

Who's Afraid of Impossible Worlds?

EDWIN D. MARES

Abstract A theory of ersatz impossible worlds is developed to deal with the problem of counterpossible conditionals. Using only tools standardly in the toolbox of possible worlds theorists, it is shown that we can construct a model for counterpossibles. This model is a natural extension of Lewis's semantics for counterfactuals, but instead of using classical logic as its base, it uses the logic **LP**.

1 Introduction The semantics of relevant logics and strong paraconsistent logics contain worlds in which contradictions and other impossibilities come true. The use of impossible worlds has provided a barrier to understanding these logics especially for philosophers trained in classical logic and traditional metaphysics. In this paper, I present a semantics for a paraconsistent logic using only the entities that have become standard in traditional semantics: possible worlds, relations, individuals, and sets. This semantics contains inconsistent worlds, but they are set-theoretic constructs. It is hoped that this construction will help to give classically-minded philosophers a way to understand at least some nonclassical logics.

The construction is fairly simple. Given a set of relations, a set of individuals, and little bit of set theory, I construct a set of states of affairs. The worlds of my semantics are just sets of states of affairs. These, of course, are "ersatz worlds" in the sense of Lewis [8]. We can distinguish among these ersatz worlds, possible and impossible worlds. A possible (ersatz) world is such that all of its states of affairs are true in some "real possible world."

I then take this set of worlds and, applying some plausible metaphysical principles, impose upon it a relation of comparative similarity. Abstracting the formal features of the set of ersatz worlds and this similarity relation, I construct a formal semantics for a logic of counterfactuals. As Fuhrmann and I argue in [10] and [9], there is a particular need for a paraconsistent logic for counterfactuals and I briefly repeat our reasons in Section 2 below.

The semantics I create is a natural extension of Lewis's semantics for counterfactuals. As I argue in Sections 2 and 3 below, the logic captures some rather strong

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intuitions about counterpossible conditionals and does so in a natural way. This semantics of Section 4 is presented to show that impossible worlds are useful elements in our semantics, and the construction of Section 3 is presented to show that there is no good reason not to include them in our semantics.

The logic of this semantics, viewed proof-theoretically,¹ is an extension of Priest's logic, **LP**. **LP** is a particularly useful logic; one might even call it "a laboratory for the paraconsistent logician." It has a simple three-valued semantics, it captures many of the inferences we want to capture, even if it does not capture everything we want. **LP** lacks some important connectives, such as a veridical implication, but this is part of what makes it simple and useful.

I do not axiomatize the logic that is complete over this semantics. That is not my interest in the present paper. Although this logic might be interesting in its own right, my eventual target (to be reached elsewhere) is to provide an understanding of other nonclassical logics, in particular relevant logics. I do not now extend the construction to provide a semantics for relevant logics because I want only to discuss the issue of impossible worlds here. The need to understand the other elements of the semantics of relevant logics will cloud this issue. The simple semantics and metaphysical construction here is sufficient to show that a good deal can be gained in semantics with no ontological cost by adding impossible worlds.

2 Semantics and metaphysics There seems to be a need in our model theory for impossible worlds. For consider the following pair of counterfactuals.

- (1) If Sally were to square the circle, we would be surprised.
- (2) If Sally were to square the circle, we would not be surprised.

As we have suggested elsewhere (Mares and Fuhrman [10]), the first of these counterfactuals seems true and the second false. The problem is that, on Stalnaker's and Lewis's semantics, both of these statements must be given the same value (on some translations false and on other translations they are both taken to be true). We find similar problems with

- (3) If water were an element, it could not be broken down into hydrogen and oxygen,

and

- (4) If water were an element, it could be broken down into hydrogen and oxygen.

Like (1) and (2), (3) and (4) both have metaphysically impossible antecedents but seem to have different truth values. In this paper, I will provide a theory that allows pairs of counterpossibles to have different truth values. The theory does this by allowing impossible worlds (or indices as I will call them) into the model.

