Notre Dame Journal of Formal Logic
Volume 34, Number 1, Winter 1993

Intermediate Quantifiers for Finch's Proportions

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Abstract In "Validity Rules for Proportionally Quantified Syllogisms", H. A. Finch gave six complex rules for determining validity and invalidity of syllogism-analogues—argument forms that contain fraction quantifiers applying to each of the terms. The algebraic method for determining validity or invalidity for the 5-quantity and "higher" quantity syllogisms can be extended to cover all the syllogism-analogues which Finch considers, clearing the way for a syllogistically-oriented approach to explaining human reasoning about numbers (wherein intermediate quantifiers are fundamental).

The traditional Aristotelian doctrine of the syllogism has been extended in a thoroughly traditional manner to *three* more quantities—so-called "intermediate" quantities occurring *in between* universal and particular. These quantities are the common (expressible by "many"), the majority (expressible by "most"), and the predominant (expressible by "almost-all" and by the negative "few" (parallel to "none")). In the new 5-quantity syllogistic, there are 4000 well-formed argument types of which 105 are valid.[1] Peterson [7], in addition to providing a brief but clear introduction to the 5-quantity syllogistic, shows how to extend the number of quantities to 6, then to 7, to 8, and so on to as "high" a number of quantities as you like. This further extension is effected by introducing "fractional" quantities which are expressed by fraction-*like* quantifiers of the form "m/n" and "More than $(n - m)/n$" for integers m and n such that $2m \leq n$. (*Fractional* quantifiers are not just all possible ratios serving as quantifiers. They are a proper subset of modified fractions—those motivated by squares of opposition like (1), (7), (11), and (13) of [7].)

Here are some examples of 5-quantity "intermediate" syllogisms and of "higher" quantity syllogisms:

Received March 24, 1992; revised August 19, 1992

(1) (a) Three intermediate syllogisms

(valid) EKG-2	No *P* are *M*	(valid) PKI-3	Almost-all *M* are *P*
	Many *S* are *M*		Many *M* are *S*
so,	Many *S* are not-*P*.	so,	Some *S* are *P*.

(invalid) TAT-3	Most *M* are *P*
	All *M* are *S*
so,	Most *S* are *P*.

(b) Two higher-quantity syllogisms (6-quantity, 8-quantity, both valid)

AFK-1	All *M* are *P*	ESO-4	No *P* are *M*
	$\frac{1}{2}$ of *S* are *M*		$\frac{1}{3}$ of *M* are *S*
so,	Many *S* are *P*.	so,	Some *S* are not-*P*.

I will demonstrate herein the fruitfulness of our (Carnes' and my) approach to the 5-quantity and higher-quantity syllogisms by considering how to apply it to "proportional syllogisms" from H. A. Finch [2]. Although Finch's proportional syllogisms do not completely reduce to higher-quantity syllogisms, they are analyzable by our methods with notable gains in clarity and simplicity.

Finch has devised rules for a "very general type of numerically definite inference which is a strong analogue of the classical syllogism" (p. 1). He is mainly concerned with argument forms that go well beyond orthodox syllogistic forms as Carnes and I understand them. *Our* extension of Aristotle's syllogism is only along *one* dimension—viz., quantity. We have *not* extended any of the other features of syllogistic forms. We adhered strictly to the concept of categorical proposition according to which it is a two-termed (subject-predicate) proposition with explicit negation or not (making the categorical a denial or not) *and exactly one quantity.* The quantity is that expressed by a quantified phrase ("all", "every", "some", "almost-all", etc.) attached to the *subject* term alone. The propositions in Finch's analogues are not categorical in this strict sense, since *each* of the two terms in them has its own quantifier. Thus, each proposition in Finch's analogue of syllogistic forms has *two* quantities (rather than one). Single-quantity propositions are limiting cases for Finch.

Finch introduces quantifiers of three different types as follows:

(2) (a) At least m/n (of)
(b) At most m/n (of)
(c) Precisely m/n (of)
for integers m and n such that $n \geq m$.

So, the *kind* of syllogism-analogue Finch investigates is illustrated in this example:

(3)

	At most $\frac{3}{4}$ (of the) *P* are at least $\frac{1}{2}$ (of the) *M*
	Precisely $\frac{1}{3}$ (of the) *S* are at most $\frac{3}{4}$ (of the) *M*
so,	At least $\frac{1}{4}$ (of the) *S* are at most $\frac{1}{10}$ (of the) *P*.

