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Information and Impossibilities

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Abstract In this paper I explore informationalism, a pragmatic theory of modality that seems to solve some serious problems in the familiar possible worlds accounts of modality. I view the theory as an elaboration of Stalnaker's moderate modal realism, though it also derives from Dretske's semantic theory of information. Informationalism is presented in Section 2 after the prerequisite stage setting in Section 1. Some applications are sketched in Section 3. Finally, a mathematical model of the theory is developed in Section 4.

How many times have I said to you that when you have eliminated the impossible, whatever remains, however improbable, must be the truth?
–Arthur Conan Doyle

You've put me in an impossible situation.
–Anonymous

[N]othing we imagine is absolutely impossible.
–David Hume

1 The granularity problem Possibilities are central to many philosophical issues, from logic (consider Sherlock Holmes's view of deduction as the elimination of possibilities) through metaphysics (take the Kripke/Putnam discovery of *a posteriori* necessities, for example) to epistemology (recall attempts to characterize knowing that p in terms of having the information that every relevant possibility is one where p is the case) to ethics, where our concepts of moral, ethical, and lawful action, indeed our whole system of law and justice, are predicated on the assumption that people are responsible for choices they make among various possibilities.

My own current preoccupation with possibilities grew out of my work on information. An oft-noted relationship between possibilities and information is that eliminating possibilities corresponds to increasing information. This observation is at the heart of Shannon's famous explication (in [10]) of the amount of information in a communication network. This inverse relationship is also central to various attempts,

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most famously Dretske's [4], to develop a semantic theory of information content on top of the notion of possibility. One can also look at Stalnaker's *Inquiry* [12] as developing a theory of information out of a theory of possibilities and how they are eliminated through inquiry. (Stalnaker uses inquiry as a prototypical intentional activity. I will do the same here.)

Much of my research over the past fifteen years has been motivated by the belief that information is undeniable and important. I have, however, been reluctant to accept any account of information that depends crucially on an unexplained notion of possibility. What are possibilities? In particular, what are unactualized possibilities? Where do they come from and how do they fit in with the rest of the world? More generally, are unactualized possibilities compatible with some kind of realism that I can find acceptable?

Lewis argued in [7] for a position now known as extreme modal realism. This is the view that possibilities are worlds, unactualized possibilities being alternative worlds to this world, similar in kind but different in the way things turned out.

> Our actual world is only one world among others. We call it alone actual not because it differs in kind from all the rest but because it is the world we inhabit. The inhabitants of other worlds may truly call their own world actual, if they mean by 'actual' what we do; . . . ([7], pp. 85–86)

In Chapter 3 of [12], Stalnaker takes on extreme modal realism. He attempts to show that Lewis's argument for extreme modal realism is fallacious (pp. 44–50) and proposes to replace the view with what he calls moderate modal realism. According to moderate modal realism, possibilities are ways things could have been. He calls these ways things could have been states, or more frequently (and confusingly), possible worlds. While he calls them possible worlds, Stalnaker's possible worlds are very different from what Lewis takes them to be:

> But the moderate realist believes that the only possible worlds there are—ways things might have been—are (like everything else that exists at all) elements of our actual world. They obviously are not concrete objects or situations, but abstract objects whose existence is inferred or abstracted from the activities of rational agents. ([12], pp. 50–51)

I am persuaded by Stalnaker's arguments, both those against extreme modal realism and those in favor of moderate modal realism. But I still want to know more than Stalnaker tells us about possibilities. Where, for example, do these ways things could have been come from? And where do they go? When we make a decision to act, for example, we choose among alternative possibilities. What happens to the possibilities not chosen?

There is another set of questions that need to be answered. Are there ways things could *not* have been? How can there be, if they are impossible? And yet, it seems, there must be impossibilities. After all, not everything is possible so anything that is not possible must be an impossibility. But if there are both possibilities and impossibilities, what kind of thing is it that turns out to be either possible or impossible, and what makes it one rather than the other? These seem to me serious questions about moderate modal realism, questions that need answering.

The most notorious problem with both extreme and moderate modal realism, though, has to do with logic, deduction, and mathematical truths. The problem has many variants, but all versions turn on the inability of possible worlds theories to make the kind of fine-grained distinctions that appear to be needed for accounts of propositional attitudes and of information. It is often called the "granularity" problem. Let's look at a nonmathematical example.

Imagine that I am getting dressed and am faced with tying my shoes. Which one shall I tie first? I can't tie each of them first; that is an impossibility. It is just not in the nature of things to be able to do two distinct things each before the other.[1]

Now consider the proposition that I can tie each of my shoes before the other. Within the possible worlds framework, this proposition is (or is adequately modeled by) a set p of possible worlds. What set is p? Well, given that it really is impossible for me to tie each of my shoes before the other, the set p is the empty set of worlds. This is the same as the set of possible worlds in which $2 + 2 = 5$ and the same as the set of worlds in which Fermat's Last Theorem is false. So, on this account, very different claims are seen as expressing the same proposition. But then believing or doubting or claiming one of these propositions should be the same as believing or doubting the others, which is obviously false.

The granularity problem poses obstacles to the inverse relation between possibilities and information as well. Consider, for example, the question of the product of two large numbers, say 535,864 and 345,131. Assuming that we have never had occasion to consider this problem before, it seems like there are lots of possibilities. We can see at a glance that the answer must be an even number in the billions, but that's about all that's immediately obvious. If we use the standard algorithm (or a calculator) to compute the product, we obtain the information that

$$535{,}864 \times 345{,}131 \;=\; 184{,}943{,}278{,}184.$$

Now according to the inverse relationship between information and possibility, that information results in the elimination of possibilities. Intuitively, the eliminated possibilities includes the possibilities that

$$535{,}864 \times 345{,}131 \;=\; 184{,}953{,}278{,}184,$$

that

$$535{,}864 \times 345{,}131 \;=\; 184{,}743{,}278{,}184,$$

and many, many others. But are these really possibilities? If so, in what sense are they possible? Surely the world could not have been such that either of these were the case any more than that it could have been such that $2 + 2 = 5$.

Stalnaker realizes that the granularity problem is a serious obstacle to his theory of attitudes, especially in regard to mathematical inquiry and belief. He offers as a proposed solution to treat mathematical propositions in a very different way than other claims. Namely, he makes the unpalatable proposal "to take the objects of belief and doubt in mathematics to be propositions about the relation between statements and what they say" ([12], p. 73). Stalnaker himself points out some serious objections to this proposal ([12], p. 76).

Dretske faces a similar problem in his information-theoretic account of knowledge. He takes it that all mathematical truths are necessary so there is no possibility that they could be false. Consequently, on his account, true mathematical propositions have null information content. This is highly problematic for his account of knowledge in terms of beliefs that carry information. As he says in a footnote, “Frankly, I do not know what to say about our knowledge of those truths that have an informational measure of zero (i.e., the necessary truths)” ([4], p. 264).

In response to the Granularity Problem, various authors (Dunn [5], Kripke [6], Lycan [8], Yagisawa [15], e.g.) have proposed that among the worlds there are, in addition to possible worlds, also *impossible* worlds, some where you can tie each shoe before the other, some where tying shoes behaves normally but $2 + 2 = 5$, and still others where Fermat’s Last Theorem is false. On this view, the claim of the impossibility of p corresponds not to the claim that the set p of worlds is empty but rather to the claim that p contains only impossible worlds. To have an inconsistent belief is, on this view, to believe of the actual world that it is among some set of worlds which, as it happens, contains only impossible worlds.

In a recent article, Stalnaker [13] finds this suggestion even less palatable than his own proposal, declaring that impossible worlds are “too much to swallow.” In this paper, I want to propose informationalism, an elaboration of Stalnaker’s moderate modal realism. This theory, however, ends up disagreeing with Stalnaker by countenancing impossible states, so (in his terminology) impossible worlds. It seems to me that this modification of Stalnaker’s moderate modal realism has all the advantages of his version while at the same time overcoming the Granularity Problem and answering many (though certainly not all) unanswered questions about possibilities.

2 *An informational account of possibility* The main idea of informationalism is to take the inverse relationship between information and possibility as a guiding tenet.

> *The Inverse Relationship Principle*: Whenever there is an increase in available information there is a corresponding decrease in possibilities, and vice versa.

The implications of this principle will depend a lot on what else one assumes about information and possibilities. My main proposal here is that a good theory of possibility and information should be consistent with this principle. As I analyze things, impossibilities are those states of the system under investigation that are ruled out by (i.e., incompatible with) the currently available information about the system. States not so ruled out are possibilities. Mathematical inquiry, like any other form of successful inquiry, necessarily decreases possibilities when it increases the available information.