We can go even further. Consider the following pair of "counterlogicals."

If Sally were to prove that *A*, we would be surprised.

and

If Sally were to prove that *A*, we would not be surprised.

where A is some long truth-functional contradiction. We have the same intuitions about this pair as in (1), (2), (3), and (4). Here then, we even have a need nontrivially to treat counterpossibles with antecedents that are contradictions according to the classical propositional calculus.

The need for an adequate theory of counterpossibles has also been felt outside philosophical semantics. For example, Field says:

It is doubtless true that nothing sensible can be said about how things would be different if there were no number 17; that is largely because the antecedent of this counterfactual gives us no hints as to what alternative mathematics is to be regarded as true in the counterfactual situation in question. If one changes the example to ‘nothing sensible can be said about how things would be different if the axiom of choice were false’, it seems wrong . . . : if the axiom of choice were false, the cardinals wouldn’t be linearly ordered, the Banach-Tarski theorem would fail and so forth. ([5], pp. 237f)

Field says that the need for nontrivial truth conditions for these sorts of counterpossibles shows that we should treat mathematical necessity as a more restrictive modality than metaphysical necessity ([5], p. 236). That is, he says that we should hold that mathematical truths are not true in all metaphysically possible worlds. Another approach to this problem is to hold that all mathematical truths are metaphysically necessary, but that metaphysical necessity is not determined by all the worlds that there are.

Lewis has suggested that we should deal with the apparent semantical difference between counterpossible conditionals as pragmatic differences ([6], p. 24). I agree in general with Dowty, Wall, and Peters when they say that a semantic theory should account for native speakers’ “judgments of synonymy, entailment, contradiction, and so on” ([3], p. 2). Although there may be cases in which we should toss out our semantic intuitions, for the most part they are our best guides to semantic theory, and there must be some severe pressure from other sources to force us to deny intuition. I suggest that the need to violate our intuitions about counterpossibles is felt only if we also feel horror at the thought of allowing impossible worlds into our semantics.

This horror should not force us to cower in the apparent safety of a semantics that includes only possible worlds. In the theory that I set out below, impossible worlds have a very different status than possible worlds. This special status of possible worlds is entailed by the theory I present below in connection with the theory of possible worlds that one chooses. If we pick, say, Lewis’s theory of possible worlds, then the difference between possible and impossible worlds is that the former are veritate real worlds (or copies of real worlds—see Section 3 below) and impossible worlds are ersatz constructions.

Far from violating our ontological intuitions, the existence of these impossible worlds is supported by our metaphysical theories. They are made from common-or-garden varieties of entities found in possible worlds semantics—relations, individuals, and sets. Impossible worlds are “made from consistent stuff” available in possible worlds. Thus, in a very real sense, the present theory does not violate our metaphysical intuitions; for all “real” things are consistent and obey all the other laws of metaphysics. Even impossible worlds, although they make impossibilities true, do not themselves have any impossible properties. Thus, I claim to have saved both our

semantical and metaphysical intuitions.

3 *Constructing impossible worlds* I begin by assuming that there is a set of all possible worlds. It doesn't matter whether these worlds are space-time continua like the actual world, or ersatz constructs of some sort. I also assume that there are individuals in these worlds and that we can collect all of these individuals into a set. Clearly, I also assume that there are sets.

An index is a set of states of affairs. I borrow, more or less, from [1], Barwise and Perry's treatment of states of affairs. A state of affairs (SOA) is a structure of the form

$$\langle R, a_1, \dots, a_n, \pi \rangle$$

where R is an n -place relation, a_1, \dots, a_n are individuals, and π is either 1 or 0. π is called 'a polarity'. The structure itself might just be a set-theoretic construct (such as a sequence) or something more vertebrate, perhaps even *sui generis*. An index i is a set of SOAs such that for each n -place relation R and each sequence $\langle a_1, \dots, a_n \rangle$ of individuals, either $\langle R, a_1, \dots, a_n, 1 \rangle$ or $\langle R, a_1, \dots, a_n, 0 \rangle$ is in i . Note that this disjunction is inclusive; it might be that both are in i .