But Finch limits his discussion (for *un*stated reasons) to slightly simpler conclusions—viz., to those with only one such quantifier, the one modifying the subject term. He gives (only!) three examples, the third of which is so simple as not to bear discussing further—viz., his analogue of Barbara. (Precisely $\frac{100}{100}$ *M* are *P*, precisely $\frac{100}{100}$ *S* are *M*; so $\frac{100}{100}$ *S* are *P*.) Here are the two others:

(4) At most $\frac{4}{7}$ *P* are at most $\frac{2}{3}$ not-*M*
At least $\frac{3}{5}$ *S* are precisely $\frac{7}{8}$ *M*
―――
At most $\frac{1}{3}$ *S* are *P*.

(5) Precisely $\frac{1}{3}$ *M* are *P*
Precisely $\frac{1}{2}$ *M* are *S*
―――
At most *r* of *S* are *P*. . . For finite size universe of discourse *N*, what is *r*?

Finch formulates six individually complex "rules of validity" for his multiple-quantity syllogisms. He "proves" these (i.e., deduces them) from four hypotheses, seven definitions, and six axioms. The proof amounts to demonstrating soundness at best, not completeness.

I shall now show that all that is needed to test validity or invalidity of multiply-quantified syllogisms—Finch's "proportionally quantified syllogisms"—is available in the algebraic methods for evaluating higher-quantity syllogisms presented in [7] (cf. Appendix II, especially (d)).[2] I shall merely extend the algebra slightly. Developing extensions of the 5-quantity syllogistic rules for the *k*-quantity syllogistic system (for some finite *k*) is not required. Neither is developing extensions of the 5-quantity Venn Diagram method required, though some use or other of Venn Diagrams seems indispensable.

The first thing to notice in approaching Finch's syllogism-analogues is that they are combinations of orthodox (one quantity per proposition) higher-quantity syllogisms. Take the first premise of (3). It is equivalent to

(6) At most $\frac{3}{4}$ of the *P* are *M* *and* at least $\frac{1}{2}$ of the *M* are *P*.

Similar premises can be made for the other propositions in (3). Listing the component propositions separately (except for the conclusion), (3) is a complex and particularly dense (or dense-appearing) version of

(7) At most $\frac{3}{4}$ of the *P* are *M*
At least $\frac{1}{2}$ of the *M* are *P*
Precisely $\frac{1}{3}$ of the *S* are *M*
At most $\frac{3}{4}$ of the *M* are *S*
―――
so, At least $\frac{1}{4}$ of the *S* are *P* & At most $\frac{1}{10}$ of the *P* are *S*.

Evaluating (7) is like evaluating two or more higher-quantity syllogisms simultaneously. Before applying the appropriate higher-quantity algebraic methods, simplification of the quantifier *types* can be instituted. This is not necessary, but it helps in utilizing the algebra since certain assumptions are already in effect. Central among these is the assumption that every quantifier in a higher-quantity syllogism is *interpreted* (in context, for the sake of evaluating the argument in question) as having a tacit "or more" rider (cf. Peterson [3], p. 157 and [7], p. 351). Notice that even an Aristotelian form like AAI-1 would not be valid without this assumption:

(8)

All *M* are *P*
All *S* are *M*

Some *S* are *P*.

For if "Some *S* are *P*" did not mean (or was not interpreted so that it meant) "Some *or more S* are *P*", then the form would not be valid. For the *reason* that the conclusion follows from the premises here (or one way of looking at it) is that the premises entail "All *S* are *P*" (via Barbara) and that proposition entails "Some *S* are *P*". But the only way that can be is if the latter is interpreted as "Some *or more S* are *P*"—which is the same thing as Finch's "At least some" (replacing his ratio with "some"). (If it were interpreted "At most some *S* are *P*" or "Exactly some (and no more) *S* are *P*", the inference would be invalid.) Since it is advisable (not just convenient, but very useful to comprehend the whole concept of an orthodox syllogistic system) to take every categorical proposition actually *in* a syllogistic argument (or form of argument) as automatically involving the "or more" rider (equivalent to an "at least" prefix), it is also advisable (at least convenient) to reduce Finch's three types of proportion quantifiers to one—where that one is the "at least" variety (equivalent to instituting the "or more" rider). So, any quantifier containing an "at least" remains as it is (even with "at least" dropped, since the "or more" is always tacitly in effect). "At most" quantifiers are reduced as follows:

(9) "At most m/n *S* are *P*" =df. "–(More than m/n *S* are *P*)"
=df. "$(n - m)/n$ *S* are not-*P*".

