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Formalizing the Logic of Positive, Comparative, and Superlative

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Abstract A formalism is introduced for symbolizing positives, comparatives, and superlatives like "old", "older", and "oldest" in such a way as to show their interdependence, as in "The youngest old man is a young old man". Formal principles are stated for determining logical properties of such propositions, e.g., whether "All young old men are young men" is logically true. These allow for the fact that adjectives like "young" and "old" may not only be vague and indefinable in terms of their corresponding comparatives, but they may also be relative, so that to be young *for an old man* is not necessarily to be young *for a young man*.

1 Introduction Even simple, or at least short ordinary language statements that involve connections between positives, comparatives, and superlatives can be 'logically opaque'. For instance, few students in my experience have been able to 'see' directly which of the following are valid:

The oldest bold pilot is old; therefore, the boldest old pilot is bold. (Inference 1)[1]

The oldest bold pilot is bolder than the boldest old pilot; therefore, the oldest bold pilot is older than the boldest old pilot. (Inference 2)

Putting aside general problems in the analysis of definite descriptions like "the oldest bold pilot" and "the boldest old pilot", and the vagueness of "old" and "bold",[2] which will be returned to briefly and inconclusively below, the special difficulty that arises in applying formal logic to the analysis of these inferences derives from the fact that its notation does not show the connections between the forms. Thus, while we might symbolize "old" and "bold" as "O" and "B", respectively, we are forced to symbolize "older than" and "bolder than" with binary relation symbols such as "R" and "S", which are formally unrelated to "O" and "B". And, if we choose to symbolize "the boldest old pilot" and "the oldest bold pilot" as individual constants, say as "o" and "b", we lose sight of their connections with "old", "older than", "bold" and "bolder than",[3] on which

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the validity of the sort of reasoning just illustrated rests. Given the ubiquity of reasoning involving these forms in ordinary life, it seems desirable to develop a symbolism suitable to analyzing it which will show connections between the positive, comparative, and superlative explicitly. This paper will make suggestions concerning this, beginning with positives and their corresponding comparatives.

*2 **Positive and comparative*** Assuming that an adjective like "old" has a corresponding comparative,[4] we propose to symbolize the comparative by writing the ordering symbol "$>$" as a superscript on the symbol that symbolizes the adjective. Thus, if "old" and "bold" are symbolized as "O" and "B", respectively, then "older than" and "bolder than" will be symbolized as "$O^>$" and "$B^>$", respectively.[5]

We will assume that any natural language *positive-comparative predicate pair* (P-C pair) that we deal with, symbolized by predicate pairs Π and $\Pi^>$,[6] satisfies the following conditions:

Schema 1

(1) $\Pi^>$ is a strict weak ordering relation in the sense of satisfying:
  (a) $(\forall x)(\forall y)(x\Pi^>y \to \neg y\Pi^>x)$, and
  (b) $(\forall x)(\forall y)(\forall z)(x\Pi^>z \to (x\Pi^>y \vee y\Pi^>z))$,[7]
(2) $(\forall x)(\forall y)((\Pi x \;\&\; \neg\Pi y) \to x\Pi^>y)$.

Condition 2 says that Π is a *cut* of the ordering $\Pi^>$, i.e., Πy holds for all y higher in the ordering than any element x for which "Πx" holds. In the case of age, this is equivalent to saying that anyone older than someone old is old, and it is plausible that satisfying this is a *requirement of consistency* for connections between positives and their corresponding comparatives. Though persons are sometimes judged not to be old even though they are older than persons judged to be old, it is not implausible that when this is pointed out to the persons making the judgments they are likely to revise them, saying, for instance, that the person judged to be old only looks old. Of course this suggestion requires careful consideration, which we are not prepared to give it here.

It is important for present purposes to make it plausible that Schema 1 captures essentially all of the 'elementary' principles that are valid for P-C reasoning in general. This will follow if it can be argued that any *Schema 2 model*, i.e., system $\langle D,\Pi,\Pi^>\rangle$ with domain D that satisfies Conditions 1 and 2, is 'elementarily equivalent' to a possible P-C system, and therefore if elementary reasoning (i.e., reasoning formulable in first-order logic) involving P-C predicates isn't formally valid in the sense of following from Conditions 1 and 2, then there is some P-C system in which its premises would be true and its conclusion would be false. A sketch of an argument for this is as follows.