With regard to relations, like my attitude toward possible worlds and SOA, I try to stay as ontologically neutral as possible. I do not assume any particular theory of relations here. As far as my current purpose is concerned, relations can be Armstrongean universals, Platonic forms, or even functions from worlds to sets of n -tuples of individuals. My aim is to convince a possible worlds theorist to adopt impossible indices. As far as possible, I do not want to tell her that she also needs to adopt other bits and pieces that she does not already accept for other reasons. (Of course, someone who rejects relations altogether will be hard to please.)

We also need a relation between indices and worlds—the relation of an index's representing a world. If for all SOA, $\langle R, a_1, \dots, a_n, 1 \rangle$ and $\langle R, a_1, \dots, a_n, 0 \rangle$,

$$\langle R, a_1, \dots, a_n, 1 \rangle \in i \quad \text{iff} \\ \langle a_1, \dots, a_n \rangle \text{ is in the extension of } R \text{ at } w$$

and

$$\langle R, a_1, \dots, a_n, 0 \rangle \in i \quad \text{iff} \\ \langle a_1, \dots, a_n \rangle \text{ is not in the extension of } R \text{ at } w$$

then i is said to *represent* w . If an index i represents some possible world, then i is called 'a possible index'.

A classical logician looking at the second of the two conditions defining the representation relation might accuse me of assuming the classical meaning of negation. For a negative state of affairs holds at a world if and only if the corresponding positive state of affairs does not. This sounds pretty much like an encapsulation of a classical notion of negation. The correctness of this accusation depends on the way we choose to understand states of affairs and indices. The following interpretation is taken from [2] and [1] and I think it will help avoid this difficulty. We can think of states of affairs as basic pieces of information. $\langle R, a_1, \dots, a_n, 1 \rangle$ is the information that R holds between a_1, \dots, a_n and $\langle R, a_1, \dots, a_n, 0 \rangle$ is the information that R fails to hold between those objects. Negative information, moreover, does not just

reduce to the absence of positive information. Having the information that it is not raining in Auckland right now does not reduce to not having the information that it is raining in Auckland right now. We can take an SOA to be a basic piece of information. Some of these pieces are positive and others are negative, and neither is to be explicated in terms of the other.

Thus, an index is a complete information state. It always contains either the information that a relation holds between a sequence of objects or the information that it does not hold between them. One might wonder why I do not also include partial information states in the semantics; for we are always in partial information states. I want to leave the subject of how properly to represent partial information to other papers. At any rate, we can represent partial information in the present model by taking sets of indices to represent the information state of a person, a computer, and so on.

One might also wonder why I have included all individuals in the domain of every index. This was just to have an easy way to ensure the completeness of all indices, to ensure that they all satisfy the principle of bivalence. Of course this does not imply that all individuals exist at every index. But how we limit domains of quantification at indices is not a topic I want to discuss here. I just want to talk about propositional logic.

In order for our set of indices to constitute a Lewisian model for counterfactuals, we must impose a comparative similarity relation on these worlds. For the purpose of the present construction, I accept Lewis's doctrine of *Humean supervenience*. The relevant version of this position is the following:

Humean Supervenience: If two worlds are identical in the matters of particular fact that they support, they are also identical in their modal and counterfactual properties. (See [7], p. 111.)

In terms of our ontology, Humean supervenience says that the SOA contained by each world determines which worlds are closer and more distant from other worlds. Thus, as in Lewis's theory of counterfactuals, given a particular context of utterance (including the purpose of the utterance and other parameters of the conversation), there will be a similarity relation (or more likely a family of similarity relations) on all indices.