"Precisely" quantifiers are reduced as follows:

(10) "Precisely m/n *S* are *P*" =df. "(m/n *S* are *P*)
& ($(n - m)/n$ *S* are not-*P*)".

Here is (7), an analysis of (3) remember, restated in reduced-quantifier form:

(11)

$\frac{1}{4}$ of the *P* are not-*M*
$\frac{1}{2}$ of the *M* are *P*
$\frac{1}{3}$ *S* are *M* & $\frac{2}{3}$ *S* are not-*M*
$\frac{1}{4}$ of the *M* are not-*S*

$\frac{1}{4}$ of the *S* are *P* & $\frac{9}{10}$ of the *P* are not-*S*.

To demonstrate the validity of (11), it is sufficient to show that the denial of the conclusion together with the premises produces a contradiction. To demonstrate the *invalidity* of (11), it is sufficient to show that there is a possibility of assigning memberships to all the relevant subclasses (including the classes of things that are non-*S*, non-*P*, and non-*M*) in such a way that the premises are true and the conclusion false.[3] Each line of the following table, (12), contains one proposition of the argument in (11) followed by two different versions of its truth conditions (the first being a notational variant useful for deriving the second, with some redundancy since either rendition of truth conditions could be used alone). (13) reveals how the relevant subclasses are notated.

(12) $\frac{1}{4}$ P are $-M$ ...... $3(P\overline{M}) \geq 1(PM)$ ...... $3(b+c) \geq (e+f)$

$\frac{1}{2}$ M are P .... $1(MP) \geq 1(M\bar{P})$ ...... $(e+f) \geq (d+g)$

$\frac{1}{3}$ S are M $2(SM) \geq 1(S\overline{M})$
& $\frac{2}{3}$ are $-M$ .... & $1(S\overline{M}) \geq 2(SM)$ .. $2(d+e) = (a+b)$

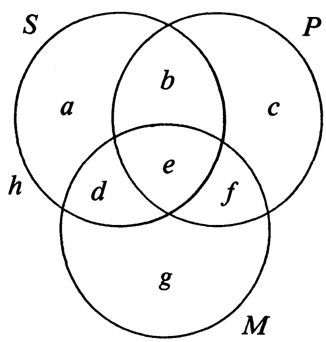
$\frac{1}{4}$ M are $-S$ ...... $3(M\bar{S}) \geq 1(MS)$ ...... $3(g+f) \geq (d+e)$

so $\frac{1}{4}$ S are P $3(SP) \geq 1(S\bar{P})$ $3(b+e) \geq (a+d)$
& $\frac{9}{10}$ P are $-S$ .. & $1(P\bar{S}) \geq 9(PS)$ ... & $1(c+f) \geq 9(b+e)$

where: for "X" and "Y" representing "S", "P", "M", "$-S$", "$-P$", and "$-M$", let "(XY)" =df. "the quantity of X that are Y" or "the size of the subclass of Xs that are Ys", and read "$m(XY)$" as "m times the quantity of subclass XY" (for any integer m); and the formula (rule) for producing truth conditions from these categoricals is:

$$m/n\ X \text{ are } Y \textit{ iff } (n-m)XY \geq mX\bar{Y}$$

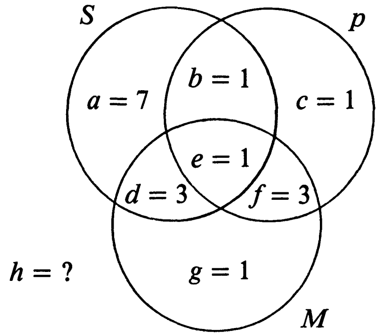
(13)



i.e., $S\bar{P} = (a+d)$, $SP = (b+e)$, etc.

