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The World, the Facts, and Primary Logic

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Thomas Hobbes rightly stated that everything done by the mind is a computation by which is understood either the addition of a sum or the subtraction of a difference. So just as there are two primary signs in algebra, '+' and '−' in the same way there are, as it were, two copulas.

-Leibniz

If, as I hope, I can conceive all propositions as terms, and hypotheticals as categoricals, and if I can treat all propositions universally, this promises a wonderful ease in my symbolism and analysis of concepts, and will be a discovery of the greatest importance.

-Leibniz

Abstract Frege gave priority to propositional logic over term or predicate logics analyzing categorical forms like 'every *A* is *B*' and 'some *A* is *B*' in terms of compound forms like '$Ax \rightarrow Bx$' and 'Ax & Bx'. Leibniz hoped to do the reverse by treating '$p \rightarrow q$' and 'p & q' as categoricals of form 'every $\{p\}$ is a $\{q\}$' and 'some $\{p\}$ is a $\{q\}$'. More generally he believed it possible to reduce all compound forms to categoricals using the old term connectives '*A*', '*E*', '*I*', '*O*'. The paper shows how Leibniz's program of treating propositions as terms and truth functions as term connectives can be realized. Where ordinary nonvacuous terms denote things in the world and signify their characteristics (e.g., 'wise' denotes wise things and signifies wisdom or being wise) propositional terms denote the world itself, signifying facts (e.g., 'There are elks' signifies the existence of elks and denotes the world characterized by their presence). False propositions are vacuous. Because all true propositions denote one and the same world (though signifying different world characteristics) 'some $\{p\}$ is $\{q\}$' (the categorical form of 'p & q' will entail 'every $\{p\}$ is $\{q\}$' ($p \rightarrow q$). The paper shows that this approach regards existence and nonexistence as world properties (facts).

1 The positive and negative copulas[1] Leibniz rightly sees that the terms of a statement such as 'Socrates is wise' or 'some Spaniard is a painter' are joined by a logical copula that has the properties of the addition operator. Take the

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predicative formula 'Being (a) *P* (thing) characterizes (an) *S* (thing)' (Aristotle's '[being] *P* belongs to (an) *S*'). This has instances like 'being (a) wise (thing) characterizes Socrates' and 'being a painter characterizes some Spaniard'. An expression that joins the two terms is a "logical copula", in this case, the old I-functor in '*P*i*S*'. Representing 'I' as '+' brings out the symmetry of terminist predication: the equivalence of 'being (a) *P* (thing) characterizes (an) *S* (thing)' to 'being (an) *S* (thing) characterizes (a) *P* (thing)'. Algebraic transcription makes it possible to express this as an equation

$$P + S = S + P.$$

Proceeding to represent other syncategorematic elements in a plus/minus way we use '−' for the negative particles that qualify terms and statements. Unlike the copulative '+', '−' is unary. Thus '$(-W) + G$' transcribes 'some Greek is unwise'; '$-((-W) + G)$' transcribes 'no Greek is unwise' (i.e., 'not: being unwise characterizes some Greek').

The use of '−' for the negative particles is natural. Another natural assignment is the use of '+' for 'and' in forming compound terms like 'gentleman and scholar'. Let angle brackets encase compound terms. Then 'some farmer is a gentleman and scholar' would be transcribed as '$\langle S + G \rangle + F$' and its equivalent, 'some farmer and gentleman is a scholar', as '$S + \langle G + F \rangle$'. The plus sign is associative as well as commutative, and the systematically ambiguous use of '+' for both conjunction and predication preserves logical equivalence. The associative equivalence is expressed as:

$$\langle S + G \rangle + F = S + \langle G + F \rangle.$$

Leibniz speaks also of a negative *copula*, and he suggests that such a copula would behave in a minus-like way, expressing "the subtraction of a difference". Now this means that in addition to the unary negative signs for the negative particles, we also have a binary subtractive sign predicatively joining the two terms of the statement. Here again Leibniz's logico-algebraic intuition is sound. Taking the unary negative operator and the binary predicative operator as primitive, we may use obversion to define a new binary predicative tie between terms: '*Being . . . characterizes every . . .*'. Algebraically expressed, the obverse equivalence reveals that this predicative tie (the A-functor in '*P*a*S*') is indeed subtractive:

$$P\mathrm{a}S =_{def} \text{not } ((\text{non-}P)\mathrm{i}S)$$

$$P - S =_{def} -((-P) + S).$$

Aristotle's formulas for the logical copulas, A and I, were 'belongs to every' and 'belongs to some'. Representing the I-functor as a plus sign and the A-functor as a minus sign brings out the logical powers of the two predicative forms '$P + S$' and '$P - S$' by representing obverse, converse and contrapositive equivalence in algebraic form.