Given this construction of indices, one might wonder why we need possible worlds as well as indices. The job that possible worlds (other than the actual world) do in the construction is to determine which indices are possible and which are impossible. In [8], Lewis argues that theories that contain only ersatz worlds are somehow defective, in particular, that they require a primitive notion of possibility. If Lewis is right, then we can avail ourselves of Lewis-style vertebrate worlds to determine which indices are possible, and we do not require a primitive notion of possibility any more than he does. If, on the other hand, some ersatzist construction allows us to get the right notion of possibility "on the cheap," then we can avail ourselves of that construction. Now, for those who do not mind taking possibility to be primitive or think that it can be determined somehow by the internal properties of indices, we can dispense with possible worlds altogether (apart, of course, from the actual one).

We can now see how the present techniques allow us to construct a model that will satisfy the linguistic intuitions discussed in Section 2 above. We can construct indices in which Sally squares the circle and in which we are surprised and some of

these will be closer than any index in which Sally squares the circle and we are not surprised. Similarly, we can construct indices in which water is an element and cannot be broken down into hydrogen and oxygen, and in which there is a universe of pure sets in which the axiom of choice does not hold.

4 The formal model To put the intended model to use in a theory of counterfactuals, we need a model theory. That is, we need a theory of how worlds satisfy statements. To this end, I shall abstract certain features of the intended model (and make certain assumptions about the intended model) and create a frame theory. In addition to this frame theory, we will add a theory of truth from an appropriate language and then show that the resulting model theory has some important virtues.

4.1 Frame theory To make our proofs easier, I will describe the frames using Lewis's notion of "spheres." A sphere around an index i is a set of indices that are at most a particular "distance" from i ; that is to say, the worlds in this sphere are all of those indices that do not differ from i by any more than some particular degree of dissimilarity.²

My frame theory is based closely on Lewis's theory of centered frames (see [6], §1.3).³ A frame is a triple $\langle I, P, \$ \rangle$ where I is the set of indices, P is the set of possible indices, and $\$$ assigns to each index a set of spheres. In addition, all frames satisfy the following postulates. Where i is an arbitrary index,

1. If $i \in P$, then $P \in \$(i)$. (Poss1)
2. $\{i\} \in \$(i)$. (Strong Centering)
3. If φ and ψ are both in $\$(i)$, then either $\varphi \subseteq \psi$ or $\psi \subset \varphi$. (Nesting)
4. $\$(i)$ is closed under unions and nonempty intersections. (Closure)

The first of these postulates is special to this semantics, whereas the latter three are standard, if sometimes controversial, postulates used in theory of counterfactuals. The closure postulate is a useful housekeeping postulate. The first four postulates are easiest to understand in terms of relative similarity. To motivate Poss1, let us first discuss Poss2 stated below. Poss2 says that, for any given possible index i , every possible index is closer to i than is any impossible index. I'm not sure how to argue for Poss2 other than saying that it seems reasonable. Poss1 says that the set of possible indices makes up a sphere around any possible index. Given Poss2, Poss1 also seems reasonable. For, if all possible worlds are closer to i than any impossible index, partitioning the possible index from the impossible indices marks a real distinction in terms of comparative similarity. Nesting says that comparative similarity is a linear ordering, which seems right. Strong centering says, in effect, that no index is as similar to i as i itself. If all indices are distinguished by what states of affairs they satisfy (as we have assumed in adopting Humean supervenience), then strong centering holds. Although most philosophers of logic have now rejected the idea that the similarity relation used in the theory of counterfactuals is an intuitive similarity relation, these motivations still seem to hold for the notion of similarity that they have adopted. From the postulates presented above, we can derive the following.

1. If $i \in P$ and $\varphi \in \$(i)$, then either $\varphi \subseteq P$ or $P \subset \varphi$. (Poss2)
2. If $\varphi \in \$(i)$, then $i \in \varphi$. (Weak Centering)

As we have said above, Poss2 says that, for any given possible index i , every possible index is closer to i than is any impossible index. Weak centering says that no world is closer to i than i .