The argument in (11), and thereby (7) and (3), is shown to be *invalid* by first assigning some sizes (amounts, quantities, measures) to the subclasses which make the conclusion false, e.g., $b = c = e = 1$, $d = 3$, $f = 3$, $a = 7$. (These values amount to overkill, since they make each conjunct of the conclusion false where only one needs to be false for the demonstration.) With these values the remaining four premises (right-hand column of (12)) amount to $6 \geq 4$, $4 \geq 3 + g$, $8 = 8$, and $3g + 9 \geq 4$. Then assigning $g = 1$ makes them all true. So there *is* an interpretation of (3)—via (7), (11), and (12)—wherein the premises are true and the conclusion false. So the argument is invalid. (One might consider whether simplifying the conclusion produces a valid argument, say dropping one of the conjuncts. It does not, due to the overkill already involved.) Here is a representation of the invalidity-making values for (11) which may help the reader check the demonstration:

(14)



Finch's first example ([2], pp. 1 and 16) is (4) above. Here is an analysis of (4) paralleling the breakdown of (3) to (7):

(15)

$$\begin{array}{l} \text{At most } \tfrac{4}{7}\ P \text{ are } -M \\ \text{At most } \tfrac{2}{3}\ -M \text{ are } P \\ \text{At least } \tfrac{3}{5}\ S \text{ are } M \\ \text{Precisely } \tfrac{7}{8}\ M \text{ are } S \\ \hline \text{At most } \tfrac{1}{3}\ S \text{ are } P. \end{array}$$

Further reducing (15) to one kind of quantifier (rather than three) replaces (15) with (16):

(16)

$$\begin{array}{l} \tfrac{3}{7}\ P \text{ are } M \\ \tfrac{1}{3}\ -M \text{ are } -P \\ \tfrac{3}{5}\ S \text{ are } M \\ \tfrac{7}{8}\ M \text{ are } S \\ \tfrac{1}{8}\ M \text{ are } -S \\ \hline \tfrac{2}{3}\ S \text{ are } -P. \end{array}$$

Here is a representation of the truth conditions for the components of (16)—in the style of (12), with two notational variants for each proposition:

(17)

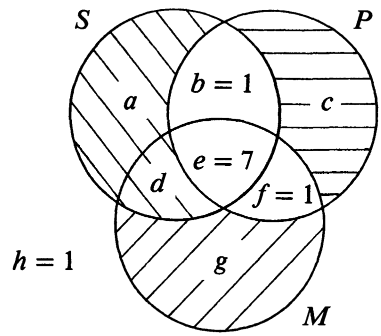
$$\begin{array}{llll} \tfrac{3}{7}\ P \text{ are } M \ldots\ldots & 4(PM) \geq 3(P\overline{M}) & \ldots\ldots & 4(e+f) \geq 3(b+c) \\ \tfrac{1}{3}\ -M \text{ are } -P \ldots & 2(\overline{M}\overline{P}) \geq 1(\overline{M}P) & \ldots\ldots & 2(a+h) \geq (b+c) \\ \tfrac{3}{5}\ S \text{ are } M \ldots\ldots & 2(SM) \geq 3(S\overline{M}) & \ldots\ldots & 2(d+e) \geq 3(a+b) \\ \tfrac{7}{8}\ M \text{ are } S & 1(MS) \geq 7(M\bar{S}) & & \\ \tfrac{1}{8}\ M \text{ are } -S \ldots & 7(M\bar{S}) \geq 1(MS) & \ldots\ldots & (d+e) = 7(g+f) \\ \hline \tfrac{2}{3}\ S \text{ are } -P \ldots\ldots & 1(S\bar{P}) \geq 2(SP) & \ldots\ldots & (a+d) \geq 2(b+e) \end{array}$$

(where subclasses referred to in the right-hand column are those of (13)).

In order to show that this syllogism-analogue is invalid, it is sufficient to show that the conclusion can be false when the premises are true. To make the conclusion false, assign subclass sizes as follows: let $a = d = 0$ and $b > 0$ and $e > 0$.

Then to make the premises all true, assign $c = g = 0$, $b = h = f = 1$, and $e = 7$. Here is a helpful representation of this assignment:

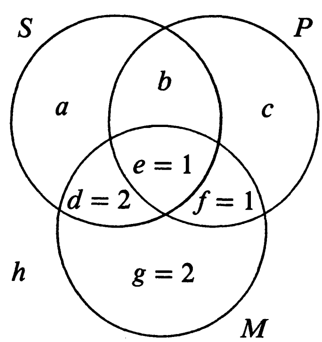
(18)



By inspecting (18) one can determine that it makes each premise of (16) true and the conclusion of (16) false. So, the argument *forms* in (16) and (15) are invalid and so, thereby, is (4)—Finch's example—invalid (as he also determines).