Equivalences	Equations
P belongs to some *S* = not : non-*P* belongs to every *S*;	$P + S = -((-P) - S)$
P belongs to some *S* = *S* belongs to some *P*;	$P + S = S + P$
P belongs to every *S* = non-*S* belongs to every non-*P*;	$P - S = (-S) - (-P)$

(Equality is only a necessary condition of logical equivalence. Let two statements of the same logical quantity (both being universal or both being particular) be called *covalent*. The necessary and sufficient conditions are *covalence* and *equality*.)

The plus/minus notation reveals that 'some' is a positive copula, and 'every' a negative copula. As for the grammatical copula 'is', it is logically superfluous, introducing no difference between 'some *X* is a *Y*' and 'every *X* is a *Y*'; the crucial difference is marked by the binary quantifiers 'some' and 'every'. Indeed, in many languages there is no grammatical copula, juxtaposition of the terms serving as an implicit positive term connective. It is perhaps odd to think of 'some' and 'every' as copulas or term connectives, but logically that is what they are. The traditional terminist implicitly recognizes this in using binary functors 'I' and 'A' as term connectives in '*P*i*S*' and '*P*a*S*'. Following Leibniz's lead, we may go on to say that I is to A as addition is to subtraction.

Leibniz treats 'Socrates is wise' as 'WiS^*' or '$W + S^*$'. (The asterisk signified singularity.) Since 'S^*' denotes uniquely, '$W + S^*$' entails '$W - S^*$'. Singular sentences generally are of form 'YiX^*', but they have "wild quantity" in the sense that they entail their own generalizations.

(The predicative relation represented by the positive copula, '+', is symmetrical but not transitive. The predicative relation represented by the negative copula, '−', is transitive and reflexive but not symmetrical. Since 'belongs to every' is transitive, conjoining '$M - S$' to '$P - M$' syllogistically entails '$P - S$'. Algebraically: $[P - M] + [M - S] = P - S$.)

The algebraic representation for the conjunction of the premises suggests that Hobbes and Leibniz were also right in thinking that conjoining two statements, like conjoining two terms, is logically the same as adding them algebraically. Taking this idea seriously one represents '*q* and *p*' as '$q + p$' and 'not both not-*q* and *p*' as $-((-q) + p)$'. It is then natural to extend the notation by defining an algebraic representation for 'if' by taking the formula for '*q* if *p*' to be algebraically equal to the formula for 'not both not *q* and not *p*'. In effect we define '*q* if *p*' "obversely" by way of 'not both not *q* and *p*':

$$q \text{ if } p =_{def} \text{not both not } q \text{ and } p.$$

By representing '*q* if *p*' as '$q - p$' one gets the algebraic form of this defining equivalence:

$$q - p =_{def} -((-q) + P).$$

That 'if', like 'every', is subtractive is borne out logically by such equations as:

$q - p = (-p) - (-q);$	*q* if *p* = not *p* if not *q*
$-(q + p) = (-q) - p;$	not: *q* and *p* = not *q* if *p*
$q - (-p) = -((-q) + (-p))$	*q* if not *p* = not: not *q* and not *p*.

Leibniz's idea that the binary term and statement connectives should be given a plus/minus representation is thus of a piece with his second idea of unifying logic by incorporating propositional logic into the classical monadic logic of terms. The thesis that propositional logic is a branch of syllogistic term logic was suggested to Leibniz by the fact that the sentential connective 'if', like the term connective '(belongs to) every', is transitive and reflexive, and by such laws of term and propositional logic as contraposition which have the common form: $y - x = (-x) - (-y)$. He therefore hoped to find a way to construe propositions as terms and to "read", say, 'if p then q' as 'every $\{p\}$ is a $\{q\}$' ('$\{p\}$' and '$\{q\}$' being propositional terms of some kind). The common plus/minus notation strongly suggests that, syntactically at least, the idea of construing the formulas of propositional logic "categorically" (with 'and' as I-functor, 'if' as A-functor) may indeed be feasible.