It is well known that any system $\langle D,\Pi,\Pi^>\rangle$ is elementarily equivalent (Chang and Kiesler [1], p. 32) to a system $\langle E,\Gamma,\Gamma^>\rangle$ in which E is a countable set; and assuming that Condition 1 is satisfied, $\Gamma^>$ must be a strict weak ordering of E. Given this, E can even be assumed to be a subset of the positive rationals,[8] which we can in turn assume are the weights of corresponding physical objects;[9] hence (equating objects and their weights), $x\Gamma^>y$ holds if and only if x is heavier than y. It follows that E and $\Gamma^>$ are elements of the P-C system of *heaviness*, and it only remains to argue that the cut Γ might correspond to the class of objects that are *heavy*. This is a complicated matter, but it becomes plausible if it is recognized that Γ could even be empty or universal—i.e., either no objects might be said to be heavy, or all of them might be. The first could arise if the domain consisted solely of light objects, say of atoms, and the second could arise if it consisted solely of large astronomical objects.[10]

Two things follow assuming 'Schema 1 completeness' in the sense that was argued for above. One is that the schema is compatible with the positive being *any* cut of the comparative, and therefore it is not necessarily definable in terms of the comparative.[11] Our logic is compatible with a looser connection between the two, and even with a good deal of vagueness in the positives to which it applies. So, for instance, we don't need to assume that "old" has a precise definition or that there is general agreement among its users, and all that we require is that on particular occasions persons should reason consistently with Schema 1.

Again, to the extent that Schema 1 captures the logic of positive and comparative, it follows that 'possible comparatives' can be adjoined to any positive predicate Π by ordering the classes of objects satisfying "Πx" and "$\neg\Pi x$" arbitrarily and putting all members of the first class above those of the second in the ordering $\Pi^{>}$. For instance, arbitrarily ordering old persons and doing the same for persons who are not old, and stipulating that persons in the first category are older than those in the second, will yield a P-C pair that satisfies Schema 1.[12] This implies that the fact noted in Footnote 4, that only certain natural language predicates are positive in the sense of having corresponding comparatives, cannot be explained 'internally' within the logic that is formulated here.

Of course, Conditions 1 and 2 do not capture reasoning that depends on features of *particular* P-C concepts, e.g., which might assume that only finitely many things are ordered, so that it could be logically true that there exists a youngest old man. Furthermore, so far any one application of our logic deals *only* with the relation between a single positive and its corresponding comparative, and not with relations between those and other concepts. This is serious because positive predicates correspond to adjectives, and adjectives are modifiers that seldom if ever stand 'logically alone', and the nouns and noun phrases that they modify are essential to their logic.[13] For example, even when an adjective like "old" stands alone grammatically, as in "He is old", what is meant is something like "He is an old *person*". And, that these adjectives normally have a degree of *relativity*, so that saying that someone is old is relative to the concept modified, means that this should be taken account of, say, in analyzing "All young old men are young men". To explain why this isn't logically true, our logic must be extended to show the dependence of "young old man" on "young" and "old man" in a way that does not treat "young" as a self-subsistent predicate. This will be discussed in Section 5, but for now we are warned that in applying the present logic to reasoning involving adjectives like "young" and "old", we must take care that they modify the same concepts throughout.

Now we will consider superlatives.

3 The logic of superlatives We need a symbolism that makes the dependence of individual referring expressions like "the oldest bold pilot" on both "old" and "bold pilot" explicit. Of course, the Russellian analysis [4] of sentences $\phi(\alpha)$ where $\alpha =$ 'the oldest bold pilot',

$$(\exists x)\{(Bx\&Px\&\phi(x)) \ \& \ (\forall z)((Bz\&Pz) \rightarrow -zO^{>}x)$$
$$\& \ (\forall y)[[(By\&Py) \ \& \ (\forall z)((Bz\&Pz) \rightarrow -zO^{>}y)] \rightarrow x = y]\}$$

(letting "P" symbolize "pilot") does this, but for present purposes a subscripting convention provides a more compact notation. Letting "O" symbolize "old" and "$Bx\&Px$" symbolize "bold pilot", the description "the oldest bold pilot" will be symbolized as a complex 'superlative individual constant':

$$c_{O;Bx\&Px},$$ [14]

which can be read as "the object highest on the O scale that satisfies '$Bx\&Px$' ". Given this, the proposition "The oldest bold pilot is older than the boldest old pilot" can be expressed both compactly and in a way that corresponds closely to the English:

$$c_{O;Bx\&Px}\,O^{>}c_{B;Ox\&Px}.$$

The following schema formulates two conditions that are the key to the logic of sentences like the foregoing:

Schema 2 For any positive predicate Π and formula $\varphi(x)$ with one free variable "x":

(3) $\varphi(c_{\Pi;\varphi(x)})$,
(4) $\neg(\exists y)(\varphi(y) \ \& \ y\Pi^{>}c_{\Pi;\varphi(x)})$.

