ \vdash\frac{Q\ ab^i}{p}$$

(where Q is a possibly null sequence of forms and p is a form and x does not occur in Q or p)

$$\text{(R4)}\quad \text{If}\ \vdash\frac{Q\ ax^u bx}{p}\ \text{then}\ \vdash\frac{Q\ ab^o}{p}$$

(where Q is a possibly null sequence of forms and p is a form and x does not occur in Q or p).

In addition, the system has eight modal axioms:

$$\text{(A7)}\ \frac{Lax}{ax} \qquad \text{(A8)}\ \frac{Lax^u}{ax^u} \qquad \text{(A9)}\ \frac{Lba^a\ ax}{Lbx} \qquad \text{(A10)}\ \frac{Lba^e\ bx}{Lax^u}$$

$$\text{(A11)}\ \frac{bx\ Lax}{Lba^i} \qquad \text{(A12)}\ \frac{Lbx^u ax}{Lba^o} \qquad \text{(A13)}\ \frac{ba^a Lbx^u}{Lax^u} \qquad \text{(A14)}\ \frac{Lbx\ ax}{Lba^i}.$$

And four rules of modal ecthesis:

(R5) If $\vdash \dfrac{Q\ ax}{Lbx}$ then $\vdash \dfrac{Q}{Lba^a}$

(where Q is a possibly null sequence of forms, and x does not occur in Q)

(R6) If $\vdash \dfrac{Q\ ax}{Lbx^u}$ then $\vdash \dfrac{Q}{Lba^e}$

(where Q is a possibly null sequence of forms, and x does not occur in Q)

(R7) If $\vdash \dfrac{Q\ Lax\ bx}{p}$ and $\vdash \dfrac{Q\ ay\ Lby}{p}$ then $\vdash \dfrac{Q\ Lba^i}{p}$

(where Q is a possibly null sequence of forms and p is a form and x, y do not occur in Q or p)

(R8) If $\vdash \dfrac{Q\ Lax^u bx}{p}$ then $\vdash \dfrac{Q\ Lab^o}{p}$

(where Q is a possibly null sequence of forms and p is a form and x does not occur in Q or p).

2.1 Three Lemmas

(L1)

$$\text{A1}\ \frac{ax}{ax} \xrightarrow{\text{R1}} \dfrac{\qquad}{\text{A2}\ \dfrac{aa^a}{aa^i}}$$

(L2)

$$\text{A9}\ \dfrac{Lba^a\ ax}{\text{A7}\ \dfrac{Lbx}{bx}} \xrightarrow{\text{R1}} \dfrac{Lba^a}{ba^a}$$

(L3)

$$\left.\begin{array}{l} \text{A5}\ \dfrac{ax \quad \text{A7}\ \dfrac{Lbx}{bx}}{ba^i} \\[2ex] \text{A5}\ \dfrac{by \quad \text{A7}\ \dfrac{Lay}{ay}}{ba^i} \end{array}\right\} \xrightarrow{\text{R7}} \dfrac{Lba^i}{ba^i}$$

2.2 Ecthetic derivation of Baroco LLL and Bocardo LLL

$$\text{A12}\ \dfrac{\text{A13}\ \dfrac{\text{L2}\ \dfrac{Lbc^a}{bc^a} \quad Lbx^u}{Lcx^u} \quad ax}{Lca^o} \xrightarrow{\text{R8}} \dfrac{Lbc^a\ Lba^o}{Lca^o}$$

$$\mathrm{A12}\ \dfrac{Lbx^u \quad \mathrm{A7}\ \dfrac{\mathrm{A9}\ \dfrac{Lca^a\ ax}{Lcx}}{cx}}{Lbc^o} \xrightarrow{\text{R8}} \dfrac{Lba^o\ Lca^a}{Lbc^o}.$$

2.3 Ecthetic derivation of Baroco XLL and Bocardo LXL

$$\mathrm{A12}\ \dfrac{\mathrm{A13}\ \dfrac{bc^a\quad Lbx^u}{Lcx^u} \quad ax}{Lca^o} \xrightarrow{\text{R8}} \dfrac{bc^a\ Lba^o}{Lca^o}$$

$$\mathrm{A12}\ \dfrac{Lbx^u \quad \mathrm{A3}\ \dfrac{ca^a\ ax}{cx}}{Lbc^o} \xrightarrow{\text{R8}} \dfrac{Lba^o\ ca^a}{Lbc^o}.$$

The ecthetic system contains the revised Aristotelian system. This is shown by (i) deriving within it the basis of the revised Aristotelian system, and (ii) constructing a semantics relative to which it is sound and according to which the axiomatic rejections of the revised Aristotelian system are invalid.