2.1 States The first question we must ask ourselves is this: what kind of thing is it that is classified as possible or impossible? Once that is settled, we can ask how they get classified the way they do.

The crucial starting point for the view of modality presented here is Stalnaker’s moderate modal realism mentioned earlier, according to which possibilities are “ways the world could be, . . . abstract objects whose existence is inferred or abstracted from the activities of rational agents." To avoid any possibility of confusion with extreme modal realism, let us follow scientific practice and refer to these abstract objects as *states* rather than as possible worlds.

It might be thought that practically speaking, not much hangs on which philosophical version of modal realism one adopts. In fact, though, a great deal hangs on it. These two views of possible worlds simply have very different ramifications and we need to be aware of these differences. While informationalism is compatible with moderate modal realism, it does not seem to be compatible with extreme modal realism.

To illustrate the differences between the two positions, consider the following analogy involving a simple card game. Two players, Max and Claire, are each given a card from a deck consisting of cards of denominations K, Q, and J, where $K > Q > J$ indicates the relative ranking of the cards. The player with the higher card wins. If they have cards of the same denomination, then the play results in a draw. We suppose that the players play this simple game many times, say 10,000. We further suppose that after these 10,000 hands, Max and Claire destroy the deck of cards and never play the game again. (The extreme modal realist can take this example in a different way: Max and Claire play the game once in each possible world, of which we assume there are 10,000.)

On the extreme view, there are 10,000 different hands, or "worlds," one for each time the game is played. Of course Max and Claire may get the identical cards in different hands, but they are still different hands, taking place at different times and places. During any one hand, that hand is the actual hand, and the other 9,999 are other possible hands.

Let us now take a moderate view of the same game, in terms of states. Intuitively there are nine possible relevant states these hands can be in. We model these states with nine ordered pairs

$$(K, K), (K, Q), (K, J), (Q, K), (Q, Q), (Q, J), (J, K), (J, Q), (J, J)$$

where, for example, a given hand h is in state (J, Q) if in h Max has a J and Claire has a Q.

A significant difference between the two versions of modal realism emerges if we consider the proposition expressed by "Claire has a Q". On the extreme approach, this will be modeled by the set p of all hands in which Claire has a Q. It will be true of a particular hand h if and only if $h \in p$. If each of the 10,000 hands is in p, then p will be deemed necessary. If p contains none of the 10,000 hands, it will be deemed impossible. If, as is more likely, p consists of some but not all of the 10,000 hands, then p will be deemed a contingent proposition.

On the moderate approach, the proposition expressed by "Claire has a Q" is modeled by a set of three states

$$\{(K, Q), (Q, Q), (J, Q)\}$$

Whether this proposition is judged as necessary, contingent, or impossible, depends on the status of the nine states. For example, if each of the nine original states is possible, then p will be contingent, containing as it does some but not all of the possible states.

A second difference emerges when we ask what makes the set of all possibilities the “right” one on the two accounts? On the extreme view of possibilities as particular worlds, they just are whatever they are, it is brute modal fact. On the moderate view, however, there is clearly something somewhat relative (some might say *ad hoc*) about the set of states. Why, for example, in discussing our card game, did we not keep track of the suit of the cards, or how old they are, or where they were manufactured? Surely these things are all part of the total state of the cards.

Two considerations went into our choice of states. First, given the rules of the game, each of these states gives all the information about the hands that is relevant to determining who wins and loses a hand. If we changed the rules so that in the case of cards of the same denomination, the winner was determined by the suit of the cards, then our states *would* need to capture not just the denomination of each player’s card, but also its suit. Thus our choice of states is determined in part by the issues raised by the rules of the game.

Second, in setting up our states, it is crucial that all possibilities be among these states. If someone had left a 10 in the deck, for example, then the above set of states would have been inadequate since Max or Claire might have gotten a hand that would not have had a state at all. So again, on the moderate approach there is an element of relativity in our choice of states that is avoided by the extremist appeal to alternative possible worlds.

2.2 Pragmatism The above discussion shows that for the moderate modal realist, what counts as a possibility is to some extent a pragmatic matter. At the same time, we note that *some* form of pragmatism is implicit in the Inverse Relation Principle. After all, this principle is not only about possibilities and available information but also about changes in each. This forces upon us a more pragmatic picture of possibility and information in inquiry. In a pragmatic picture where possibilities and available information change during the course of an inquiry, one is forced to recognize that both depend on context in some way or other. In what way can that be?

Stalnaker recognizes a pragmatic dimension to possibility. He writes, for example:

> One may say that in particular contexts of inquiry, deliberation and conversation, participants distinguish between alternative possibilities, and that they should do so is definitive of those activities. It does not follow from this that there is a domain from which all participants in inquiry, deliberation and conversation must take the alternative possibilities that they distinguish between. ([12], p. 58)

Dretske, in his discussion of knowledge, information, and communication, also recognizes a pragmatic dimension:

> When a possibility becomes a *relevant* possibility is an issue that is, in part at least, responsive to the interests, purposes, and, yes, values of those with a stake in the communication process. ([4], p. 133)

And later, in relating the notion of relevant possibility to his account of information, he writes:

> To know, or to have received information, is to have eliminated *all relevant alternative possibilities.* These concepts are absolute. What is not absolute is the

> way we apply them to concrete situations—the way we determine what will qualify as a relevant alternative. ([4], p. 133)

Dretske's discussion of what makes a possibility relevant is less than satisfying, though. It is hard to see in practical terms what Dretske's relativity really comes to, that is, *how* the context determines the relevant possibilities.

I suggest that the pragmatic move can profitably be factored into two steps, corresponding to the successive answers to the following two questions:

> What issues are relevant to the given inquiry?
> What information is currently available concerning these issues?

The quote from Stalnaker deals with the first; the issues at stake in one inquiry are typically not the same as those at stake in another. The quotes from Dretske have to do with the second; sometimes the given context makes it clear that some states that would have been considered possible in some other context are not possible in the present context. (I will say that this is because different information is available in the two contexts. Dretske could not put things this way even if he were inclined to since he wants to use possibilities to define information.)

In a given intentional context (an inquiry, communication, or modal judgment) certain issues are relevant. This set of issues is typically quite limited compared with the class of all issues about everything there is (a problematic notion on set-theoretic grounds alone). Which issues are relevant is determined by the context at hand in no doubt complex and subtle ways.[2]

Given the relevant issues, though, whatever they are, one can consider all the various ways of resolving these issues. This is what we shall mean by a state: *a way of resolving all the relevant issues.*[3] Some of these ways of resolving issues will correspond to ways the world could be, the others may correspond to ways the world could not be. The former states are possibilities, the latter impossibilities.

To give an example involving our card game, it may be that the context makes it clear that the relevant issues in a hand are simply the denominations of the cards held by each player and that the only denominations available are K, Q, and J. In this case, there are nine ways of resolving the issues and our ordered pair model above is an adequate way to model these nine states. However, if the context makes it the case that the suit of the cards is also an issue, then there will be more than nine ways of resolving the issues and our nine ordered pairs do not suffice.

To summarize: at any stage of a given inquiry there will be certain relevant issues. Any way of resolving all these issues is a state. We now turn the issue of what divides these states into possibilities and impossibilities.

2.3 Possibilities and impossibilities In a given inquiry, certain information about the relevant issues is available. Some of this information can be general, in the form of laws or regularities governing all (or a wide class of) situations, while some of the information can be very specific to a single situation or to a small set of situations. (In physics this division corresponds to a differential equation in the first case and initial conditions in the second.) What matters for the present discussion, however, is simply that the relevant available information will in general be consistent with some states, that is, with some ways of resolving all the issues, inconsistent with others.

Relative to the available information, then, certain states are possibilities, others are impossibilities.

In probability theory, physics, and applied mathematics, it is common practice to successively relativize to the set of possible states as more and more information becomes available. It is important not to completely lose track of the impossible states, however, for they are ways things cannot be, given the available information. If, for some reason, we come to believe that some of that information is unreliable, then it is no longer available and some states that were impossible must now be deemed possible.[4]

2.4 Availability By saying that whether or not a given state is possible is relative to the available information, we take out a large pragmatic mortgage that needs to be paid off. What does it mean for a piece of information to be available in a given inquiry? What makes it available or unavailable?