For example, in application to age, symbolized "O", and to bold pilots, expressed by $\varphi(x) =$ "$Bx\&Px$", Condition 3 implies "$Bc_{O;Bx\&Px}\&Pc_{O;Bx\&Px}$", i.e., that the oldest bold pilot is bold and a pilot. Of course this implies that there exist bold pilots; therefore, the mere use of "$c_{O;Bx\&Px}$" has a factual presupposition, and it should not be used when this is questionable. Similarly, Condition 4 implies that there is no bold pilot who is older than the oldest bold pilot, hence "$c_{O;Bx\&Px}$" also should not be used in cases where this is doubtful.[15]

It will prove important that Conditions 1-4 are essentially *universal*, because this implies that if the premises and conclusion of an invalid inference are quantifier-free and involve just the individual constants $c_{O;\varphi(x)}, \ldots, c_{B,\Phi(x)}, \ldots$, then there is a counterexample in a domain, all of whose members correspond to these constants. This in turn provides a decision procedure for determining the validity of these inferences—though the graphical representation to be described in Section 4 provides more insight into this reasoning.

Models relevant to assessing the validity of such reasoning combine Schema 1 models $\langle D,O,O^{>}\rangle$ and $\langle D,B,B^{>}\rangle$ together with interpretations of constants $c_{O;\varphi(x)}$ and $c_{B;\Phi(x)}$ corresponding to the formulas $\varphi(x)$ and $\Phi(x)$ that appear in the superlatives that enter into the reasoning. Thus, a *Schema 2 model* takes the form $\langle D,O,O^{>},B,B^{>},c_{O;\varphi(x)},\ldots,c_{B,\Phi(x)},\ldots\rangle$, subject to the requirement that $c_{O;\varphi(x)}\ldots$ and $c_{B;\Phi(x)}\ldots$ satisfy Schema 2. An inference formulated in the language of such a model is valid if and only if there is no Schema 2 model of that language in which the premises of the inference are true and the conclusion is false.

Let us apply the foregoing analysis to assess the validity of the two inferences noted in Section 1. In symbolizing them we can omit the predicate "pilot", symbolized above by "P", since it is constant throughout. Then, writing premises above and conclusions below horizontal lines, Inference 1 becomes:

$$\frac{Oc_{O;Bx} \text{ ("the oldest bold pilot is old")}}{Bc_{B;Ox} \text{ ("the boldest old pilot is bold")}}$$

and Inference 2 becomes:

$$\frac{c_{O;Bx}B^{>}c_{B;Ox} \text{ ("the oldest bold pilot is bolder than the boldest old pilot")}}{c_{O;Bx}O^{>}c_{B;Ox} \text{ ("the oldest bold pilot is older than the boldest old pilot")}}.$$

As it happens, Inference 1 is valid, and the following derives its conclusion from its premise by a natural deduction argument:

1. $Oc_{O;Bx}$ Given.
2. $Bc_{O;Bx}$ Instance of Condition 3.
3. $\neg(Oc_{O;Bx} \,\&\, c_{O;Bx}B^{>}c_{B;Ox})$ By Condition 4.
4. $\neg c_{O;Bx}B^{>}c_{B;Ox}$ From 1 and 3.
5. Suppose $\neg Bc_{B;Ox}$.
6. $c_{O;Bx}B^{>}c_{B;Ox}$ From 2 and 5; instance of Condition 2.
7. 6 contradicts 4.
8. $Bc_{B;Ox}$ From 5, by reduction to absurdity.