2 Primary logic The doctrine that term logic is primary logic has ancient roots. Historically, term logic came first, having been discovered and developed by Aristotle; propositional logic, primarily a Stoic innovation, came later. Before the twentieth century, the teaching of logic followed the historical order: term logic including syllogistic was taught first, propositional logic second. But today the vast majority of logicians accept Frege's revolution in logic which accords primary status to propositional logic. Where Leibniz had hoped to give a categorical construal to the conditional 'if p then q', treating its component propositions as terms and reading it as a universal statement of form 'every X is Y', Frege went the other way, construing the universal form 'every S is P' to say something like 'for any thing x: if x is an S then x is a P'. In this reformulation Frege also reversed Leibniz by treating the *terms S* and *P* as *sentences* of form 'x is an S', 'x is a P'. Frege's analysis of categorical propositions precludes a categorical interpretation of 'p & q' and 'if p then q': if Frege's reformulation is the proper way to construe 'every S is P', then, in order to understand the logical behavior of this and other universal statements, one must first understand how the sentential connective 'if then' behaves. And indeed today's students of logic first study the logical behavior of the sentential connectives 'if then', 'and', 'or', etc. after which they may go on to study the categorical forms that enter into syllogistic reasoning. The categorical forms, once the starting point of logical study, have given pride of place to the truth functions of primary logic. Learning that conjunction is commutative the student understands *why* 'belongs to some' is symmetrical. Learning that 'p if p' is tautologous and that 'r if p' follows from 'r if q and q if p' shows the student *why* 'belongs to every' is reflexive and transitive.

Yet even Frege had his Leibnizian weak moments; for example, he explains 'if p then q' by saying 'no case of p standing for the True is a case of q standing for the False', thereby construing 'if p then q' as a categorical statement. He might similarly have said that 'p & q' could be understood as 'some case of p standing for the true is also a case of q standing for the true'. But we know better than to take any such explanation seriously; Frege himself achieved the revolution that gave priority to propositional logic by parsing 'No A is B' as '$-\exists x(Ax \& Bx)$'. Thus 'no case of p standing for the True is a case of Q standing for the False' will be construed as '$-\exists x$(x is a case of 'p' standing for the True & x is a case of 'q' standing for the False') and we are back to the standard analysis that finds compound forms like 'x is P and x is Q' and 'if x is P then x is Q' to be internal to the "elementary" categoricals that enter into syllogistic logic.[2]

But is Leibniz's program realizable? It faces two difficulties. The first is the difficulty of properly construing 'p & q' as a categorical statement of form 'some

X is Y' and of construing 'if p then q' as 'every X is Y'. A more serious difficulty is that a terminist parsing of the truth functional compound forms will not always give us the expected results in propositional logic. Suppose, for example, that we somehow construe 'p & q' and '$p \rightarrow q$' as statements, respectively, of form 'some X is Y' and 'every X is Y', where 'X' and 'Y' are propositional terms of some kind. Now in term logic 'some X is Y' does not entail 'every X is Y'. But in statement logic 'p & q' does entail '$p \rightarrow q$'. A second disanalogy is that any statement 'p' is equivalent to 'p & p' which parses terministically as 'some X is X'. Similarly '-p' parses as 'some non-X is non-X'. Now in term logic two statements of form 'some X is X' and 'some non-X is non-X' are not jointly inconsistent. But where 'X' and '$-X$' represent propositional terms, they are inconsistent. It is not clear that the terminist who would treat propositional logic as a branch of term logic can explain these and other discrepancies.

I have argued elsewhere that one ought to reject the primacy of statement logic to term or predicate logic. The isomorphisms revealed by the plus/minus notation for the logical formatives, suggest that the two logics may be taken on equal terms. On the other hand, if priorities *are* to be assigned, there is reason to favor Leibniz's idea of treating statements as terms and compound forms as categoricals. By taking the terminist route of incorporating propositional logic within general term logic one achieves logical unity and economy. Any statement consists of material and formative expressions. According to Leibniz, the formative expressions are functors with familiar algebraic properties. A simple statement consists of two material expressions (two ordinary terms or two sentential terms) and a commutative binary functor that joins them. This binary (plus) functor and the unary (minus) functor that operates on a material expression to change it into its contrary or contradictory are all the logical primitives we need; other logical formatives are definable from just these two. By contrast, a standard system of modern logic discriminates, in addition to negation, three separate operations: primitive predication (asymmetric in atomic sentences), conjunction between sentences and existential quantification. In what follows we will see how Leibniz's program may be realized and examine some of the implications.

3 How statements are like terms Let us tackle the construal problem first. We shall find that a proper solution to it provides a natural explanation for the special features of statement logic understood as a branch of term logic. But, equally important, a correct understanding of how propositional terms are like and unlike ordinary terms casts terminist light on some cardinal issues in philosophical logic.