4.2 Theory of truth and falsity Now that we have a class of frames, we need a language and a theory of truth in order to have a model theory. The language is a standard propositional counterfactual language with propositional variables p, q, r, \dots , connectives \sim, \wedge, \vee , and $\Box \rightarrow$, and parentheses. Standard formation rules apply. A model is a quadruple $\langle I, P, \$, v \rangle$ where $\langle I, P, \$ \rangle$ is a frame, v is a function from propositional variables and worlds into the set $\{t, b, f\}$, such that, for all propositional variables p , and all $i \in P$, $v(p, i) \neq b$. Intuitively, t is the value *true*, f is the value *false*, and b is the value *both true and false*.

Given a model we can define two relations between indices and well-formed formulas: \models and \models . We understand \models to be a satisfaction relation, that is, when $i \models A$, A is true at i . \models , on the other hand, is a dissatisfaction relation, that is, when $i \models A$, A is false at i . (Note that $\not\models$ just means “not \models ”—it implies but is not equivalent to \models . Similarly, $\not\models$ is just the negation of \models .) Since, on this semantics, well-formed formulas are sometimes both true and false at worlds, we cannot treat falsehood merely as failing to be true. I borrow my presentation of the truth conditions from [4] and [14].

Since, as we have said, we cannot merely treat falsity as the absence of truth, we must state both truth and falsity conditions for propositional variables and the various connectives. For propositional variables, the conditions are quite straightforward:

$$i \models p \quad \text{iff} \quad (v(p, i) = t \text{ or } v(p, i) = b).$$

In words, i satisfies p if and only if p gets the value true or the value both true and false at i . Similarly,

$$i \models p \quad \text{iff} \quad (v(p, i) = f \text{ or } v(p, i) = b).$$

So, p is false at i if and only if p gets the value false or the value both true and false at i .

We extend the theory of truth and falsity to the other connectives as follows:

$$\begin{aligned} i \models A \wedge B & \quad \text{iff} \quad i \models A \quad \text{and} \quad i \models B. \\ i \models A \wedge B & \quad \text{iff} \quad i \models A \quad \text{or} \quad i \models B. \end{aligned}$$

$$\begin{aligned} i \models A \vee B & \quad \text{iff} \quad i \models A \quad \text{or} \quad i \models B. \\ i \models A \vee B & \quad \text{iff} \quad i \models A \quad \text{and} \quad i \models B. \end{aligned}$$

$$\begin{aligned} i \models \sim A & \quad \text{iff} \quad i \models A. \\ i \models \sim A & \quad \text{iff} \quad i \models A. \end{aligned}$$

The truth condition for counterfactual implication is the same as it is in Lewis [6], namely,

$$\begin{aligned} i \models A \Box \rightarrow B & \quad \text{iff} \\ \exists S(S \in \$ (i) \ \& \ \exists j \in S(j \models A) \ \& \ \forall k \in S(k \models A \text{ or } k \models B)) \\ \text{or } \neg \exists j(S \in \$ (i) \ \& \ \exists j \in S \ \& \ j \models A) \end{aligned}$$

The falsity conditions, however, are more complicated. There are two conditions under which a counterfactual implication is false. First, it can fail to be true. Second, there can be a counterexample to it. That is, a world in which the antecedent of the counterfactual is true and the consequent false. Putting these two conditions together we get the following:

$i \models A \Box \rightarrow B$ iff either:

(i) $i \not\models A \Box \rightarrow B$

or

(ii) $i \models A$ and $i \models B$.

Now we need a concept of validity on models. The most obvious one is to define all and only well-formed formulas true at all indices in all models to be valid. And this is the definition that I will set for the purposes of this paper. Yet, following Kripke's semantics for nonnormal modal logics and Routley and Meyer's semantics for relevant logic, we could take the possible indices to be the determiners of validity. That is, we could define all and only those well-formed formulas true in all possible indices in all models to be valid. I am not sure about the relative merits of each of these proposals at this time, but I choose the first, perhaps arbitrarily.