On the other hand, the following Schema 2 model is a counterexample to Inference 2, which shows that it is invalid:[16]

Symbol	Denotation of symbol in model
D	$\{1,2\}$
O	$\{1\}$
$O^{>}$	$\{\langle 1,2\rangle\}$
B	$\{2\}$

Symbol	Denotation of symbol in model
$B^{>}$	$\{\langle 2,1\rangle\}$
$c_{O;Bx}$	2
$c_{B;Ox}$	1

The correspondence between the members of D and the constants involved illustrates the point made above, that in looking for counterexamples to quantifier-free inferences we only need to consider models whose domains consist of objects corresponding to the constants involved. The counterexample represents a situation in which the only bold pilot is Pilot #2, who is therefore the oldest bold pilot, and he is bolder than the boldest and only old pilot, who is Pilot #1. On the other hand, Pilot #2 is not older than Pilot #1.

We will now show how counterexamples like the one just described can be constructed graphically.

4 A graphical representation We will continue to restrict ourselves to interrelations of the P-C pairs "old; older than" and "bold; bolder than" and their associated superlatives. Assuming that the conditions of Schemas 1 and 2 are satisfied, we can represent ages on a horizontal scale or 'axis' and degrees of boldness on a vertical axis, and picture individuals and classes of individuals by points and regions in the corresponding 'space':

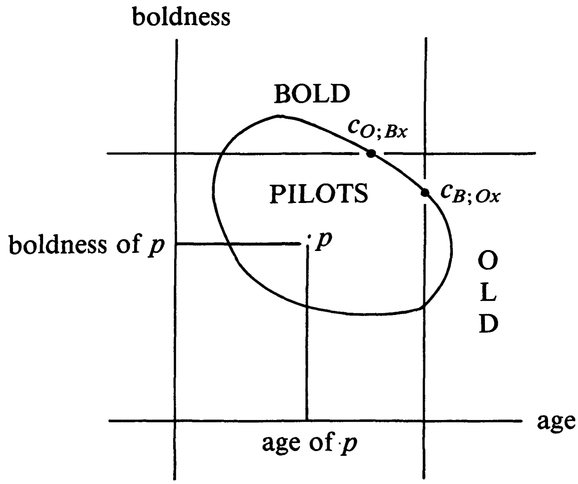

Diagram 1

Increasing age corresponds to positions farther to the right in the graph, and increasing boldness corresponds to positions higher up in it. An individual, say pilot p, is represented by a point with coordinates corresponding to his age and boldness, as shown, and classes of individuals are represented by the regions of points to which they correspond. The diagram represents the class of pilots as an oval-shaped region labeled 'PILOTS'. The region labeled 'BOLD' at the top of the diagram represents the positions of all individuals high enough up on the boldness scale to be called *bold,* while the region labeled 'OLD' on the right represents the positions of individuals sufficiently old to be called *old.* Note that while PILOTS overlaps both BOLD and OLD, it does not overlap their intersection; i.e., there are old pilots and there are bold pilots, but there are no old bold pilots.

The oldest bold pilot and the boldest old pilot, $c_{O;Bx}$ and $c_{B;Ox}$, are also represented. $c_{O;Bx}$ corresponds to the point farthest to the right in the BOLD PILOTS region, or the point that is highest on the age scale in that region. $c_{B;Ox}$ corresponds to the highest point in the OLD PILOTS region, or the highest point in the boldness scale in that region. $c_{O;bx}$ is shown above and to the left of $c_{B;Ox}$, which means that the oldest bold pilot is represented as being bolder but not older than the boldest old pilot. Therefore Diagram 1 is a *graphical counterexample* to Inference 2, which is turned into a Schema 2 model simply by taking the domain to be the two 'points' $c_{O;Bx}$ and $c_{B;Ox}$.

On the other hand, the diagram shows that there could not be a counterexample to Inference 1, that "$Oc_{O;Bx}$" entails "$Bc_{B;Ox}$". That is, because for "$Oc_{O;Bx}$" to be true the PILOTS region would have to intersect the intersection of OLD and BOLD regions, and therefore $c_{B;Ox}$ would also have to lie inside OLD.

5 Comments on restricted positives By a restricted positive we will mean an expression like "young old man" meaning "young *for* an old man".[17] In order to symbolize this predicate we need to make its dependence on both "young" and "old man" explicit, and that can be done by subscripting the basic positive predicate "Y" by a formula corresponding to "old man"—which in this case may be taken to be "$Ox\&Mx$". Thus, the formal predicate symbolizing "old young man" can be taken to be "$Y_{Ox\&Mx}$",[18] where the subscript "$Ox\&Mx$" is treated as a quotation-name of itself and "x" does not occur free in it.

Though the subscript notation begins to be unwieldy, we can also use it to symbolize superlatives corresponding to restricted positives, e.g., to symbolize "the youngest old man" as "$c_{Y;Ox\&Mx}$".