An ordinary term like 'wise' signifies a characteristic, state, or attribute (the state of being wise, the characteristic or attribute of wisdom) and denotes something that has the signified characteristic. More generally, a term 'φ' signifies *being* φ and what it denotes has the attribute of being φ (φ-ness). Similarly, if we are to get a terminist interpretation of such compound forms as 'p & q', a statement 'p' must signify an attribute and it too must denote something that has that attribute. For then we could construe the conjunction as a categorical claim to the effect that something possessing the attribute signified by 'p' also possesses the attribute signified by 'q'.

What sort of attribute or state does a statement signify? Consider the statement 'some Swede is poor' which we canonically render as 'something is a poor Swede'. This signifies *something being a poor Swede*. Equivalently, we may say that it signifies *the existence of a poor Swede*. Similarly 'no Swede is poor' signifies *nothing being a poor Swede* or the *nonexistence of poor Swedes*.

We want to say that a statement, like a term, signifies an attribute or characteristic and that it denotes something that has the characteristic signified. But this way of likening statements to terms seems interdicted since we have been led to say that the attribute in question is the existence or nonexistence of something (poor Swedes, trillionaire Swedes). If some Swede is poor, being poor is an attribute of the poor Swede and being a Swede is another attribute, but we know better than to say that the existence of the Swede is yet another such attribute. More generally if a φ thing exists, then being φ is an attribute of it, but existence is not. (We should certainly be foolish to attempt to trespass in this regard. One familiar consequence of treating existence as an attribute of what exists is our inability to cope with attributions of nonexistence. If Hercules was guileless then he was characterized by a lack of guile. But suppose Hercules never existed; shall we also say that nonexistence characterizes him?)

Treating statements as we treat terms seems to have led us to a familiar impasse. The discussion that follows is a necessary detour on the way to the goal of incorporating propositional logic within the classical logic of terms. Briefly, our way around the problem that confronts us when taking the existence and nonexistence of things to be the states of affairs signified by statements is to note that, though we are properly prohibited from attributing existence and nonexistence to the things that are said to exist or not exist, this does not mean that we may not attribute their existence or nonexistence to something *else*.

4 Existence and nonexistence as attributes In claiming that something is a φ, I have in mind a domain of discourse or world. The domain in question is the "domain of the claim". By a domain we mean the (nonempty) totality of things that is under consideration when a given assertive claim has been made. The actual world is the domain of 'there are no elves'. The natural numbers constitute the domain of such statements as 'there is an even prime number' and 'there is no greatest prime number'. The domain of the claim may be quite small. The objects in my hall closet may be the domain of 'There's no red tie'.

Any domain is characterized by the presence of certain things and the absence of certain things. For example, the domain consisting of the things currently in my hall closet is characterized by the presence of mops and the absence of red ties. The set of natural numbers is characterized by the presence of an even prime number and the absence of a greatest prime number. In general, a domain D that has a φ-constituent but no ψ-constituent is characterized by the presence of φ things and the absence of ψ things.

Domains are special and we characterize them in special ways: Let a domain, D, be called "$\{\varphi\}$ish" (read: "Fye-ish") if it contains a φ-constituent. Let it be called un$\{\psi\}$ish ("un-Sye-ish") if it has no ψ-constituent. $\{\varphi\}$ishness (φ-presence) and un$\{\psi\}$ishness (ψ absence) are *constitutive* or *existential* attributes. An existential predicate characterizes a domain or Universe of Discourse "constitutively", by informing us of the kind of constituents it has or lacks. The actual world is the domain under consideration for most of our assertions. The world is {milkmaid}ish but un{mermaid}ish, {elk}ish but un{elf}ish. For example it is characterized by the existence of Canadian elks and the nonexistence of Canadian elves.

{φ}ishness—the existence of a φ thing—and un {ψ}ishness—the nonexistence of a ψ thing—are attributes peculiar to domains. But also, they are *the kinds of attributes that statements signify*. Thus where an ordinary term like 'billionaire' signifies the attribute of being a billionaire, the statement 'someone is a billionaire' signifies an existential attribute: the existence of a billionaire. To be in a domain is to exist. But φ-existence is not an attribute of anything in the domain; it is an attribute of the domain itself. We also speak of existence and nonexistence as states of affairs. For example, the existence of poor Swedes is a positive state of affairs and the nonexistence of Swedish billionaires is a negative state of affairs. In this way of speaking, a state of affairs is a positive or negative existential attribute of a domain or world.