5 Some theorems Nesting makes the following inference hold throughout all models:

$$\begin{array}{l} i \models A \Box \rightarrow B \\ i \models A \Box \rightarrow C \\ \hline \Box \rightarrow \wedge. \\ \therefore i \models A \Box \rightarrow (B \wedge C) \end{array}$$

Weak centering gives us

$$\begin{array}{l} i \models A \Box \rightarrow B \\ i \models A \\ \hline MP \Box \rightarrow . \\ \therefore i \models B \end{array}$$

And strong centering yields

$$\begin{array}{l} i \models A \wedge B \\ \therefore i \models A \Box \rightarrow B. \end{array}$$

The following is *not* valid:

$$(A \wedge \sim A) \Box \rightarrow B.$$

But, its "contrapositive" is valid:

$$B \Box \rightarrow (A \vee \sim A).$$

6 Some other theorems One property that we want to hold of possible indices is that they are consistent. The theorem below shows that they are.

Theorem 6.1 *If i is a possible index, then for all well-formed formulas A , it is never the case that both $i \models A$ and $i \models \sim A$.*

Proof:

Case 1: A is atomic. This follows directly from the definitions of v and the theory of truth and falsity.

Case 2: A is a conjunction, for example, $B \wedge C$. This follows from the truth and falsity conditions for conjunction and inductive hypothesis.

Case 3: A is a disjunction, for example, $B \vee C$. This follows from the truth and falsity conditions for disjunction and inductive hypothesis.

Case 4: A is a negation, for example, $\sim B$. This follows from the truth and falsity conditions for negation and inductive hypothesis.

Case 5: A is a counterfactual implication, for example, $B \Box \rightarrow C$. Suppose that $i \models B \Box \rightarrow C$. We show that $i \not\models B \Box \rightarrow C$. Suppose otherwise. Then, by the hypothesis of this case and the falsity conditions for negation, $i \models B$ and $i \models C$. But, by weak centering, and $i \models B \Box \rightarrow C$, if $i \models B$, then $i \models C$. So, $i \models C$ and $i \models C$, contrary to the inductive hypothesis. This concludes the proof of Theorem 6.1. \square

Corollary 6.2 *For any possible index i and any well-formed formula A , $i \models \sim A$ if and only if $i \not\models A$.*

The above theorem and corollary together with the truth and falsity conditions for \sim , \wedge , and \vee show that these extensional connectives act classically at possible worlds. Now we will turn to the behavior of the counterfactual at possible indices.

We do so by constructing an “inner model.” Where $\langle I, P, \$, v \rangle$ is a model, let us call $\langle P, (\$ \upharpoonright P), (v \upharpoonright P) \rangle$ “the Lewis model embedded in $\langle I, P, \$, v \rangle$.” $(v \upharpoonright P)$ determines a satisfaction relation \models' such that the following hold.

1. $i \models' p$ iff $(v \upharpoonright P)(p, i) = t$.
2. $i \models' A \wedge B$ iff $i \models' A$ and $i \models' B$.
3. $i \models' A \vee B$ iff $i \models' A$ or $i \models' B$.
4. $i \models' \sim A$ iff $i \not\models' A$.
5. $i \models' A \Box \rightarrow B$ iff $\exists S(S \in (\$ \upharpoonright P)(i) \ \& \ \forall j \in s(j \not\models' A \text{ or } j \models' B))$ or $\neg \exists S \exists j(S \in \$ (i) \ \& \ j \in S \ \& \ j \models A)$.

These are just Lewis’s own truth conditions for the various connectives.

The following theorem shows that the counterfactual conditional saves much (in fact, I would say all) of what is good about Lewis’s counterfactual. In order to understand it, we need one more definition. A formula A is *impossible* on a Lewis model $\langle P, (\$ \upharpoonright P), (v \upharpoonright P) \rangle$ if and only if there is no index $i \in P$ such that $i \models' A$.