Three logical restrictions must be placed on restricted positives, of which Conditions 6 and 7 below are direct generalizations of Conditions 3 and 4:

Schema 3 For any P-C pairs symbolized by "Π" and "$\Pi^{>}$" and "Γ" and "$\Gamma^{>}$", and any formula $\varphi(x)$ with one free variable "x":

(5) $(\forall y)(\Pi_{\varphi(x)}y \rightarrow \varphi(y))$.[19]
(6) $\varphi(c_{\Pi;\Gamma;\varphi(x)})$
(7) $-(\exists y)(\Gamma_{\varphi(x)}y \; \& \; y\Pi^{>}c_{\Pi;\Gamma;\varphi(x)})$.

In application to "young", symbolized by "Y" and "old man" symbolized as "$Ox\&Mx$",[20] Condition 5 says that all young old men are old men. This is perhaps only a first approximation, but, slightly changing the example, it does explain such obvious things as that while all old old men are old, not all old men are old old men,[21] as well as the fact that "All young old men are young men" is not logically true.

Assuming that both of the P-C pairs "young-younger" and "old-older" satisfy Schemas 1-3, the following model would be a counterexample:

Symbol	Denotation
Domain	{1,2,3,4}
M	{1,2,3,4}
$Y = Y_{Mx}$	{1,2}
O	{3,4}
$Y_{Ox\&Mx}$	{3}

It is easily verified that the above model satisfies Conditions 1-7, and that "$(\forall y)(Y_{Ox\&Mx}y \rightarrow Yy)$", symbolizing "All young old men are young men", is false in it.

Other applications and extensions of our formalism might be considered,[22] but these must be omitted in the present sketch.

NOTES

1. Inspired by the WWII saying "There are old pilots and there are bold pilots, but there are no old, bold pilots".
2. Or, possibly, the *relativity* of these adjectives, as is seen in the fact that the same person can be said to be old for a man but young for an old man.
3. The cases of "good; better; best" and "*bon*; *meilleur*; *le meilleur*" show that natural languages do not always show these connections explicitly.
4. Obviously not all ordinary language expressions that are symbolized as unary predicates have corresponding comparatives. While adjectives like "old" and "bold", which as we will later see ought not to be symbolized as predicates at all, 'support' "older than" and "bolder than", nouns like "pilot" do not have 'natural' corresponding comparatives. However, we will see below that they can be assigned 'formal' comparatives that stand to them in certain respects as "older than" and "bolder than" stand to "old" and "bold".
5. Similarly, "$O^{<}$", "$O^{\leq}$", "$O^{\geq}$", and "$O^{=}$" can be used to symbolize "less old than", "not older than", "at least as old as", and "as old as", respectively. We will confine attention to "$O^{>}$", symbolizing "older than", since all of the related comparatives can be defined in terms of it.
6. "Π" will used ambiguously, both as a metalinguistic predicate variable for which such predicate symbols as "B", "O", and "P", symbolizing "bold", "old", and "is a pilot" may be substituted, and as a variable ranging over possible extensions of these symbols.
7. These conditions defining strict weak orderings are less well known than others, but they are simpler to state (cf. Roberts [3], p. 32). They entail transitivity, but asymmetry and transitivity alone are not sufficient to assure that (1) is satisfied.
8. This assumes that the reasoning does not involve identity. When identity is involved, E may have to be a system of 'replicas' of positive rationals.
9. That there could be objects with weights corresponding to all rational numbers might be inconsistent with Quantum Theory, but we do not want to assume that that theory's laws are logical truths.
10. Thus, while heavy and light are properties of actually existing objects, they might be determined 'relative' to classes of 'possible objects'.
11. Thus, we do not necessarily have to accept the common view that the positive can be defined 'statistically' in terms of the comparative: e.g., that "old" might be defined as "older than the mean of ages in the population" (cf. Wallace [5]).
12. The simplest way to do this is to define $x\Pi^{>}y$ to hold if and only if $x \in \Pi$ and $y \notin \Pi$: e.g., all old persons are defined to be equally old and older than all persons who are not old, who are also equally old.
13. Our use of old fashioned grammatical terminology like "modifier", as against, say, "verb phrase", is deliberate. We want to stress the fact that, while a word like "old"

can stand alone grammatically, its *logical* function is to modify the extensions of nouns like "man".