5 To be or not to be The besetting temptation to attribute existence to things that exist comes from a use of 'to be' that Aristotle called 'being *haplos*' which has been variously rendered 'being without qualification', 'being *tout court*', 'being *simpliciter*'. Thus I might understand 'Tame tigers are' to say 'Being characterizes tame tigers', thereby taking a bite from Anselm's Apple by attributing existence to tame tigers. The constitutive conception of existence prohibits all talk of existence (nonexistence) *tout court* or *simpliciter*. Instead we understand 'tame tigers are' or 'tame tigers exist' to say that something is a tame tiger. 'Something is a φ' signifies, not existence *tout court*, but the existence of a (something being a φ). Here we use 'being' as 'being so and so' (what the Scholastics called 'being *secundum quid*'). Abjuring all talk of existence *tout court*, while allowing talk of the existence (nonexistence) of a φ-thing in the sense of {φ}ishness or un{φ}ishness as properties of the domain under consideration, interdicts the attribution of existence ('being' *haplos*) to things in the domain. For we are now confined to talk of φ-presence (φ-absence) as attributes possessed by a domain in virtue of having a (not having any) φ-constituent.[3]

[There is the temptation to say that because tame tigers are present in the world, their presence is in the world. But that is either trivially true or wrong. It is trivially true because, 'present in the world' and 'exists in the world' are pleonasms; as Kant might have said, to think of a thing as being in the world and to think of it as existing are one and the same thing. It is wrong if the presence of a thing in the world is understood to be situated in the world. Though tame tigers are in the world, their existence, their being in the world, is a state *of* (not *in*) the world. The existence of tame tigers obtains, characterizes the world, is a fact. But facts are not in the world. The existence of tigers that are tame is no more a constituent of the world than is the nonexistence of tigers that can fly.]

6 The domain of the claim Any statement is a claim made with respect to a domain or "universe of discourse". We refer to this as the domain of the claim (DC). Where several statements are involved, for example as conjuncts in a conjunction or as premises of an argument, the DC is jointly determined; it is the universe common to the statements in that conjunction or argument. The domain of a statement may be fictional. In discussing the fauna of Greek mythology I might say 'there are flying horses' and then add 'there are no flying kangaroos' and my statements will have expressed propositions that correctly characterize the domain in question. The domain of 'some prime number is even' is the set of natural numbers; the claim is that this domain is {even-prime}ish or, equivalently, that the existence of an even prime number obtains. (To "obtain" is to characterize the domain of the claim.) For most statements the domain of the claim is the real world or some spatio-temporal part thereof. Saying that there are no longer any saber-toothed tigers I express an existential claim; my statement is true if the nonexistence of saber-toothed tigers is a characteristic of the (contemporaneous) world. If (looking into a drawer) I say 'there's no screwdriver (here now)', the domain of the claim consists of the objects currently in the drawer and the claim is that the domain in question is characterized by the nonexistence of screwdrivers, that it is an un{screwdriver}ish domain. The existential characteristics of the relevant domain are the *facts*. {Elk}ishness (the existence of elks) is a fact; by contrast, {elf}ishness (the existence of elves) is not a fact. On the other hand, un{elf}ishness is a negative fact. The classical terminist doctrine of truth is *correspondence to facts*. If my statement signifies a fact then my statement is true. Facts are negative or positive. For example, given the facts (the world's {elk}ishness, its un{elf}ishness), 'there are elks' and 'there are no elves' are true statements.

7 What statements denote I say that some Swedes are poor and my statement makes a claim that the existence of poor Swedes characterizes the world. What, if anything, does my statement denote? In general, a term or a statement will only denote that which has the characteristic signified. So the answer here depends on whether the domain under consideration, in this case the real world, is characterized by the presence of poor Swedes. As it happens, the world *is* {poor-Swede}ish: the existence of poor Swedes is a fact. Thus 'some Swedes are poor' denotes the world. The world is again denoted by 'there are elks' and 'there are no mermaids' and generally by any other statement that signifies a fact. This gives us another way of understanding the truth of a statement:

A statement that is true is a statement that denotes its domain.

True statements denote the world. False statements are vacuous; they fail to denote the world.

Different true statements, like different nonvacuous terms, signify different characteristics. 'Philosopher' and 'Athenian' signify different characteristics but both denote Socrates. So too, 'some Eskimo is a United States citizen' and 'no Albanian is an astronaut' signify different existential characteristics, different facts, but both statements denote one and the same world.

7.1 Denoting, signifying and expressing A term like 'wise' *signifies* wisdom and *denotes* what has wisdom. But an attribute does not exist unless something has it. So if no thing is perfect, there is no perfection and 'perfect' is doubly vacuous: it neither denotes a thing nor signifies an attribute. It does however *express* a concept, the concept of PERFECTION or BEING PERFECT. [Note: I use upper case for the concept *expressed* and lower case for the property, if any, that is *signified*.] A meaningful term may be vacuous with respect to denotation and significance but not with respect to expressing a concept. The same is true of statements. Consider 'some man is immortal'. Since the world is not characterized by the existence of an immortal man, this statement both fails to signify a fact and fails to denote the world. But it does express a sentential concept—the thought or proposition *that some man is immortal* or SOME MAN BEING IMMORTAL. The proposition expressed does not characterize—is not true of—the world. Similarly if no one is immortal the term 'immortal' expresses a concept (IMMORTALITY, BEING IMMORTAL) that does not characterize—is not true of—anything in the world.