These questions are closely related to issues discussed at length in Chapter 5 of [4], to which we refer the reader. Paraphrasing the first passage from Dretske displayed above, we can say that when a piece of information is available is, in part at least, responsive to interests, purposes, and values of those with a stake in the inquiry. We can go further, though, for by admitting the interdependence of information and possibility, we achieve added purchase on the questions. This purchase comes from the fact that whereas a state is a unitary thing, information comes in pieces. A given inquiry may have this piece of information available, that piece unavailable, and so on. If states are taken as primitives, then their possibility is a holistic issue. By recognizing the mutual dependence of information and possibility (as expressed by the inverse relation principle) we have a more local, issue-sensitive notion of relevance. This allows us to recognize some simple mechanisms (among, no doubt, countless others that are less simple) by which information may or may not be available.

In the space available, we can only look briefly at some examples. However, even this brief look will show us that the informational approach to possibility allows us to bring many apparently competing notions of possibility into a unified theory. To put it differently, we can see many apparently divergent intuitions about possibility as having to do with what information the bearers of those intuitions take as available in inquiry.

2.4.1 Epistemic possibility When an agent claims that something is possible, he or she often means that it is "possible for all I know." In this case, the available information is simply what the agent happens to know about the issues. The epistemically available information comes in two forms: (i) general laws and (ii) facts specific to the situation or situations under consideration. In the case of our card game, (i) includes the rules of the game, while (ii) might include specific facts that Max has noticed, that he has a J, say. If this is so, then it is for him (epistemically) impossible for him to have a Q, whereas for Claire that is a possibility.

A variant on epistemic availability is doxastic availability, where we consider not what the agent knows, but what the agent believes. Of course this will not, in general, be what Dretske and I mean by information; some of it may well be misinformation. If so, some of the states deemed impossible may in fact be possible, something that cannot happen with full-fledged information.

2.4.2 Legal possibility In the United States the laws of the land are considered available information; responsibility for getting the information (learning the relevant laws) is put on us as citizens. For example, it is not legally possible to have two spouses at the same time, even if the people involved are not aware of the law prohibiting bigamy. As the saying goes, “ignorance of the law is no excuse.” So in determining the legally possible states in the U.S., all applicable laws count as available information.

2.4.3 Physical possibility When we ask whether something is physically possible, we are relativizing to the set of physical laws. What makes information “available” in this context is simply that it be a physical law or regularity. It does not have to be a known physical law. Of course it is also possible to combine this with the epistemic notion by considering as available only the known physical laws.

This characterization of physically possibility sidesteps the difficult question as to why some pattern in nature is a law rather than a mere accidental generalization. Intuitively, accidental generalizations are patterns that are not physically necessary, that is, such that it would be possible for them to be contravened. Given our characterization of possibilities, this just brings us back in a circle. This circularity does not show that the account is wrong, since we are not trying to reduce information to possibilities (or vice versa). It only shows that the account is not particularly helpful in understanding the nature of the difference between physical laws and accidental generalizations, at least as far as I have been able to see.

2.4.4 Metaphysical possibility One kind of possibility of great recent interest to philosophers is metaphysical possibility. Here the information that matters consists of the general metaphysical laws. I take these to be the regularities that fall out of the way humans individuate objects, properties, and relations. For example, if the issues involve the relations of sibling and child, then there is an available metaphysical regularity that children of the same individuals are siblings.[5] What makes this count as being available information in determining which states are metaphysically possible is simply that it is a structural consequence of the metaphysical nature of the issues in question. Of course there are active debates about just what is and what is not metaphysically possible. Our claim here is simply that this debate can be seen as a debate about what does and what does not count as a metaphysical regularity.

2.4.5 Mathematical possibility Another form of possibility is mathematical possibility. Here the available information consists of the mathematical truths. It is mathematically possible to divide ten people into two teams of five each. It is not mathematically possible to divide ten people into three teams of three each. The first is a possibility because $2 \times 5 = 10$, the latter isn't because $3 \times 3 \neq 10$. In a given epistemological context it might be possible that

$$535{,}864 \times 345{,}131 \ = \ 184{,}953{,}278{,}184$$

but it is certainly not mathematically possible.

2.4.6 Logical possibility In the case of logical possibility, the available information consists of the laws of logic. Just what the laws amount to, and how they relate to metaphysical and mathematical laws, is a contentious question, however, one we will not attempt to answer. How one answers it will determine what one counts as a logical possibility. As a convenient and simple place holder for an answer we assume here that the laws of logic are the laws of classical propositional logic. If we take these laws as available, then each possibility will be compatible with the laws of propositional logic. It is this position that we will incorporate in our mathematical model, with the notion of a Boolean information context.

Example 2.1 Let's work out three examples. First, let us return to our card game. The issues, we suppose, determine that the relevant states are adequately represented by the nine ordered pairs displayed earlier. Assume there is only one queen in the deck. Is the state (Q, Q) possible or not?

There are different stories one could tell, stories that would push our intuitions in different directions. The claim here is that the stories push us this way or that by making different information available.

One story is where the rules of the game dictate that there is only one Q in the deck; if it is not played with that kind of deck, then its a different game. On this story, it seems that (Q, Q) is an impossible state because the rules of the game would naturally be considered as available to all players of the game.

Another story, though, is where it so happens that three of the four Q's have been been lost from one deck, the deck that happens to be selected at random and used in a given play of the game. Of course *we* have this information and so (Q, Q) is an epistemic impossibility. For Max and Claire this information is unknown so (Q, Q) is a possibility.

Example 2.2 Inquiry (and I now mean it literally, not just as a place-holder for a family of related propositional attitudes) has to do with the interplay between various notions of available information. For our second example, let's address the question of how inquiry can lead us from something being possible to its being impossible. In order to make the case rather stark, we take a mathematical example, namely, the possible values of $535{,}864 \times 345{,}131$. How, one might wonder, is it that an inquiry could ever determine that it is impossible for 535,864 times 345,131 to be anything other than 184,943,278,184. If some other value is (epistemically) possible at the start, how can it become impossible as a result of inquiry? After all, if the agent A knows the laws of arithmetic, how can any other value be even epistemically possible? On the other hand, if A does not know them, then how could A's inquiry be successful?

To answer this, let's consider two forms of inquiry, one where A uses a calculator, the other where A uses the familiar multiplication algorithm. We start with the calculator. We suppose A knows how to input numbers, how to choose the operation of multiplication, how the output is represented, and how overflow is indicated. We also suppose that the calculator is working properly and that A knows this to be the case. That is, we suppose all this information is available to A at the start of the inquiry. Notice, though, that this kind of information does not preclude $535{,}864 \times 345{,}131$ having some value other than 184,943,278,184. It is only when A actually inputs the correct values and hits the $=$ key that the output display carries the information that the value is 184,943,278,184. It is at the point in the inquiry where this information becomes available that any other value becomes epistemically impossible. The point here is that A can have a general knowledge of the working of the calculator without having knowledge of what the specific outcome will be as the result of a specific calculation.

Now let's look at the problem when A is using the multiplication algorithm. This algorithm represents general knowledge A has about integers and how to multiply them. It is analogous to A's knowledge of the calculator. Only if A correctly employs the algorithm, though, does the process result in the information that 535,864 $\times$ 345,131=184,953,278,184; it is at this point that any other outcome becomes epistemically impossible.

Example 2.3 For a final example, we give one illustrating logically impossible states. Let us suppose that the relevant issues of an inquiry include whether some mathematical domain M satisfies the first-order sentence θ as well as first-order sentences $\varphi_1, \ldots, \varphi_N$ and that it is already established that each φ_i is not the case. In other words, each $M \models \neg\varphi_i$ is included in the available information. Let us further suppose that the sentence

$$\theta \vee \varphi_1 \vee \cdots \vee \varphi_N$$

is a theorem of first-order logic. The fact that M satisfies the displayed sentence may not be included in the available information, and even if it is there is no reason to suppose that $M \models \theta$ is included in the available information, even though it is a logical consequence of the available information. For example, if we are dealing with epistemic possibility and N is very large, the agent may not realize that the set $\{\theta_i \mid i = 1 \ldots N\}$ *exhausts* the remaining disjuncts of the displayed logical truth. Checking that this is so is a step that must be gone through before $M \models \theta$ becomes available. Until this, our (epistemically) possible states include the (logically impossible) states where $M \models \neg\theta$ is the case alongside each of $M \models \neg\varphi_i$.

2.4.7 Summary We close this section with a summary of the pragmatic theory of possibility propounded here.

Issues:	The set of all states of a given inquiry depends on the system under investigation and on the issues regarding the system relevant to the inquiry.
States:	A state is a way of resolving all the relevant issues.
Impossibilities:	The set of possible states at a given point in the inquiry depends on the information concerning the issues currently available. The impossible states are those incompatible with the currently available information; the others are possible.
Available information:	What information is available at any point in an inquiry is a context-sensitive matter, depending on the kind of possibility one is considering and on the progress of the inquiry up to that point.