2.4 Derivation of the laws of apodeictic conversion

La conv

$$\mathrm{A11}\ \dfrac{ax \quad \mathrm{A9}\ \dfrac{Lba^a ax}{Lbx}}{Lab^i} \xrightarrow{\text{R3}} \dfrac{\mathrm{L1}\ \dfrac{}{aa^i}\ Lba^a}{Lab^i}$$

Le conv

$$\mathrm{A10}\ \dfrac{bx\ \ Lba^e}{Lax^u} \xrightarrow{\text{R6}} \dfrac{Lba^e}{Lab^e}$$

Li conv

$$\left.\begin{array}{l} \mathrm{A11}\ \dfrac{Lbx\quad ax}{Lab^i} \\[2ex] \mathrm{A14}\ \dfrac{Lay\quad by}{Lab^i} \end{array}\right\} \xrightarrow{\text{R7}} \dfrac{Lba^i}{Lab^i}.$$

2.5 Derivation of Aristotle's axiomatic LXL syllogisms

Barbara LXL

$$\mathrm{A9}\ \dfrac{Lcb^a \quad \mathrm{A3}\ \dfrac{ba^a\ ax}{bx}}{Lcx} \xrightarrow{\text{R5}} \dfrac{Lcb^a\ ba^a}{Lca^a}$$

Celarent LXL

$$\mathrm{A10}\ \dfrac{\text{Le conv}\ \dfrac{Lcb^e}{Lbc^e} \quad \mathrm{A3}\ \dfrac{ba^a\ ax}{bx}}{Lcx^u} \xrightarrow{\text{R6}} \dfrac{Lcb^e\ ba^a}{Lca^e}$$

Darii LXL

$$\mathrm{A14}\ \dfrac{ax \quad \mathrm{A9}\ \dfrac{Lcb^a\ bx}{Lcx}}{Lca^i} \xrightarrow{\text{R3}} \dfrac{Lcb^a\ ba^i}{Lca^i}$$

Ferio LXL

$$\dfrac{\text{Le conv}\ \dfrac{Lcb^e}{Lbc^e} \quad bx}{\text{A10}\ \dfrac{}{\text{A12}\ \dfrac{Lcx^u \quad ax}{Lca^o}}} \quad \xrightarrow{\text{R3}} \quad \dfrac{Lcb^e\ ba^i}{Lca^o}\ .$$

2.6 Derivation of Aristotle's axiomatic LLL syllogisms

From Barbara LXL, Celarent LXL, Darii LXL, Ferio LXL, together with L2 and L3.

It is also clear that the ecthetic system is not contained in the revised Aristotelian system, since the former includes theses of at least two types which the revised Aristotelian system lacks: (i) formulas like A7 in which non-modal conclusions follow from premises some of which are modal, and (ii) (other) formulas containing singular terms.

3 Semantics The basic idea here is from Johnson [3]. There, more than one class is assigned to each term '*a*', namely a class of *a*'s, a class of 'essential' *a*'s, and a class of 'essential' non-*a*'s. The effect is that Aristotle's 'apodeictic' propositions become non-modal propositions stating ordinary class-relations. This is also the effect of the semantics to be developed here; but we differ from [3] in (i) not requiring that the class of 'essential' *a*'s be non-null and (ii) not postulating any class of 'essential' non-*a*'s.

1. We postulate a nonempty domain **D** of individuals.

2. We postulate a nonempty set **F** of nonempty sets whose only members are members of **D**. These sets will be the extension of the terms in formulas.

3. We postulate a nonempty set $\mathbf{F}^*$ of possibly empty sets whose only members are members of **D**. The members of $\mathbf{F}^*$ will be referred to as *star sets*.