Let '@' be a term or statement, let [@] be the concept or proposition expressed by '@' and let ⟨@⟩ be the property (fact), if any, that '@' signifies. Then either '@' is doubly vacuous in failing to signify and failing to denote or else:

1. '@' signifies ⟨@⟩.
2. '@' denotes what has the property ⟨@⟩.
3. [@] "corresponds" to ⟨@⟩.
4. [@] is true of what is @.
5. '@' is true.

Saying that ⟨@⟩ is not a fact is like saying the present king of France does not exist. The latter is understood to say there is no such thing as the present king of France; the former is understood to say there is no such fact as ⟨@⟩. For example 'some man is immortal' expresses [some man is immortal] but fails to signify ⟨some man is immortal⟩ because the existence of an immortal man is not a fact, which is to say: there is no such fact as the existence of an immortal man.

These relations between statements and the propositions they express provide a common way of formulating the truth conditions of a statement:

A statement is true iff the proposition it expresses characterizes the world.

The following are some alternative ways of stating the truth conditions for a statement '@':

'@' is true iff [@] is true of the domain.
'@' is true iff [@] corresponds to ⟨@⟩.
'@' is true iff '@' denotes the domain.
'@' is true iff '@' signifies a fact.
'@' is true iff '@' expresses a true proposition (a FACT).

The last formula shows that the word 'fact' is used either for the state of affairs that makes a statement true or for the true proposition that corresponds to that state of affairs. It would be difficult to exaggerate the amount of confusion generated by these nonidentical twins. I follow the practice of using upper and lower case letters, using 'FACT' to talk about a true proposition and 'fact' to talk about a positive or negative existential attribute of the world.

Our detour is at an end. It has garnered us a conception of the special attributes that statements signify and so puts us on the road that leads to treating propositional logic as a special branch of term logic in which conjunctions, conditionals, and other compound statements are construed as categoricals.

8 Elementary and compound states of affairs Every elementary statement can be canonically paraphrased as a statement of form 'something is φ' or 'nothing is φ'. Sometimes 'φ' may be a bit complex. For example, 'every man is born of some woman' can be paraphrased as 'nothing is a man and non(born of some woman)' which signifies the nonexistence of a man born of no woman. States of existence or nonexistence ($\{\varphi\}$ishness, un$\{\varphi\}$ishness) signified by elementary statements may be called *elementary* states. Since all compound statements have elementary statements as their components, we may define the states they signify in recursive fashion. Compound statements are then seen to signify states that are conjunctions or disjunctions of elementary states. Assume, for example, that 'p', an elementary statement, signifies $\{\varphi\}$ishness (denoting a $\{\varphi\}$ish "p-world") while 'q' signifies $\{\psi\}$ishness (denoting a $\{\psi\}$ish "q-world"). Then 'p & q' signifies the compound state of a world that is both $\{\varphi\}$ish and $\{\psi\}$ish, '$p \rightarrow q$' signifies the state of being either $\{\psi\}$ish or un$\{\varphi\}$ish and so forth for the other compound forms. Generally, any true statement, r, denotes an r-world, i.e., a world characterized by the (compound) state that 'r' signifies. Expressions like "p-world" or "r-world" are propositional terms that may take subject or predicate position in categorical statements. For example, the predicative form of 'p & q' is 'Being a q-world characterizes some p-world'; the predicative form of '$p \rightarrow q$' is 'Being a q-world characterizes every p-world'.

8.1 Terminized propositional logic The propositional terms of a terminized logic denote worlds. Using terms like "p-world" and "q-world", we can give a categorical formulation to any well formed statement of propositional logic, rewriting, say, 'if p then q' as 'every p is a q'. (Within a categorical formula, 'p' is read as the propositional term 'p-world'. Thus, 'every p is q' is read as 'every p-world is a q world'.)

Standard Formula		Categorical Version
p	$\Rightarrow$	some world is a p
$-p$	$\Rightarrow$	some world is a non-p
p & q	$\Rightarrow$	some p is a q
$p \rightarrow q$'	$\Rightarrow$	every p is a q.
$p \vee q$	$\Rightarrow$	every non-p is a q. or: every world is either a p or q.
$p \rightarrow (q$ & $r)$	$\Rightarrow$	every p is a q and an r.
$p \leftrightarrow q$	$\Rightarrow$	every world is either a p and a q or a not-p and a not-q.