Increases in information:	The correct elimination of any nonempty set of possibilities corresponds to a strict increase in the information available at the next stage in the investigation.
Decreases in possibilities:	Conversely, the acquisition of any new information corresponds to a strict decrease in the states that are possible.

3 Applications In this section we discuss in a cursory manner several applications of the theory outlined above.

3.1 Impossible worlds As we noted earlier, Stalnaker has written attacking the idea of impossible worlds. Ironically, while Stalnaker's arguments are persuasive on the extreme modal realist view of possible worlds, they do not seem persuasive on the moderate view. Why should there not be ways that things cannot be as well as ways things can be; that is, states the system could not be in (given currently available information) as well as states it could be in? In our elaboration of the moderate view, the impossible states of the world are simply those states that are incompatible with the available information.

3.2 Metaphysical impossibilities One of Stalnaker's examples of something that cannot happen in any "world," that is, in any state of this world, is for there to be a round square. Other examples are worlds where some claim and its denial are both true.

It is instructive to see why Stalnaker thinks that there can be no state in which there is a round square. The reason, he says, is that the information that an object is round automatically precludes its being square. But to say this is just to say that in the context where we have the information that squares cannot be round, the alternative of something being a round square is not a relevant one to consider. A different context might not have this information. For example, suppose we were concerned not with metaphysical possibility but with epistemic possibility and all the agent knew about squares and circles were how they appeared from the side. In such a case, the possibility of a round square (as depicted somewhat facetiously in Figure 1) is something that would need to be considered if one is to avoid modal error.[6]

Figure 1: A square circle (side view)

To be a bit more concrete, let's set up two systems of states to model Stalnaker's example. Suppose the relevant issues have to do with physical shapes with various attributes, such as area, shape, color, weight, and so on. We might set up a system of states to classify these objects by taking as our set Ω_C of states those functions which assign appropriate values to these various attributes. One such state $\omega \in \Omega_C$ would have

$$\begin{array}{lcl} \omega(\text{area}) & = & \text{4 sq meters} \\ \omega(\text{shape}) & = & \text{round} \\ \omega(\text{color}) & = & \text{red} \\ \omega(\text{weight}) & = & \text{2 kg} \\ & \vdots & \end{array}$$

If we set things up in this way, then there is no state $\omega \in \Omega_C$ corresponding to a round square, simply because we have built the information that there are no round squares into the state space.

It is easy enough to find a larger state space where this assumption is not built in. We could, for example, use arbitrary relations, rather than functions, to model the set of states. Relative to this larger state space Ω, the proposition that the object in question was a round square would be modeled by the set of all states where the attribute shape was related to both the values *round* and *square*. Relative to the information that squares are not round, each such relation would model an impossible state.

Something similar happens with logical impossibilities. We can preclude such things from our state space in the first place, or we can have a state space that admits of states which, relative to the laws of logic, are seen to be impossible. (This idea is illustrated in Example 2.3 in the preceeding section and is taken up in more formal detail in Section 4, where we consider non-Boolean (as well as Boolean) information contexts.) There is nothing incoherent about this notion of state and it can be useful for characterizing the beliefs of those of us who are less than logically omniscient. (The epistemic nature of the move was stressed in the proposal to allow impossible situations in [5].)

3.3 Impossible situations We have seen above that it is perfectly coherent to allow impossible states into our account. What about impossible situations? What does one mean when one says that they have been put in an impossible situation?

One might argue that this sort of language is metaphorical, that a real situation can be embarrassing or unpleasant but it cannot be literally impossible. And no doubt such language is often metaphorical. Still, within the theory of possibility proposed here where one recognizes that what counts as possible is inevitably relative to the available information, a different answer can be given.

Suppose we are in a context where certain information is available. On one view of information, the one we build into our model in the next section, information is always about some situation or some range of situations. Assume this is what we mean by information. On this account, our notion of what is possible will apply to that same situation or range of situations. But what if the situation *s* under discussion happens to lie outside that range? In such circumstances it could easily happen that the state of *s* is not compatible with the available information. In such a case, one would perhaps be justified in saying that the situation *s* is impossible.

One might object that in a case like this, the available "information" relative to which the state is impossible is really misinformation, not information. This is certainly a defensible line. However, it is not the only defensible line. Suppose, for example, that we are interested in legal possibility, so that the available information consists of all the applicable laws. In this case a situation being impossible means that people are not acting as the laws dictate. For example, if one is put in a position where one is required by law to act, but any action one takes would violate some law, then one has been put in a legally impossible situation.

3.4 Hume's Maxim Hume's Maxim, the claim (quoted at the start of the paper) that nothing we imagine is absolutely impossible, has a very curious status within philosophy. It is frequently used in the thought experiments that we as philosophers carry out as part of some argument. Think of Putnam's Twin Earth example, where we are asked to imagine a very far-fetched situation, one involving earth and twin earth, and the indistinguishable liquids H_2O and XYZ. From this imagined situation we conclude that such a situation is possible and draw conclusions about meaning and minds. But while it is a standard methodological tool in philosophy, Hume's Maxim has also had its detractors. What, one might ask, do our human imaginative capabilities have to do with genuine possibility? Just because we can imagine a perpetual motion machine, or traveling faster than light, does not show that either is really possible.

A version of Hume's Maxim is ably defined in Yablo [14]. Yablo argues that if one can *imagine a situation of which one can truly believe that p* then generally p is possible. As Yablo characterizes this form of imagining, it is rather strong. It takes much more than simply imagining that p. We adopt Yablo's understanding of this notion. People may say that they can imagine that p, or prove that p, but be wrong about it. It takes imagining a situation, or producing a proof, respectively, to do these things.

It seems to me that Yablo is correct, but that he does not go as far as he could. For while Yablo takes imagining a situation of which one can truly believe that p to be a reliable guide to the possibility of p, he does not take it to be an infallible guide. I think if we use Yablo's notion of imagining a situation, combined with the informationalist concept of possibility advocated here, then full-fledged imagining is an infallible guide to at least a certain kind of possibility.

As purported counterexamples to this claim, Yablo takes certain *a posteriori* impossibilities. He argues, for example, that the ancients might have imagined Hesperus outlasting Phosphorus, but that this does not show that Hesperus really could have outlasted Phosphorus. After all, they are the same planet and nothing can possibly outlast itself. If an ancient had imagined Hesperus outlasting Phosphorus and so judged this to be possible, he or she would, Yablo claims, have made a modal error.

There is a shift in contexts going on here, between that of the ancient's imagining and our judgment as to impossibility. In what might the ancient's imaginative act have consisted? There are many stories one could tell. The stories would naturally differ in just what the issues were and in what information was taken as available, and so (on our conception) in what was possible. For one story, the ancient might have imagined that the heavenly body called Hesperus would outlast that called Phosphorus. Or she might have imagined that the brightest heavenly body in the morning sky would outlast the brightest heavenly body in the evening sky. These are both certainly states the world might have been in. Relative to the information that these heavenly bodies are one and the same object, though, the state in question is impossible. But the ancient did not have access to that information.

Since this point is controversial, seeming to question orthodox attitudes about metaphysical possibility, let's go over it in a bit greater detail. Consider the sentence

(∗) Hesperus will outlast Phosphorus.

There are many different propositions that we might use (∗) to express. One of them, p which we might symbolize as

$$Outlast(Venus, Venus)$$

does indeed violate a very basic metaphysical constraint, namely, that nothing can outlast itself. But surely p is not what the ancient imagined.

Another proposition, call it q, that might be expressed by (∗) could be expressed more pedantically as: the heavenly body called "Hesperus" will outlast the heavenly body called "Phosphorus." We might symbolize q by:

$$Outlast(The\ x(x\ is\ called\ \text{Hesperus}), The\ y(y\ is\ called\ \text{Phosphorous}))$$

If, in addition to the above metaphysical constraint involving the relation of outlasting, the empirical information that "Hesperus" and "Phosphorus" co-refer is available, then q is impossible. However, without this additional piece of information as part of the context, proposition q is both imaginable and possible.

These considerations convince me that Yablo's counterexample to Hume's Maxim is flawed. Still, the skeptic has a good question. How could an act of the imagination guarantee that something is possible?

By way of analogy, consider going to an architect wanting to know whether it would be possible to build a house to fit your needs on a particular piece of land. The architect will probably try to convince you that it is possible by drawing a scale drawing of the proposed house. Perhaps he will even build a scale model of it. Based on this drawing or scale model, you may decide that it is indeed possible to build a house. How does this work?