Theorem 6.3 *Where $\langle I, P, \$, v \rangle$ is a model, if A contains no subformulas impossible on $\langle P, (\$ \upharpoonright P), (v \upharpoonright P) \rangle$, then for all $i \in P$, $i \models A$ if and only if $i \models' A$.*

Proof:

Case 1: $A = p$. Then, $i \models p$ if and only if $i \models' p$ by the definitions of \models and \models' .

Case 2: $A = B \wedge C$.

$$\begin{aligned} i \models B \wedge C &\iff (i \models B \ \& \ i \models C) \\ &\iff (i \models' B \ \& \ i \models' C) \quad \text{by inductive hypothesis} \\ &\iff i \models' B \wedge C \end{aligned}$$

Case 3: $A = B \vee C$. Similar to Case 2.

Case 4: $A = \sim B$.

$$\begin{aligned} i \models \sim B &\iff i \not\models B && \text{Corollary 6.2} \\ &\iff i \not\models' B && \text{by inductive hypothesis} \\ &\iff i \models' \sim B \end{aligned}$$

Case 5: $A = B \Box \rightarrow C$.

Suppose first that $i \models B \Box \rightarrow C$. Since B is possible on $\langle P, (\$ \upharpoonright P), (v \upharpoonright P) \rangle$, there is some $i' \in P$ such that $i' \models' B$. By the inductive hypothesis, $i' \models B$. So, $B \Box \rightarrow C$ is not vacuously true at i . Then there is some $\varphi \in \$(i)$ such that $i' \in \varphi$, $i' \models B$ and $i' \models C$, and for all $i'' \in \varphi$, if $i'' \models B$, $i'' \models C$. Now by nesting, either $\varphi \subseteq P$ or $P \subset \varphi$. Suppose that $P \subset \varphi$. Then for all $j \in P$, if $j \models B$, $j \models C$. But as we have said, $i' \in P$, $i' \models B$ and $i' \models C$. So by the inductive hypothesis, for all $j \in P$, if $j \models' B$, $j \models' C$ and there is an $i' \in P$, $i' \models' B$ and $i' \models' C$. So, since $P \in (\$ \upharpoonright P)(i)$, $i \models' B \Box \rightarrow C$.

On the other hand, suppose that $\varphi \subseteq P$. Then it follows straightforwardly from the inductive hypothesis that $i \models' B \Box \rightarrow C$.

Now suppose that $i \models' B \Box \rightarrow C$. It follows directly from the construction of the models and the inductive hypothesis that $i \models B \Box \rightarrow C$. \square

7 Comments If we pay for ideology in the coin of ontology, then the doctrine of impossible worlds can be bought with loose change lying around the house of almost any possible worlds theorist. As we have seen, all we need are relations (of whatever brand), individuals, and a little set theory to construct impossible worlds. Adding impossible worlds to our semantics gives us the tools with which to deal systematically and nontrivially with counterpossible conditionals. Thus, there seems no reason why we should stop at the limits of the possible and not accept also the impossible.

That is not to say that I find the logic characterized by the present semantics perfect. As I said in the introduction, my ultimate goal is to provide a philosophical basis from which to understand relevant logic. In future papers I will argue that not much more is needed to provide a philosophically acceptable semantics for relevant logic. But arguing for the acceptability of impossible worlds is an important first step toward that goal.

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NOTES

1. Viewed as a Hilbert-style axiom system, this logic is an extension of classical propositional logic. But, like **LP** and unlike classical logic, the present logic rejects the inference from a contradiction to anything.

2. In this presentation of the formal material, I use classical first-order logic as my metalanguage. Because I am trying to sell paraconsistent logic and nonnormal worlds to classical logicians, I thought that I had better speak their language.
3. Note that, as Lewis points out in [6], §1.4, this semantics is neutral with regard to the limit assumption. These frame conditions allow that there be an infinitely descending sequence of spheres around i in which some index satisfies the antecedent in question. Then again, the frame conditions do not demand that there be such a sequence.

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Department of Philosophy
 Victoria University of Wellington
 P.O. Box 600
 Wellington
 NEW ZEALAND
edmares@matai.vuw.ac.nz