[The general form of statement (in the algebraic notation for term logic) is a dyad in which two positive or negative terms are joined by addition or subtraction the whole being qualified by a unary plus or minus sign of affirmation or denial:

$$+(+(+y)+(+x)) \qquad \text{yes/no (some/every } x\text{/non-}x \text{ is } y\text{/non-}y).$$

The terms x and y are either ordinary (denoting things in the world) or propositional (denoting the world itself). The unary plus signs of affirmation are normally omitted. By the rules of formation, if 'x' is a term, then '$-x$' is a term, if x and y are terms then $+ \langle y + x \rangle$ is a term. Again if x and y are terms, $+(y + x)$ is a statement. Thus any sentence is a dyad consisting of two terms that may themselves be dyads. For example, the terminist form of 'q if $(r \,\&\, -s)$' is '$(q-(r + (-s)))$', i.e., every r and non-s is a q.]

9 Saying and claiming 'Some Swedes are poor' and 'Socrates is poor' say something about things *in* the domain under consideration, namely that some of them are poor Swedes or that one of them, Socrates, is poor. But they are claims about the domain itself, namely, that the domain is characterized by the existence of Swedes who are poor or by Socrates who is poor. Any statement may also be read as saying what it claims. I say 'Some Swedes are poor' and one may interpret this in two ways: (1) as being about something *in* the world (Swedes, poor people); (2) as being about the world, *claimsaying* that it is a {poor-Swede}ish world. The first is the usual interpretation, the second reading interprets the statement to say what we should normally be taking it to claim.

The distinction between saying and claiming is idle in propositional logic since the statements we are there concerned with are represented by statement letters that give no clue as to the internal contents of the statements represented (nor is any needed for the purpose at hand). All statements of statement logic are understood as being about the world. Given 'p' we interpret it as *asserting its truth claim*, viz., that the world is a p-world. Given 'p & q' we interpret it to say that the world is both a p-world and a q-world and so on for other compound forms.[4]

9.1 The singular universe of propositional logic There is a crucial semantic difference between any statement understood as being about the world and any statement about things *in* the world: the world is full of many things, and statements about things in the world will normally have one or more general terms denoting more than one thing. But any propositional term that is not vacuous will uniquely denote the world of the truth claim—*and nothing else*. Consider 'p & q' in its categorical form 'some p-world is a q-world'. That this makes a claim about the one world is evident from its equivalence to '*The* world is both a p-world and a q-world'. In particular 'p & p' whose categorical form is 'some p-world is a p-world' is equivalent to 'p' or to 'The world is a p-world'. In effect all propositional terms are uniquely denoting terms and all propositional statements are singular statements. This may be disconcerting: why talk of "some p-world" or "every p-world" when only *one* world is under consideration? But of course we *don't* really talk that way; we actually say 'p & q', not 'some p-world is a q-world'. All the same, 'p & q' or its categorical paraphrase, 'some p (world) is a q (world)', is a semantically singular statement in disguise.

Having said this, one must immediately add that Leibniz followed the scholastic practice of treating *all* singular sentences as disguised "general" categoricals. For example, 'Jungius is great' would be construed by him as 'some Jungius is great' a statement that entails 'every Jungius is great' (there being only one Jungius). Quite generally, whenever the subject term, S^*, is uniquely denotative, 'some S^* is P' entails 'every S^* is P' (there being only one S^*). The converse is not true however. If 'Atlantis did not sink into the sea' is asserted by someone who denies the existence of Atlantis, then the claim made is negative: that the world is characterized by THE NONEXISTENCE OF ATLANTIS, A CONTINENT THAT SANK INTO THE SEA. It would be perverse to put this negative claim in the form of an affirmation ('every Atlantis never sank into the sea' but even if we did so, it would still not entail 'some Atlantis did not sink into the sea').

In effect Leibniz held that singular statements are disguised general statements. Since all propositional terms denote uniquely or not at all, the terminist parsing of 'p & q' as 'some p is q' is yet another way of removing the disguise, this time from common forms like 'p and q' and 'if p then q' exposing them as "general" categoricals with uniquely denoting terms.