Examining the plan or scale model you see that the house is two stories tall, has three bedrooms, a large kitchen, and so on. Notice that scale drawings and scale models do not really have these properties. We interpret them as though they do by using properties of buildings to classify representations of buildings—even when there is no actual building that the representation represents. The important point, though, is that to the extent that the drawing or scale model honors the constraints that physical buildings honor, under the above interpretation, the model *does* show that a kind of building is possible. That is why we don't need to build a building to know that a certain type of building is possible.

The use of scale drawings and models to show that a house is possible is no different in principle from the use of set-theoretic structures to show that first-order theories are consistent. Our theory may be about physical objects, but we reinterpret the physical predicates of our theory in terms of certain sets of abstract objects. If we can do this in such a way that we can find a set-theoretic structure that satisfies the theory, we are justified in concluding that the theory is consistent.

Imagining is a mental activity that results in a particularly vivid mental image. By "vivid" I mean to require full-fledged imaginings, what Yablo calls "imagining a situation.” Such imagined situations share with drawings, pictures, and scale models the property that they can be treated as surrogate situations supporting, under reinterpretation, their own properties. Just as we use properties of buildings to classify drawings of buildings, we also use properties of external situations to classify these imagined situations. As long as all the contextually available information is honored under such a reinterpretation, it shows that some state is indeed possible. Or, to put it a different way, the act of imagining takes place relative to its own context, and relative to that context, at least, what is imagined is possible.

An act of imagination can be impoverished or rich. One impoverished form of imagining occurs when the imaginative act fails to result in an imagined situation. Just saying or thinking “I want to take a trip to Paris” without the accompanying mental imagery would be such a failure. These kinds of imaginings, if they are really imaginings at all, are not the sort that Yablo is talking about when he speaks of imagining a situation.

There are ways in which even an act of imagining a situation can be also impoverished, though. This occurs if the imagined situation fails to satisfy very familiar information. Picture a fan blowing air at a small windmill, which in turn generates electricity which in turn powers the fan. This seems to me something I can imagine, and if so, then it seems that perpetual motion machines are possible. However, my imagining is impoverished in that it does not satisfy constraints of modern physics, constraints that preclude the existence of perpetual motion machines. Only if the context somehow leaves out some of the constraints of modern physics does the imagined situation show that a perpetual motion machine is possible. In the same way, an architect’s plan may not include available information about the finances of the client, and so be impoverished (and leave the future home owner impoverished).

The question here boils down to this: in an act of imagination, what information counts as being available? If the imagining is going to be the basis of action, like building a house, then much more needs to be considered available than if the act of imagination is an end in itself, as with a piece of science fiction. Indeed, it seems that part of what we often do in science fiction is precisely this—suspend or alter certain common information so as to make it unavailable in the context at hand.

The final part of [14] is devoted to offering a dialectic model of how people come to realize that modal judgments are mistaken. On the theory presented here, there are two very different ways to fall into modal errors. Given the available information, someone can certainly be mistaken about a given state, thinking it is compatible with the information when it is not. That is, they could make what seems like a logical error. On the other hand, and a much more common source of modal error, the agent might be missing a piece of information relative to which a state is impossible. That is ignorance. Filling the agent in on a piece of available information is very different than helping the agent to see that a state is incompatible with earlier accepted information. In the case of the ancients, the error comes about from ignorance. We can see this since to correct them we inform them that Hesperus and Phosphorus are the same heavenly body.

3.5 Consciousness One currently fashionable philosophical application of Hume’s Maxim is to the problem of consciousness. We are asked to imagine a zombie-like creature exactly like us in all physical regards, but without conscious experience. Some people, at least, claim that they can imagine such a creature. According to Hume's Maxim, the ability to imagine such a creature shows that it is possible for two individuals to be alike physically but only one of them to be conscious. But then, the argument goes, consciousness cannot be explained in terms of physical properties.

There is an obvious flaw in this argument. The demonstrated possibility is relative to the information available during the imagining. In order for the last step of the argument to be correct, the purported theory relating consciousness to physical properties would need to be part of this available information. Only if the imagined situation with its zombie satisfied this theory would it show that the theory did not give an account of consciousness in terms of physical properties. But clearly in our act of imagination, this information is not available. Saying it is would be like saying that a planned house is financially possible simply because we have a scale model and have no information about the finances of the would-be owner.

3.6 Inference and information In various writings (see [1], pp. 68–69 or [2], for example) Etchemendy and I have proposed a view of inference and problem solving that goes beyond the sentential, allowing diagrams and other forms of representations into reasoning. The basic idea is to see deduction in terms of exploring spaces of possibilities, eliminating some and further exploring others.

Suppose, for example, that one is trying to prove that $\sqrt{2}$ is irrational. One begins by considering the possibility that it is rational, shows that this possibility leads to an inconsistency, and so concludes that it is not really possible after all. In [1], we use this metaphor to suggest good strategies for certain kinds of problem solving.

On the standard view of possibilities, either the extreme or moderate realism toward possible worlds, this idea simply can't be right. On either Lewis's and Stalnaker's understanding of possibilities, there is no possibility that $\sqrt{2}$ is rational, there never was such a possibility and never could be such a possibility. So what are we doing when we think we are considering this possibility.

On the view presented here, there is such a possibility. For the mathematical neophyte, it is *epistemically* possible that $\sqrt{2}$ is rational since it is compatible with the information available to the agent, that is, with what he or she knows. It is the space of epistemic possibilities that is explored and narrowed using the laws of logic and mathematics available to the neophyte in problem solving.

4 An information-based modal logic The familiar and elegant Kripke frame approach to modal logic starts with the possible world account of possibility and uses it to develop a logic of possibility. In this section we propose an alternative framework based on the theory of possibility discussed above. We will show that it includes, but is more general than, the Kripke frame approach. In so doing, some strong assumptions built into the Kripke frame approach come to light.

First a note on methodology. I am personally an adherent of strict realism, the view (implicit in the first quote from Stalnaker) that there is only one world and that everything that exists is part of it. For this reason, one of my goals is to construct a modal framework that is consistent with strict realism. On the other hand, I do not limit the framework to this assumption, since I want the model to be a generalization of the Kripke frame approach. So there are instances of the model that adhere to strict realism, others that adhere to Lewis's extreme modal realism.

This section builds on ideas from my recent book with Seligman [3] though the presentation will be largely self-contained. In [3], we develop three mathematical models of information (channels, local logics, and state spaces) and study their relationships. This section reviews one of these models (that of a local logic) and shows how to use it to model states, possibilities, and impossibilities. This modeling should not be confused with an attempt to define these notions in terms of information. I could equally well have started with the notions of state space and subspace and used them to develop a model of information. As I have stressed earlier, informationalism above does not attempt to *define* possibilities in terms of information. (I am skeptical of all attempts at reduction.) Rather, the informal theory presented above attempts to explicate the relationship between possibilities and information and the model attempts to illuminate this relationship in a different way.

4.1 Modeling information We begin this section by reviewing (with somewhat different terminology) some material from Part 2 of [3]. A *classification* $A = \langle S, \Sigma, \models\rangle$ consists of a nonempty set S of objects called *situations*, a nonempty set Σ of objects called *situation types*, and a binary relation $\models$ on $S \times \Sigma$. The notation "$s \models \sigma$" is read "s is of type σ" or "s supports σ" or "σ holds of s".

A classification A can be used to model the relevant issues concerning the situations of A. The types of a classification may be linguistic (words or sentences, say), conceptual (concepts, say), their objective counterparts (properties or types of situation, say), or things like heights, weights, and so forth. They can also be sets of situations. The situations of a classification may be real situations, but they may also be ordinary objects or anything else. To relate this to more traditional approaches, we present two additional examples.

Example 4.1 Let W be some set of (Lewis-style, perhaps) possible worlds and let $\Sigma = \text{pow}(W)$ be the set of all subsets of W. Define $s \models \alpha$ if and only if $s \in \alpha$. For ease of reference, we call this the Lewis classification on W.

Example 4.2 Let Σ be the set of first-order sentences in some vocabulary L and let S be some set of L-structures, with $M \models \alpha$ if and only if α is true in M.

In [3], situations and situation types are called "tokens" and "types" respectively.

A *Boolean* classification $A = \langle S, \Sigma, \models, \wedge, \neg, \rangle$ consists of a classification together with binary operation $\wedge$ on situation types and a unary operation $\neg$ on situation types satisfying the following conditions:

1. $s \models \sigma_1 \wedge \sigma_2$ if and only if $s \models \sigma_1$ and $s \models \sigma_2$;
2. $s \models \neg\sigma$ if and only if $s \not\models \sigma$.

Both of the numbered examples are Boolean. Classifying people by their height is not Boolean.