Finch's other nontrivial example ([2], p. 16) is (5) above. This is not exactly a syllogism-analogue to test for (in)validity. Rather, it is an underspecified argument form of which Finch asks, what is the value of r given some specific size of universe N. Finch derives the rule: $r = 1 - 1/(N - 3)$ ([2], p. 17). Six members is the smallest number of members that M can have to make both premises of (5) true. This is most easily verified by inspecting a representation like

(19)



If we also set $N = 6$ (so that there is nothing else in the universe of discourse besides these members of M), then $r = \frac{2}{3}$. Then the resulting syllogism-analogue (substituting "$\frac{2}{3}$" for "r" in (5)) is:

(20)

$$\frac{\begin{array}{l}\text{Precisely } \frac{1}{3}\ M \text{ are } P\\ \text{Precisely } \frac{1}{2}\ M \text{ are } S\end{array}}{\text{At most } \frac{2}{3}\ S \text{ are } P.} \qquad \text{given } N = 6$$

That is, if we deplete the assignments in (19) in any way, the premises of (20) cannot both be true. Of course, N can be indefinitely larger; e.g., simply add the same number of zeros, for as many as you like, to each integer in (19) to preserve premise-to-conclusion entailment in (20) for N consisting entirely of members of M. From my perspective, what this means is that (20) is actually *invalid* when N is ignored. This is easily shown (shortcutting somewhat the leisurely procedure used above):

(21)
$$\begin{array}{ll} \text{Precisely } \tfrac{1}{3}\ M \text{ are } P \ldots\ldots & 2(e+f) = (d+g) \\ \text{Precisely } \tfrac{1}{2}\ M \text{ are } S \ldots\ldots & (d+e) = (g+f) \\ \hline \text{At most } \tfrac{2}{3}\ S \text{ are } P \ldots\ldots\ldots & 2(a+d) \geq (b+e). \end{array}$$

Simply adding to (19) the assignment $SP\overline{M} = 32$—i.e., $b = 32$—assuming $a = c = h = 0$, produces a complete assignment wherein the premises of (21) are true and the conclusion false. (The first premise reduces to $2 \times 2 = 2 + 2$, the second to $2 + 1 = 2 + 1$, but the conclusion to $4 \geq 33$, which is false.) But look at (19) again. If all the unmarked subclasses of S and P are empty (i.e., $a = b = c = h = 0$), then the premises and conclusion of (20) are all true. This does *not* show that (20) is, without restriction to $N = 6$, valid. For it is invalid (since just proved so). However, under certain conditions (e.g., N restricted to 6, distributed as in (19)), the premises and the conclusion are both true.

To *change* (21) into a correct entailment for the assignment just mentioned—$b = 32$, $a = c = h = 0$, $d = g = 2$, and $e = f = 1$ (i.e., (19) with $b = 32$ added)—use Finch's formula to derive a proposal for the value of r. Presuming no other objects in the universe of discourse, $N = 38$. Then $r = 1 - 1/(38 - 3) = \frac{34}{35}$. So, the following premises entail the conclusion (for the assignment just given):

(22)
$$\begin{array}{ll} \text{Precisely } \tfrac{1}{3}\ M \text{ are } P & \text{for } N = 38 \\ \text{Precisely } \tfrac{1}{2}\ M \text{ are } S & \\ \hline \text{At most } \tfrac{34}{35}\ S \text{ are } P \ldots\text{i.e., } \tfrac{1}{35}\ S \text{ are } -P. & \end{array}$$

That is, the first premise reduces again to $2 \times 2 = 2 + 2$ and the second to $2 + 1 = 2 + 1$, but the conclusion reduces to $34 \times 2 \geq 32 + 1$, which is also true.

Thus any given value of N produces a restricted syllogism-analogue which *can* be tested for (in)validity by our methods. So nothing in this kind of example (or "problem" as Finch calls it) outstrips, evidently, straightforward application of the algebraic methods associated with the higher-quantity syllogistic.