In saying that a claimsaying is about "the world" we are also maintaining that claimsaying is not to be interpreted as a claim about a *domain* of worlds, saying of that domain that one of its constituent worlds is a p-world. For we may hold that the actual world is the only world. The "actualist" (unlike Leibniz, I am one) does not think of the world as one of many possible worlds. Nevertheless, although it is true that Leibniz was no actualist, it is probably true that he regarded all nonmodal propositional statements as being about the (one and only actual) world.[5]

9.2 The propositional branch of term logic Propositional terms are uniquely denotative and all denote one and same domain: this explains why and how a terminized propositional logic differs from the rest of term logic. Consider that in general 'some A is B' does not entail 'every A is B' whereas 'some p is q' does entail 'every p is q'. To explain this we note that 'p & q' and its terminist equivalent, 'some p is a q', are disguised singular statements. Where 'X^*' and 'Y^*' denote uniquely, 'some X^* is Y^*' does entail 'every X^* is Y^*' as well as 'every Y^* is X^*'. So, given the premise that the world is both a p-world and a q-world, it follows that *every* p-world is a q-world. Moreover since, all sentential terms denote the *same* world, we see why 'some q is p' is incompatible with 'some q is not p' and in general why 'some world is p', is incompatible with 'some world is non-p'. Indeed, the feature distinguishing propositional logic as a special branch of term logic is just this: because of the singularity of the universe of discourse, 'something is p', 'something is non-p' cannot both be true. In a consistent and thoroughgoing term-theoretic interpretation, the propositional law of contradiction '$-(p \,\&\, -p)$' reverts to its old Aristotelian form as just another instance of the prohibition against attributing contrary attributes to one and the same subject, in this case contrary existential attributes to the domain of the claim.

10 Summary of the argument for terminizing propositional logic Statements as well as terms signify characteristics and denote what has the characteristic signified. The most plausible candidate for what an elementary statement signifies is the existence (nonexistence) of a certain kind of thing. We have argued for a conception of existence that treats it as a property of domains. Elementary statements (i.e, statements that are canonically of form 'something/nothing is

φ') signify the existence (nonexistence) of φ things and denote the domain so characterized.

The doctrine that existence and nonexistence are world properties that true statements signify provides a clear conception of the *facts* that makes true statements true. Construing true statements as expressions that signify facts and denote the common domain of the truth claim enables the term logician to give the well formed statements of propositional logic the categorical reading that Leibniz adumbrated. The categorical transform of 'p & q' is 'some p-world is a q-world', of '$p \rightarrow q$', 'every p-world is a q-world'. The doctrine that *all true statements denote one and the same domain* (though signifying different facts) is the key to understanding why all of the "general categorical" statements of a terminized propositional logic are semantically singular. That all nonvacuous propositional terms denote the universe of discourse fully accounts for the atypical features of propositional logic as a special branch of term logic.

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NOTES

1. For further discussion of the logical copulas in a term/functor logic see Sommers [3].

2. The widespread but baseless belief that the logic of terms is essentially weaker in inference power than modern predicate logic (because it is unable to deal with multiply general statements like 'some boy loves every girl') has been largely responsible for the unseemly abandonment of a terminist tradition that is one of the intellectual glories of medieval philosophy. Schools that attempted to retain a tradition of teaching logic in the terminist way were dubbed "Colleges of Unreason" by P. T. Geach, one of the more adamant and committed Fregeans who, along with Father Bochenski, promoted the doctrine that modern predicate logic was canonical by claiming for it illusory advantages over term logic. For how term logic extends to relational inference see Sommers [2], [3]. See also [2] and Chapters 1 and 6 of [3] for accounts of the terminist doctrine that singular statements are of form '*PiS*'.

3. For an account of the scholastic distinction between being *simpliciter* and being *secundum quid* see Williams [4], p. 4.

4. Note that as soon as we move to truth functional forms, the truth claim interpretation is the *normal* one. For example, even an elementary categorical like 'No trillionaire is immortal' (which negates a statement purporting to be about things *in* the world) will normally be interpreted as being about the world itself, saying of it that it lacks immortal trillionaires. Frege's way of interpreting 'p' as asserting a truth claim is found at the beginning of *Begriffschrift*, where he remarks that any statement 'p' can be construed as saying 'that p is a fact'.

5. Actualism (which abjures possible worlds) is classical terminist doctrine. According to Norman Kretzmann and Eleonore Stump [1], Aristotle's modal logic is "best understood as an attempt to characterize relations between accidental and necessary properties of things in the actual world." In developing an actualist account of modality one forsakes explanations that appeal to "all possible worlds"; instead one ascribes

to the world itself certain preclusive properties. For example, to say that necessarily water boils when superheated (to above 100C, in standard conditions) is to say the world is proof against the existence of superheated water that fails to boil (*Pace* Anselm, the necessary existential properties of the world are all *negative* and "preclusive"). See [1], p. 312f.

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