In our approach to modeling pieces of information, we follow Gentzen's lead and use two-sided sequents. A (Gentzen) *sequent* is a pair $I = \langle \Gamma, \Delta\rangle$, where Γ and Δ are sets of situation types. A sequent $I = \langle \Gamma, \Delta\rangle$ *holds* of a situation s provided that if s

supports every type in Γ then it supports some type in Δ. A sequent I is *information about* a set S of situations if it holds of each $s \in S$. In the following result we use standard conventions about writing sequents.

Proposition 4.3 *Let A be a classification, let S be a set of situations. Let us write $\Gamma \vdash \Delta$ if the sequent $I = \langle \Gamma, \Delta \rangle$ is information about S. This relation satisfies the following conditions:*

Identity: $\alpha \vdash \alpha$;
Weakening: *If* $\Gamma \vdash \Delta$ *then* $\Gamma, \Gamma' \vdash \Delta, \Delta'$;
Global Cut: *If* $\Gamma, \Sigma_0 \vdash \Delta, \Sigma_1$ *for each partition* $\langle \Sigma_0, \Sigma_1 \rangle$ *of some set* Σ', *then* $\Gamma \vdash \Delta$.

Proof: Identity and weakening clearly preserve the relation in question. Global cut is almost as obvious, but we will prove it. Let $s \in S$. Toward a contradiction, assume that $\Gamma \cup \Sigma_0 \vdash \Delta \cup \Sigma_1$ for each partition $\langle \Sigma_0, \Sigma_1 \rangle$ of Σ' and that s is of every type in Γ but not of any type in Δ. Let Σ_0 consist of those types $\alpha \in \Sigma'$ such that $s \models \alpha$ and let $\Sigma_1 = \Sigma' - \Sigma_0$. But then $\Gamma \cup \Sigma_0 \vdash \Delta \cup \Sigma_1$, whereas s satisfies all the types on the left and none on the right, which is a contradiction. □

This proposition shows that the information about any set of situations is always closed under identity, weakening, and global cut. In [3], Seligman and I prove a representation theorem (Theorem 9.33) that shows these closure conditions, thought of as rules of inference, to be complete.[7] Hence, they constitute the natural set of closure conditions on sequents viewed as modeling pieces of information about some set of situations.

Definition 4.4 An *information context* $C = \langle A, \vdash, N \rangle$ consists of a classification A together with a binary relation $\vdash$ relating sets of situation types, and a set $N \subseteq S$ of situations called the *normal situations*, satisfying the following additional conditions.

Entailment:	The sequents in the relation $\vdash$ satisfy identity, weakening, and global cut.
Normal situations:	Each sequent I in the relation $\vdash$ is information about the set of normal situations.

In [3] information contexts are called *local logics*. There we give a number of results indicating natural ways in which information contexts arise. In this paper we use information contexts to model the available information relative to which judgments of possibility are made. The set of situation types of an information context represents the relevant issues. The relation $\vdash$ of an information context represents the information available in the given context.

A word of explanation is called for to motivate the use of the distinguished set N of normal situations. Intuitively, the normal situations are the situations the available information is about. They must be among the situations that satisfy the information; they might be all such situations or just some of them. One example would be where we start with a given set of normal situations, representing one's experience to date and let the information consist of all sequents satisfied by this experience. Another example, fitting more the case where the information is conventional or regulatory, would be to start with the consequence relation $\vdash$ and take the normal tokens to be those satisfying the relation. A third example is where we are modeling inquiry about some particular situation s, in which case we would take it to be normal, modeled by letting $N = \{s\}$. (If further motivation is desired, the reader should consult Theorem 14.8 of [3], a theorem showing how information contexts are related to information channels. The use of normal situations is crucial to this result.)

A *Boolean* information context is an information context on a Boolean classification such that the information relation $\vdash$ is closed under the usual classical Gentzen introduction rules for $\wedge$ and $\neg$; see Section 11.2 of [3] for details. Boolean information contexts build in the standard meaning of the Boolean connectives, so that states that are possible relative to Boolean contexts respect these connectives. Thus Boolean information contexts model contexts in which complete information about classical propositional logic is available.

An information context C is *sound* if every $s \in S$ is normal. C is *complete* if for all sequents $I = \langle \Gamma, \Delta \rangle$, if $\Gamma \nvdash \Delta$ then it is not the case that I holds of every normal situation, that is, there is at least one normal situation that provides a counterexample to I. C is *consistent* if there is at least one sequent $I = \langle \Gamma, \Delta \rangle$ such that $\Gamma \nvdash \Delta$. In view of weakening, this is equivalent to the condition that $\varnothing \nvdash \varnothing$. Notice that as long as there is at least one normal situation, C is consistent, since no situation can satisfy the sequent $\langle \varnothing, \varnothing \rangle$.

Example 4.5 Let W be a set of possible worlds and A be the Lewis classification on W. Define $\Gamma \vdash \Delta$ if and only if every world satisfying every $\alpha \in \Gamma$ satisfies some $\beta \in \Delta$. Let every world be normal. This is a Boolean context. It is both sound and complete. We call it the Lewis information context on W.

Example 4.6 Let A be the classification of some set of first-order structures by first-order sentences, as in Example 4.2, and let $\vdash$ be the usual Gentzen sequent calculus. Let every structure be construed as normal. This too is a Boolean information context. It is sound, but whether it is complete depends on what set of structures one starts with. If, for example, it consists of all finite and countable structures, then it is complete. However, if one starts with just the finite structures, then it is not complete.

Many other examples of information contexts, some Boolean, some not, are given in [3].

4.2 Modeling states We now turn to the issue of modeling Stalnakerian possible worlds, that is, possible states. Given a set Σ of types, one straightforward way to model states relative to Σ is by means of truth assignments, that is, set-theoretic functions from Σ into the set $\{T, F\}$. This form of modeling builds in an extensionality assumption about ways of settling the relevant issues that has not come up in our discussion until now. Namely, it assumes that distinct ways of settling the issues must settle some one issue in distinct ways. This is analogous to the Lewis-Stalnaker assumption that any two distinct propositions must differ on some state. We are not particularly troubled by this assumption, but point out that it is simply one way of modeling the theory and not part of informationalism itself.

Actually, rather than use truth assignments, we use an equivalent method that is slightly more convenient for our purposes. The convenience stems from the fact that it meshes better with [3]. We model states by binary partitions $\langle \Gamma, \Delta \rangle$ of Σ. Think of

Γ as the set of types with value T under the assignment and Δ as the set with value F.

Definition 4.7 Let C be a fixed information context.

1. A *state* consists of a binary partition $\langle \Gamma, \Delta \rangle$ of the situation types of C. Let Ω be the set of all states.
2. The state of a situation s is the partition $\langle \Gamma_s, \Delta_s \rangle$ where $\Gamma_s = \{\alpha \in \Sigma \mid s \models \alpha\}$ and $\Delta_s = \Sigma - \Gamma_s$. We sometimes denote the state of a situation s by $\text{state}(s)$.
3. A state $\omega \in \Omega$ is *realized* by the situation s if $\text{state}(s) = \omega$.
4. A state $\omega = \langle \Gamma, \Delta \rangle$ is *impossible* if $\Gamma \vdash \Delta$. (This makes intuitive sense since $\Gamma \vdash \Delta$ means that a normal situation cannot satisfy every type in Γ without also satisfying at least one type Δ.) Otherwise it is *possible*. We use Ω_C to denote the set of all states that are possible in the information context C.
5. A situation s is *impossible* if and only if its state $\text{state}(s)$ is impossible.

Notice that, in agreement with the first quote from Dretske, the notion of which states are possible, in a given information context, is absolute. What is not absolute is which information is available when making a given modal judgment, that is, which information context is in force. The following are immediate consequences of these definitions.

Proposition 4.8 *Let C be a fixed information context.*

1. *A sequent $I = \langle \Gamma, \Delta \rangle$ holds of a situation s if and only if the following holds, where* $\text{state}(s) = \langle \Gamma_s, \Delta_s \rangle$*: if $\Gamma \subseteq \Gamma_s$ then $\Delta \cap \Delta_s = \varnothing$.*
2. *Any state realized by a normal situation is a possible state, and hence every normal situation is a possible situation.*
3. *If the information context is sound, then every realized state is a possible state and every situation is a possible situation.*
4. *If the information context is complete, then every possible state is realized by some normal situation.*

In connection with (3), note that if the context is *not* sound, then it may happen that some situations are impossible, that is, have a state that is not possible relative to the given context.

Example 4.9 In the Lewis information context on W of Example 4.5 the possible states are each uniquely determined by a single possible world in that for each possible state ω there is a world w such that ω is the partition $\langle \Gamma_w, \Delta_w \rangle$ where Γ_w contains those sets that do contain w and Δ_w contains those that do not. Conversely, every world $w \in W$ determines such a possible state.