4. An assignment of values to the variables in a wff associates with each individual variable in the wff a member of **D**, and with each general variable ***a*** in the wff an ordered pair consisting of a member of **F** followed by a member of $\mathbf{F}^*$. These sets I will refer to as the *a*'s and the a^*'s, respectively. The a^*'s can be thought of as a privileged set of individuals standing in some ontological relation to the *a*'s. In the Aristotelian context, the a^*'s might be those individuals that are *really*— or *per se*—***a***. For Aristotle, some terms, but not all, belong to their subjects *per se*: thus, there is a non-null class of *per se* horses, but no non-null class of *per se* musicians. On this interpretation, the a^*'s—the real *a*'s—are in every case included in the *a*'s. Thus:

5. If the pair $\langle \alpha_1, \alpha_2 \rangle$ is assigned to a variable then $\alpha_2 \subseteq \alpha_1$. (This is weaker than the requirement in [8] (p. 139) that either $\alpha_1 = \alpha_2$ or $\alpha_2 = \varnothing$.) On the suggested reading, this says that the real *a*'s, if there are any, are among the *a*'s. Suppose that to ***a*** is assigned the pair of sets $\langle \mathbf{f}_1, \mathbf{f}_1^* \rangle$, to ***b*** the pair of sets $\langle \mathbf{f}_2, \mathbf{f}_2^* \rangle$, and to ***x*** the individual **d**, where ***x*** is a singular variable, $\mathbf{f}_1 \in \mathbf{F}$, $\mathbf{f}_2 \in \mathbf{F}$, $\mathbf{d} \in \mathbf{D}$, $\mathbf{f}_1^* \in \mathbf{F}^*$, $\mathbf{f}_2^* \in \mathbf{F}^*$. Then:

6. $\boldsymbol{ax}$ is true iff $\mathbf{d} \in \mathbf{f}_1$, i.e., iff the a's include x. Diagrammatically:

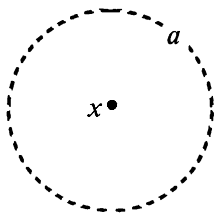

Figure 1

7. $\boldsymbol{ax^u}$ is true iff $\mathbf{d} \notin \mathbf{f}_1$, i.e., iff the a's do not include x. Diagrammatically:

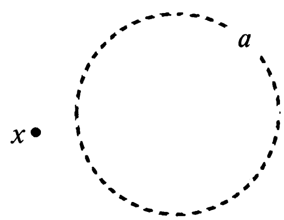

Figure 2

8. $\boldsymbol{ba^a}$ is true iff $\mathbf{f}_1 \subseteq \mathbf{f}_2$, i.e., iff the b's include the a's. Diagrammatically:

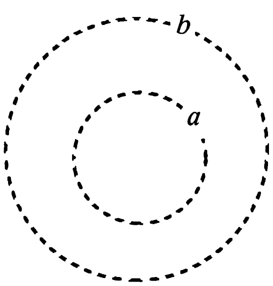

Figure 3

9. $\boldsymbol{ba^e}$ is true iff $\mathbf{f}_1 \cap \mathbf{f}_2 = \varnothing$, i.e., iff the a's exclude the b's. Diagrammatically:

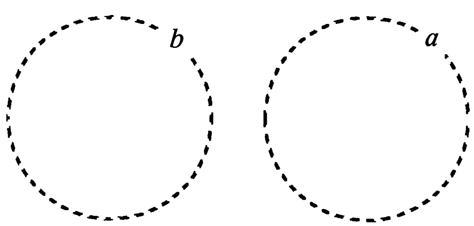

Figure 4

10. $\boldsymbol{ba^i}$ is true iff $\mathbf{f}_1 \cap \mathbf{f}_2 \neq \varnothing$, i.e., iff the a's intersect the b's. Diagrammatically:

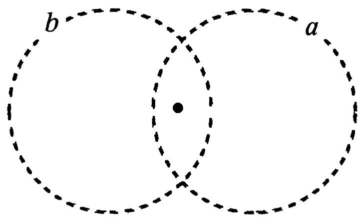

Figure 5

11. $\boldsymbol{ba^o}$ is true iff $\mathbf{f}_1 \not\subseteq \mathbf{f}_2$, i.e., iff the b's do not include the a's. Diagrammatically:

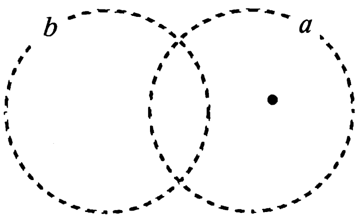

Figure 6

12. $\boldsymbol{Lax}$ is true iff $\mathbf{d} \in \mathbf{f}_1^*$, i.e., iff the a^*'s include x. On the suggested reading, this says that x is one of the real a's. Diagrammatically:

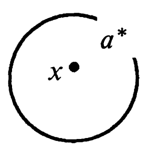

Figure 7

(Starred sets will be represented by unbroken lines, unstarred sets by dotted lines.)

13. $\boldsymbol{Lax^u}$ is true iff there are members $\mathbf{f}_3^*, \mathbf{f}_4^*$ of $\mathbf{F}^*$ such that $\mathbf{d} \in \mathbf{f}_3^*$ and $\mathbf{f}_1 \subseteq \mathbf{f}_4^*$ and $\mathbf{f}_3^* \cap \mathbf{f}_4^* = \varnothing$, i.e., iff there are mutually exclusive star sets, one of which contains x, while the other includes the a's. (x is really something which excludes something that the a's really are.) Diagrammatically:

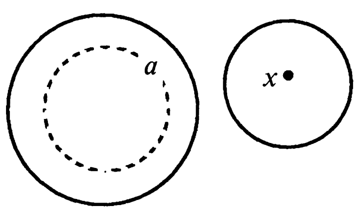

Figure 8

14. $\boldsymbol{Lba^a}$ is true iff $\mathbf{f}_1 \subseteq \mathbf{f}_2^*$, i.e., iff the b^*'s include the a's. (All a is really b.) Diagrammatically:

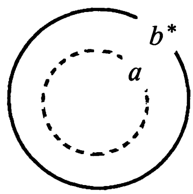

Figure 9

15. $\boldsymbol{Lba^e}$ is true iff there are members $\mathbf{f}_3^*, \mathbf{f}_4^*$ of $\mathbf{F}^*$ such that $\mathbf{f}_1 \subseteq \mathbf{f}_3^*$ and $\mathbf{f}_2 \subseteq \mathbf{f}_4^*$ and $\mathbf{f}_3^* \cap \mathbf{f}_4^* = \varnothing$, i.e., iff there are two mutually exclusive star sets which respectively include the a's and the b's. (All a's are really something exclusive of something which all b's really are.) Diagrammatically:

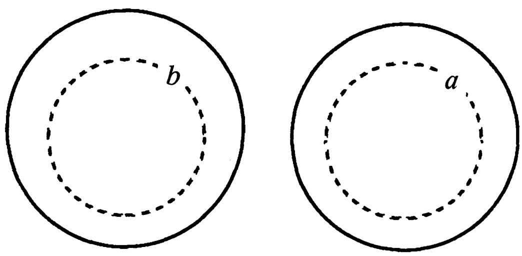

Figure 10

16. $\boldsymbol{Lba^i}$ is true iff either (i) $\mathbf{f}_1^* \cap \mathbf{f}_2 \neq \varnothing$; or (ii) $\mathbf{f}_2^* \cap \mathbf{f}_1 \neq \varnothing$, i.e., iff the a^*'s intersect the b's, or the b^*'s intersect the a's. (Either some a is really b or some b is really a.) Diagrammatically:

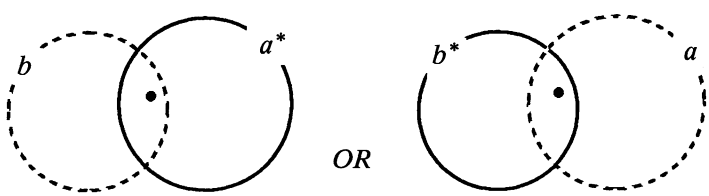

Figure 11

17. $\boldsymbol{Lba^o}$ is true iff there are members $\mathbf{f}_3^*, \mathbf{f}_4^*$ of $\mathbf{F}^*$ such that $\mathbf{f}_1 \cap \mathbf{f}_3^* \neq \varnothing$ and $\mathbf{f}_2 \subseteq \mathbf{f}_4^*$ and $\mathbf{f}_3^* \cap \mathbf{f}_4^* = \varnothing$, i.e., iff there are two mutually exclusive star sets, one of which intersects the a's, while the other includes the b's. (Some a is really something exclusive of something which every b really is.) Diagrammatically:

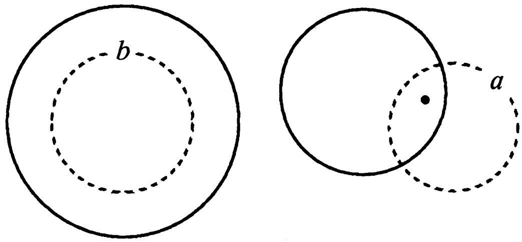

Figure 12

18. A formula:

$$\frac{Q}{p}$$

is true iff some member of Q is false or p is true.

Validity is defined in the standard way.

The ecthetic system is sound with respect to this semantics. The non-modal base is readily shown to have only valid axioms and validity-preserving rules on this interpretation.

The validity of the six modal axioms can also be shown, using elementary set theory. The rules are validity-preserving. This is readily shown for the non-modal rules of Substitution, Permutation, and Cut. That it also holds for the rules of ecthesis can be shown as follows. Consider R1. If its RHS is not valid, there is a domain $\mathbf{D}$ and an assignment of values on which $\boldsymbol{ba}^{\boldsymbol{a}}$ is false. In $\mathbf{D}$, suppose the values assigned to $\boldsymbol{a}$ and $\boldsymbol{b}$ are $\langle \mathbf{f}_1, \mathbf{f}_1^* \rangle$ and $\langle \mathbf{f}_2, \mathbf{f}_2^* \rangle$; then there must be a $\mathbf{d}_1$ in $\mathbf{D}$ such that $\mathbf{d}_1 \in \mathbf{f}_1$, $\mathbf{d}_1 \notin \mathbf{f}_2$. So there is a domain and an assignment of values on which $\boldsymbol{ax}$ is true and $\boldsymbol{bx}$ false—namely any domain in which $\langle \mathbf{f}_1, \mathbf{f}_1^* \rangle$, $\langle \mathbf{f}_2, \mathbf{f}_2^* \rangle$ are assigned to $\boldsymbol{a}, \boldsymbol{b}$ and $\mathbf{d}_1$ is assigned to $\boldsymbol{x}$ and $\mathbf{d}_1 \in \mathbf{f}_1$, $\mathbf{d}_1 \notin \mathbf{f}_2$. Thus, if the RHS of R1 is not valid neither is the LHS. Similarly with the other ecthetic rules.

We now show that the formulas which are axiomatically rejected in the revised Aristotelian system are invalid on our semantics.

Lo conversion

Suppose there are just two non-null star sets (the c^*'s and the d^*'s, which exclude one another), with the a's intersecting the c^*'s, some a's not being c^*'s, and the b's included in the d^*'s:

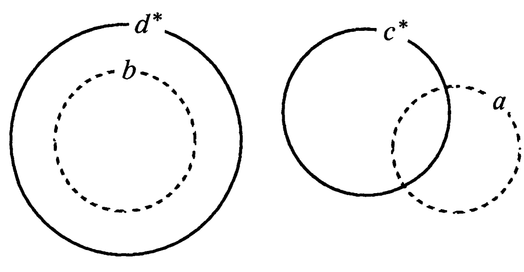

Figure 13

Then, while it is true that some *a* cannot be *b*, it is not true that some *b* cannot be *a* since there is no star set including the *a*'s.

Barbara XLL

Suppose there is just one non-null star set (the *b**'s), where the *c*'s include the *b*'s, and the *b**'s include the *a*'s:

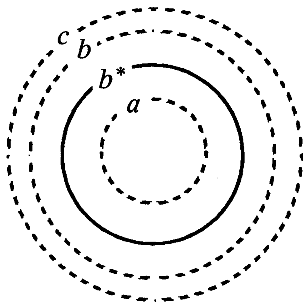

Figure 14

Then, while it is true that all *a* must be *b* and that all *b* is *c*, it is not true that all *a* must be *c* since there are no *c**'s.