14. The subscript "$Bx\&Px$" in "$c_{O;Bx\&Px}$" is to be understood as the name of itself, and therefore "x" is not a free variable in it (i.e., "$c_{O;Bx\&Px}$" is a constant distinguished by its special relation to the P-C pair "old; older than" and to the formula "$Bx\&Px$"). The reason for representing the ordering predicate by "$O^>$" without a variable in the subscript "$O;Bx\&Px$", while the class whose highest element in the ordering is to be picked out is represented by the complex formula "$Bx\&Px$", is that orderings are symbolized as atomic predicates whereas the classes whose highest members are picked out can be the extensions of arbitrary propositional functions.

15. Note that Condition 4 does not guarantee the uniqueness of the oldest bold pilot, though it does guarantee that all of the oldest bold pilots are equally old. This means that "$\varphi(c_{O,Bx\&Px})$" together with Conditions 3 and 4 only imply

$$(\exists x)(Bx\&Px\&\varphi(x))\ \&\ (\forall z)((Bz\&Pz)\rightarrow \neg zO^>x)$$

and not the following part of the Russellian analysis of a sentence that we should symbolize as "$\varphi(c_{O,Bx\&Px})$":

$$(\forall y)\,[(By\&Py)\ \&\ (\forall z)((Bz\&Pz)\rightarrow \neg zO^>y)]\rightarrow x=y.$$

As is well known, of course, since the mere use of "$c_{O,Bx\&Px}$" entails "$(\exists x)((Bx\& Px\&\varphi(x))\ \&\ (\forall y)((By\&Py)\rightarrow\neg yO^>x))$", both "$\varphi(c_{O,Bx\&Px})$" and "$\neg\varphi(c_{O,Bx\&Px})$" entail this, and so these sentences are not true logical contradictories, which is why "$c_{O,Bx\&Px}$" should only be used when "$(\exists x)((Bx\&Px)\ \&\ (\forall y)((By\&Py)\rightarrow\neg yO^>x))$" is known to be the case—i.e., when it is known that there is an oldest bold pilot.

16. The invalidity of this inference is also an obvious consequence of the fact that the inference from the same premise to the converse conclusion, "$c_{B;Ox}O^>c_{O;Bx}$", is valid. This can easily be derived along the same lines as used for Inference 1, but the method described in Section 4 will 'show' this graphically.

17. As with the formation of comparatives, only adjectives can be combined with other 'predicative expressions' like "old man" to yield restricted positives like "young old man" (similarly, "old" must be an adjective to combine with "man", which is not an adjective, to yield "old man").

18. Or, since "old man" means old *for a man*, this could be written with a double subscript, thus, "$Y_{O_{Mx}}$", which we will later simplify to "$Y_{O;Mx}$". Using this notation, "$Y_{O;Y;Mx}$" symbolizes the predicate "young old young man".

19. Because restricted positives like "old young man" do not support comparatives, there is no need for a corresponding symbolism; i.e., there is no need for a complex relational predicate like "$O^>_{Yx\&Mx}$".

20. This is wrong in the case of adjectives like "alleged", since alleged criminals are not always criminals—as Montague pointed out ([2], p. 393, Condition (3)). But it is noteworthy both that "alleged" does not support a natural comparative and also that Montague did not deal with the relation between positives and comparatives.

21. This would be symbolized as:

$$(\forall y)(O_{Ox}y\rightarrow Oy)\ \&\ \neg(\forall y)(Oy\rightarrow O_{Ox}y),$$

the first conjunct of which is logically true since it is an instance of Condition 5. That the second conjunct is logically consistent, though it would be *inconsistent* if symbolized in the usual way, can be seen because it is true in the Schema 3 model, which satisfies Conditions 1-7:

Symbol	Denotation of symbol in model
D	$\{1,2\}$
O	$\{1\}$
O_{Ox}	Λ

22. Four extensions that would be worth study are: (1) to the *converses* of many natural language comparatives, (2) to inter-relational expressions like "The older the bolder", (3) to adverbs like "very" as in "he is a *very* old man", and (4) to 'Boolean' compounds of adjectives as in "not very old", whose 'logic' is not strictly Boolean. As to (1), let it simply be noted that a relation like "younger than", which is the converse of "older than", can be symbolized by the converse ordering $0^{<}$, as in Footnote 5, but it is interesting in its own right because it generates its own cut, corresponding to the adjective "young", which is positive relative to "younger than" but negative relative to "older than".

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