Example 4.10 In the first-order context, the possible states are determined by consistent, complete, first-order theories.

4.3 Changing information contexts We want to explore two ways in which information contexts can change that have to do with topics discussed in the preceding section. There are many more that we will not take up here. We begin with some general considerations.

Definition 4.11 The partial ordering $\sqsubseteq$ on information contexts C_1 and C_2 on a fixed classification A is defined by: $C_1 \sqsubseteq C_2$ if and only if

1. for all sets Γ, Δ of situation types, $\Gamma \vdash_{C_1} \Delta$ entails $\Gamma \vdash_{C_2} \Delta$ and
2. every situation of A that is normal in C_2 is normal in C_1.

It is important to notice that the inclusions go in opposite directions. As one goes down in the ordering, the less information one has, but the more normal situations one has. We write $C_1 \sqsubset C_2$, which is read "C_1 is less informative than C_2", if $C_1 \sqsubseteq C_2$ and $C_1 \neq C_2$. The following result is not difficult but is important for our purposes here.

Theorem 4.12 *The information contexts on a given classification form a complete lattice under the ordering* $\sqsubseteq$. *If* $C_1 \sqsubseteq C_2$ *then* $\Omega_{C_2} \subseteq \Omega_{C_1}$.

Proof: The first statement is proved in [3]. The second is obvious from the definitions. □

This allows us to combine and move between information contexts representing different background assumptions. It also allows one to adjust an information context so as to take into account newly discovered impossibilities, or to admit a new situation as normal.

4.3.1 Information from impossibilities As we have noted, one way to get new information is to eliminate possibilities, to discover somehow or other that they are in fact not the case. We can use the above results to show how this gives rise to a new information context.

Let C be an information context and let Ω_0 be some nonempty set of possible states not realized by any normal situation. Being possible means they do not conflict with the available information. Being unrealized by any normal situation means that they are candidates for being eliminated through some form of inquiry. Let us imagine that some inquiry has in fact shown these states to be unrealized by any normal situation. How should we increase the information in C to account for this?

By eliminating a given state $I = \langle \Gamma, \Delta \rangle$, we know that no normal situation realizes I, hence, any normal situation that satisfies every type $\alpha \in \Gamma$ satisfies some type $\beta \in \Delta$. In other words, we should be able to add I as a new piece of information. Of course this new piece of information may interact with information already available, especially through the global cut closure condition. What we end up with, by countenancing all the states in Ω_0 as impossible, is the least information context C' such that every state in Ω_0 is a piece of information in C'. This amounts to closing the old information, plus Ω_0, under global cut, since identity and weakening come along for free. This information context has the same normal situations as the original (that is why we insisted that the states in Ω_0 should not be realized by any normal situation) but additional information. In particular, $C \sqsubset C'$.

4.3.2 Throwing away misinformation We now want to show that if we are in a given information context and find ourselves faced with an impossible situation, there is a canonical way to adjust the available information so as to admit the new situation as normal (and hence possible, of course).

Definition 4.13 Let C be an information context and let s be any situation of C. We define $C[s]$ to be the greatest lower bound of those information contexts $C' \sqsubseteq C$ such that s is normal in C'.

It is not true that the greatest lower bound of information contexts all with some property necessarily has that property, so we need to show that s is indeed a normal situation in the context $C[s]$.

Theorem 4.14 *Let C be an information context and let s be any situation of C.*

1. *$C[s]$ is the largest context $C' \sqsubseteq C$ such that s is normal in C'.*
2. *The constraints of $C[s]$ are those constraints of C satisfied by s.*
3. *The normal situations of $C[s]$ consist of those of C together with the situation s.*

Proof: Let A be the classification part of C. Define an entailment relation on the types of A by $\Gamma \vdash_0 \Delta$ if and only if $\Gamma \vdash_C \Delta$ and s satisfies $\langle \Gamma, \Delta \rangle$. Similarly, define N_0 to be $N_C \cup \{s\}$. We will show that $C_0 = \langle A, \vdash_0, N_0 \rangle$ is an information context. From this it is clear that $C[s] = C_0$ and so our theorem is an immediate consequence. It is clear from the definition of C_0 that all $s \in N_0$ satisfy all constraints of C_0. What we need to verify is that the consequence relation $\vdash_0$ satisfies identity, weakening, and global cut. This follows from Proposition 4.3. □

4.4 Modal logic Finally, we want to discuss modal logic within the current framework. The general perspective on inquiry suggested by informationalism calls out for a more dynamic approach than the traditional one, one where we could model the kinds of inquiry involved in our multiplication example, as well as account for shifts in issues and in what counts as available information. This is not the line pursued here. Here we take up the less exciting but still worthwhile task of casting traditional modal logic within the current perspective.

Given a classification A, we follow Stalnaker and define a *proposition* to consist of a set p of states. We say that p is true in a situation s if the state of s is in p; otherwise we say that p is false in s. In this way, each classification A gives rise to an associated Boolean classification Boole(A), where situations are classified by the propositions they make true. The Boolean operations are intersection (for conjunction), union (for disjunction) and complement (for negation). The Boolean classification Boole(A) is studied in [3] starting in section 7.3, but we will not need any of those results here.

Implicit in our discussion has been the idea that a proposition p of Boole(A) is possible (relative to the available information) provided that some state in p is possible (relative to that information). But what about the associated modal proposition *that p is possible*, usually denoted by $\Diamond p$? What set of states models this proposition?

In order to find for each proposition p an associated modal proposition $\Diamond p$ with the desired content, additional assumptions are needed. On the one hand, a situation s should make $\Diamond p$ true if and only if the state of s is in $\Diamond p$. Consequently, the possibility of p in s can depend only on the state of s. On the other hand, the claim that p is possible is equivalent to the claim that some state in p is possible relative to the available information. Thus the possibility, relative to the situation s, of any state in p must depend only on the state of s. This leads us to the following definition.

Definition 4.15 An *informational modal framework* $\mathcal{M}$ ("information frame" for short) consists of a classification A and, for each situation s of A, an information context C_s on A, called *the context of* s, satisfying the condition that if s and s' have the same state, then C_s and $C_{s'}$ have the same information. The situations that are normal in C_s are called *s-normal*. The states that are possible relative to C_s are called s-possible.

Example 4.16 Let $\langle W, R\rangle$ be a Kripke frame, that is, W is a set of possible worlds and R is a binary "accessibility" relation on W. Such a frame naturally gives rise to an information frame. For the classification, we use the Lewis classification on A. Given $w \in W$, the information context C_w is a modification of that given in Example 4.5. It has as normal situations the worlds w' R-accessible from w (i.e., wRw') and has information given by: $\Gamma \vdash_w \Delta$ if and only if for every world w' R-accessible from w, if $w' \in \bigcap \Gamma$ then $w' \in \bigcup \Delta$. Under the identification of possible states and worlds from Example 4.9, states that are possible in C_w are identified with the worlds accessible from w.

This example shows that our notion of information frame is as general as that of Kripke frame. (It is actually more general, as we will see below.) However, from the point of view of the philosophical picture of possibility presented here, there is a fairly high overhead in these assumptions, and so in the Kripke frame model. First, it requires that we are working in a single classification, so that there is a single way of classifying things that makes sense in all information contexts. (In terms of the account in [3], this seems quite restrictive.) Second, it requires that the information available in a given situation is completely determined by the state of the situation relative to the classification scheme at hand. This has the effect of imposing a rich classificatory scheme on the situations.

Still, as long as we are modeling a single inquiry, these assumptions are perhaps not so stringent. In any case, given them we can develop modal logic. Given an information frame $\mathcal{M}$, we first reconstruct the familiar notion of what it means for one state to be accessible from another.

Definition 4.17 Let $\mathcal{M}$ be a fixed information frame.

1. A state ω' is *accessible* from state ω provided that for some situation s with state ω, ω' is s-possible. (It follows that ω' is s-possible for *every* s with state ω.)
2. Given a proposition p, let

$$\Diamond p \;=\; \{\omega \in \Omega \mid \text{ some state } \omega' \in p \text{ is accessible from } \omega\}$$

and

$$\Box p = \neg\Diamond\neg p.$$

Let's check that these definitions behave as expected.