These developments do *not* disprove anything in Finch's rules (or associated discussion). However, it is now evident that it is unlikely there is anything in Finch's analysis that casts doubt on the nature and utility of the higher-quantity syllogistic systems (or the 5-quantity syllogistic). Especially with regard to determining validity and invalidity of Finch's syllogism-analogues—the multiply-quantitied, proportional syllogisms—it appears that Finch's rules are dispensable. In particular, although they do lead to quicker answers, they do not (it appears) lead to perspicuous or easily comprehended demonstrations. On the other hand, analyzing such syllogism-analogues from *our* (Carnes' and my) perspective (as I have just illustrated) *does* lead to absolutely convincing results. Further, I suspect that only by instituting these kinds of methods would anything like a completeness proof of Finch's rules be possible.

Our approach, in contrast to Finch's, has a more general advantage as well. A large part of the *philosophical* motive for investigating quantity, quantifiers, proportions, etc., in logic is to contribute to an eventual explanation of (i) how we humans reason (correctly!) and (ii) what we humans (*qua* philosophers) *know* about how we reason. The Aristotelian approach appears helpful in building this explanation – *very* helpful I would claim, even if others think it is only marginally helpful. Many investigators today would say that explaining the indefinite, vague, or general features of thought, language, and/or reality requires basing the explanation on something absolutely specific and definite. (For example, *vis à vis* ontological inquiry, explain reasoning about *generalities* via a theory which at bottom refers only to, or assumes to exist, very *specific* particular objects and equally specific classes of them – the nominalistic tactic.) I conjecture that the *reverse* can sometimes produce the better explanation, e.g., in *this* case of explaining (especially epistemologically) allegedly vague or indefinite quantities (expressed by, say, "few", "many", and "most"). Finch, for one, *might* claim that the way to base a theory of such vague and/or general quantities on something that is epistemologically secure is to show how it is a departure from (or construction on) the very definite logic of proportions as *he* has begun investigating it. If he or someone else did claim so, then I would reply that the reverse might be a better possibility. That is, Carnes and I have begun to provide parts of the explanation of how something quite definite, specific, and particular about our reasonings (such as concerned with *definite* proportions such as are expressed by "precisely $\frac{1}{3}$ of") is actually explained in terms of more general and/or *in*definite features of basic (orthodox) syllogistic reasoning (viz., the 2-quantity traditional syllogistic extended first to 5 quantities and, then, to k quantities). The *virtue* of the allegedly vague "few", "many", and "most" is that the "logic" of these quantities turns out to be *very* systematic, thereby revealing the real nature and wide explanatory power (vs. its alleged idiosyncrasy and limitations) of syllogistic reasoning. In the end, the *intermediate* quantity terms and concepts are not vague at all but merely generic. The final explanation of human reasoning (logically, mathematically, and otherwise) may well rest on the very exacting ways we can and do (correctly!) reason *with* these generic (nonvague) concepts.

NOTES

1. The detailed account is given in an unpublished paper by Peterson and Carnes, whose contents have been presented in Peterson [4], [5], [6], [7], [8], [9], and Peterson and Carnes [11]. The unpublished paper was originally produced as a corrective to Thompson [12], being written well before the latter was even published (cf. [4]). Thompson [13] exceeds in erroneousness [12]. Thompson [13] probably cannot be corrected, as is explained in Carnes and Peterson [1]. See Peterson [10] for an algebraic approach to Thompson's [13] data.

2. Some typographical errors occurred in [7]. On pp. 358–9, the numeral "2" should be deleted in three proofs in which it occurs: (i) of EKG-2 in line (6), (ii) of PKI-3 in line (6), and (iii) of AFK-1 in line (5). Also, concerning the discussion on p. 355, considerations not introduced there will require that $i + j$ cannot be greater than 100. So, $i + j = 100$.

3. "$\frac{9}{10}$ of the S" is *not* a higher-quantity quantifier, for it is not a "fractional" (since all fractionals greater than $\frac{1}{2}$ have a "more-than" prefix). So strictly speaking these arguments are not being reduced to higher-quantity syllogisms. Indeed, they outstrip

them. But it is interesting that the (semantically oriented) algebraic technique for higher-quantity syllogisms is completely applicable to Finch's analogues. (Indeed, here lies the clue for future development of a much broader syllogistic of proportions.)

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