Celarent XLL

Suppose there are just two non-null star sets (the *d**'s and the *e**'s, which exclude one another), where the *c*'s include the *e**'s, the *e**'s include the *b*'s, and the *d**'s include the *a*'s:

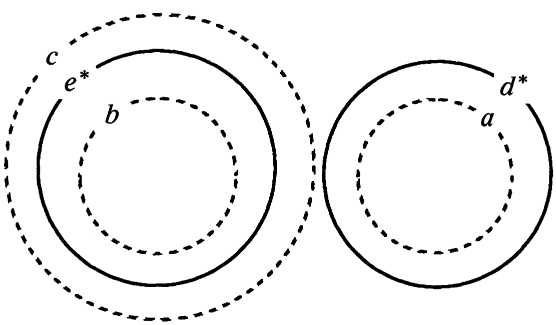

Figure 15

Then, while it is true that no *a* can be *b*, and that all *b* is *c*, it is not true that no *a* can be *c* since the *c*'s are not included in any star set.

Darii XLL

Suppose there is just one non-null star set (the *b**'s), where the *c*'s include the *b*'s, and the *b**'s intersect the *a*'s:

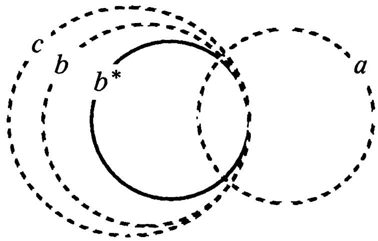

Figure 16

Then, while it is true that some *a* must be *b*, and that all *b* is *c*, it is not true that some *a* must be *c* since neither the *a**'s nor the *c**'s is non-null.

Felapton XLL

Suppose there is just one non-null star set (the *b**'s), where the *c*'s exclude the *b*'s, and the *b**'s include the *a*'s:

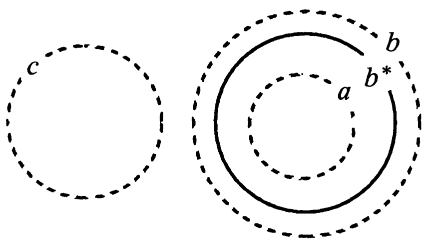

Figure 17

Then, while it is true that all *a* must be *b*, and that no *a* is *c*, it is not true that some *b* cannot be *c* since no star set includes the *c*'s.

Baroco LXL

Suppose there is just one non-null star set (the *b**'s), where some *a* is not a *b* and the *b**'s include the *c*'s:

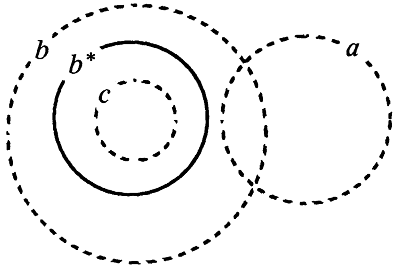

Figure 18

Then, while it is true that some a is not b, and that all c must be b, it is not true that some a cannot be c since there is only one star set.

The ecthetic system is, however, not complete relative to our semantics. This is not surprising. Syllogistic logics are in general not semantically complete with respect to validity as standardly conceived. Thus, a formula like:

$$\frac{\text{All } a \text{ must be } b \quad \text{Some } c \text{ is not } d}{\text{All } a \text{ must be } b}$$

while valid, is not a syllogistic thesis. Moreover, our ecthetic syllogistic has the syntactic peculiarity that the contradictory of a form which is expressible in it is not always itself so expressible. Thus, a formula like

$$\frac{\text{All } a \text{ is } b \quad \text{Not (all } a \text{ must be } c\text{)}}{\text{Not (all } b \text{ must be } c\text{)}}$$

while valid, is not expressible in our system, because the system does not contain the contradictory of any apodeictic proposition.

Nor is it the case that every nonredundantly valid formula expressible in the system is a thesis. Consider the formula

$$\frac{Lab^a\ Lcd^a\ ac^e}{Lac^e}.$$

This is valid, and not redundantly so, but it is not provable.

4 Philosophical interpretation (1) In our ecthetic system, the 'necessarily' in apodeictic propositions does not play a uniform role. In affirmative singulars it is clearly a predicate-modifier—certainly it cannot be taken as a propositional operator—however, in other apodeictic forms it cannot be read either as a predicate-modifier or as a propositional operator.