Proposition 4.18 *Let $\mathcal{M}$ be an information frame. For any situation s and proposition p:*

1. *$s \models \Diamond p$ if and only if there is an s-possible state ω such that $\omega \in p$;*
2. *$s \models \Box p$ if and only if every s-possible state is in p;*
3. *if the information context of s is complete, then $s \models \Diamond p$ if and only if there is an s-normal situation s' such that $s' \models p$ and $s \models \Box p$ if and only if for every s-normal situation s', $s' \models p$.*

Proof: Let $\omega_0 = \text{state}(s)$. To prove (1), first assume that $s \models \Diamond p$, that is, that $\omega_0 \in \Diamond p$. By the definition of $\Diamond p$, there is a state $\omega \in p$ accessible from ω_0. But then ω is possible relative to C_{s_0} for some situation s_0 whose state is ω_0. But then, by the defining condition of information frames, s and s_0 have the same available information, so ω must also be possible relative to C_s. To prove the converse, suppose that there is a state ω possible relative to the context C_s such that $\omega \in p$. Then ω is accessible relative to ω_0. Hence $\omega_0 \in \Diamond p$, so $s \models \Diamond p$, as desired. The proof of (2) is routine, given (1). The proof of (3) is immediate from (1) and (2) and the definition of complete information context. □

It would be interesting to understand the modal principles that are valid in all information frames. One way to start such an exploration is to examine the more familiar modal axioms and rules of inference to see what conditions on information frames are needed to insure their validity. As a first step in this direction, we present the following simple results. We use $p \to q$ as shorthand for $\neg p \vee q$. We say that a proposition p is *valid in* $\mathcal{M}$ and write $\mathcal{M} \models p$ if and only if $s \models p$ for every situation s of $\mathcal{M}$.

Proposition 4.19 *Let $\mathcal{M}$ be an information frame. For all propositions p, q, the proposition*

$$\Box(p \to q) \to (\Box p \to \Box q)$$

is valid.

Proof: The proof is straightforward from Proposition 4.18. □

By contrast with the above result, note that the seemingly weak familiar axiom $\Box p \to p$ is not universally valid. For example, it may be legally necessary for no one to kill another person but it is not true in every situation, only in those that honor the available legal information. So let us examine the conditions under which $\Box p \to p$ holds. While we are at it, we will also examine the related axiom $\Box p \to \Diamond p$.

An information frame is $\mathcal{M}$ *sound* (*complete, consistent*) if each of the information contexts C_s of $\mathcal{M}$ is sound (complete, or consistent, respectively). The information frame corresponding to any Kripke frame (Example 4.16) is clearly sound, complete, and consistent.

Proposition 4.20 *Let $\mathcal{M}$ be an information frame.*

1. *For every proposition p, the propositions $p \to \Diamond p$ and $\Box p \to p$ hold in every situation s that is s-possible.*
2. *If $\mathcal{M}$ is sound then for every proposition p, the propositions $p \to \Diamond p$ and $\Box p \to p$ are valid in $\mathcal{M}$.*
3. *C_s is consistent if and only if for every proposition p, the proposition $\Box p \to \Diamond p$ holds in s.*
4. *If $\mathcal{M}$ is consistent then $\Box p \to \Diamond p$ is valid in $\mathcal{M}$.*

Proof: To prove (1), assume that s is s-possible. To show that s satisfies $\Box p \rightarrow p$ we need to show that if the state ω of s is in $\Box p$ then it is in p. So assume $\omega \in \Box p$. By Proposition 4.18, every state possible relative to the context C_s is in p. But by assumption, ω is possible relative to the context C_s. The other part of (1) is similar. (2) follows immediately from (1). The proof of the left to right half of (3) is similar to that of (1). Let us prove the other direction. Assume C_s is not consistent. Then there are no s-possible states, so for every proposition p, $s \models \Box p$. But if there are no s-possible states, $s \not\models \Diamond p$. (4) follows from (3). □

We now turn our attention to the familiar rule of necessitation: from the *validity* of p infer the validity of $\Box p$. We say that necessitation *preserves validity* in an information frame $\mathcal{M}$ if for all propositions p, if $\mathcal{M} \models p$ then $\mathcal{M} \models \Box p$. It is easy to see that there are information frames where necessitation does not preserve validity. All one needs is a framework with a single situation whose logic is not complete. Since every Kripke frame validates the rule of necessitation, this observation shows that not all information frames correspond to Kripke fames. To put it differently, the approach to modal logic through information frames is a strict generalization of that using Kripke frames. The next result establishes a relationship between necessitation and complete information frames.

Proposition 4.21 *Let $\mathcal{M}$ be an information frame. If $\mathcal{M}$ is complete, then the rule of necessitation preserves validity in $\mathcal{M}$.*

Proof: Assume that p is valid in $\mathcal{M}$ and let s be any situation. We need to show that $s \models \Box p$. Let ω be any s-possible state. We need to see that $\omega \in p$. By completeness of C_s, there is a normal situation s' of C_s with $\text{state}(s') = \omega$. By the validity of p, $s' \models p$. Hence $\omega \in p$ as desired. □

As a consequence of these results we see that the familiar modal logic **K** is valid in all complete information frames. It is interesting to note that there is a partial converse to Proposition 4.21.

Proposition 4.22 *If the information frame $\mathcal{M}$ is sound and the rule of necessitation preserves validity in $\mathcal{M}$, then $\mathcal{M}$ is complete.*

Proof: Suppose $\mathcal{M}$ is sound but not complete. We want to find an instance of the rule of necessitation that is not valid in $\mathcal{M}$. Since $\mathcal{M}$ is not complete, there is a situation s and a sequent $I = \langle \Gamma, \Delta \rangle$ such that I is not information in C_s but I is satisfied by every s-normal situation. By the closure of information under global cut, there is a state $\omega = \langle \Gamma', \Delta' \rangle$ that is possible in s but not the state of any s-normal situation. Let $p = \{\omega\}$. By the soundness of the information context C_s, every situation is s-normal so $\neg p$ is valid in $\mathcal{M}$. However, $s \not\models \Box\neg p$, since ω is s-possible and is in p. □

To someone coming out of the standard modal logic tradition, the failure of the rule of necessitation might seem like a serious flaw. However, to a strict realist the failure of necessitation is just what the doctor ordered. To the strict realist, the rule of necessitation expresses a Humean view of necessity: any proposition that happens to hold of all situations is necessary. It does not seem that such a controversial claim should be part of logic.

This is perhaps as good a place as any to mention the relation of our notion of a normal situation to the idea of a normal world in Kripke's study (in [6]) of nonnormal modal logics. Kripke introduce his notion to, among other things, obtain completeness results for logics that did not obey necessitation. Still, the idea here is based on a different idea than Kripke's. This is evident from the fact that no necessity $\Box p$ is ever true in one of Kripke's nonnormal worlds, even if p is a tautological proposition, something that cannot happen in our setting.

We leave open the problem of characterizing those propositions that hold in all information frames. An even more intriguing line is the connections hinted at earlier between the informational approach to possibility and nonmonotonic logic. Chapter 19 of [3] can be seen as an exploration of this idea.

Acknowledgments Situation semantics arose, in part, from a need to avoid the Granularity Problem of possible worlds semantics, in part from a commitment to strict realism, and in part from trying to cope with the importance of context in semantics. The reader will note that these issues are all central to the present essay. Barbara Partee, in her 1985 essay [9] on *Situations and Attitudes*, challenged Perry and me to give a situation semantics account of the modalities. It has taken me this long to begin to sort out the various issues at stake in meeting this challenge. The central importance of the inverse relationship between possibilities and information only became clear to me in the course of working on [3] with Jerry Seligman (see especially Section 16.3). The invitation from the editors to write an article on impossible worlds prompted me to try to give some order to my then confused thoughts on these matters. Fred Dretske, David Israel, and Rob Koons gave insightful criticisms of earlier drafts, criticisms that resulted in very substantial changes and (one hopes) improvements.

NOTES

1. From a relativistic point of view, it is possible to have two events, each of which proceeds the other, from different perspectives. Thus to make what we say correct, we insist that each shoe tying precedes the other from a single perspective, mine.

2. It seems to me that part of what is happening in the phenomena usually called the "nonmonotonicity" of common sense reasoning is that various things can happen in an inquiry that cause the relevant set of issues to change. Something one thought was a given suddenly comes under question, so becomes a relevant issue.

3. We do not attempt to reduce these ways of resolving issues to anything else. However, in the mathematical model developed at the end of the paper, we model these ways by (something equivalent to) functions from issues to resolutions of issues.

4. This, like the shifting issues mentioned in note 2, is connected to defeasible reasoning and nonmonotonic logic.

5. We are not speaking here of regularities involving the words "sibling" and "child" but of regularites involving children and siblings.

6. The joke embodied in Figure 1 is borrowed from Sloman [11], p. 12, where it is used to make a different point.

7. The same result was established by Dunn and Hardegree independently but earlier in an unpublished manuscript.

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