Nonetheless, the proposed interpretation can fairly be described overall as a *de re* interpretation, insofar as the truth-conditions of all apodeictic forms reduce to those of affirmative apodeictic singulars, which are clearly *de re*. This reduction happens as follows: $\boldsymbol{Lax^u}$ is true iff for some $\boldsymbol{c,d}$: $\boldsymbol{Lca^a}$ and $\boldsymbol{Ldx}$ and $\boldsymbol{cd^e}$ are true; $\boldsymbol{Lba^a}$ is true iff for all $\boldsymbol{x}$: if $\boldsymbol{ax}$ is true then $\boldsymbol{Lbx}$ is true; $\boldsymbol{Lba^e}$ is true iff for some $\boldsymbol{c,d}$: $\boldsymbol{Lcb^a}$ and $\boldsymbol{Ldb^a}$ and $\boldsymbol{cd^e}$ are true; $\boldsymbol{Lba^i}$ is true iff EITHER for some $\boldsymbol{c}$: $\boldsymbol{Lbc^a}$ and $\boldsymbol{ac^a}$ are true OR for some $\boldsymbol{c}$: $\boldsymbol{bc^a}$ and $\boldsymbol{Lac^a}$ are true; $\boldsymbol{Lba^o}$ is true iff for some $\boldsymbol{c,d,f}$: $\boldsymbol{Lcb^a}$ and $\boldsymbol{Ldf^a}$ and $\boldsymbol{af^a}$ and $\boldsymbol{cd^e}$ are true.

The alternative suggestion of [4] (§2) that the 'necessarily' be taken as modifying the copula is correct only at the level of surface syntax. It may be true at that level that:

> One forms the (modally) different types of proposition by simply adding one copulative expression to the terms rather than another. ([4] p. 14)

But this cannot be sustained at a deeper level of syntax.

(2) Is Aristotle's system a modal logic in the modern sense? No. The reason is not (as [11] has it) that there is no implication from apodeictic propositions

to their corresponding assertorics (for we do have A7, L2, L3 — and the derivation of negative assertorics from their apodeictics follows readily). Rather, it is that our semantics does not appeal to possible worlds, but only to sets of individuals in the actual world. And this is desirable in an interpretation that seeks to cohere with Aristotelian metaphysics; for, it is quite unclear how an ontology of possible worlds could be integrated into Aristotelian ontology. The present semantics could easily be developed into an ontology in which realities are defined not simply as a sub-class of certain classes, but as being so in all possible worlds. This might be more interesting to contemporary metaphysicians, but it would be no truer to Aristotle's modal syllogistic.

Could a semantics in the style of [11] be generated from the semantics of this paper? No. At present the only theses dependent on A7 are the LLL syllogisms in Figure 1. It is true that these could be added axiomatically if A7 were dropped. But if Barbara LLL is to be valid, there must be an assumption that star sets are included in non-star sets: supposing that all a is b^* and that all b is c^*, the only way it will follow that all a is c^* is if all b^* is b. Thus our approach is inconsistent with that of [11], while agreeing with its general position that Aristotle's modal syllogistic is not a modern-style modal logic.

(3) It follows that our interpretation defends an extensionalist reading of Aristotle's apodeictic syllogistic. This is in keeping with the generally extensionalist tendency of Aristotle's early thought, and with the fact that in his assertoric logic it is never necessary — as I argue in Thom [9] — to posit intensional entities.

(4) On the question of the relation between the modal syllogistic and Aristotelian metaphysics, our interpretation makes the apodeictic syllogistic a formalization of a certain kind of statement relative to Aristotle's metaphysics of the *Posterior Analytics*, namely statements to the effect that some individual or class is or is not included in some class of realities. It is significant here that we do not claim that all of Aristotelian metaphysics is in any sense captured in the modal syllogistic. The ecthetic system developed in this paper in particular posits only an unsorted collection of individuals, whereas Aristotelian metaphysics supposes that individuals are sorted. The amount of Aristotelian metaphysics involved could be increased by developing an ecthetic system in which the singular terms are sorted. The adaptation from the system of this paper is quite straightforward.

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