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# Book Review

John P. Burgess and Gideon Rosen. *A Subject with No Object: Strategies for Nominalistic Interpretation of Mathematics.* Oxford University Press, Oxford, 1997. x + 259 pages.

***1 Introduction*** A *nominalist* is a philosopher who holds that abstract objects do not exist. The *realist* opponent retorts "Oh yes they do," and the debate is off and running. In recent decades, a number of these controversies concentrated on *mathematical* objects, typically numbers and sets, assuming that those are abstract objects par excellence. If nominalists are correct, then mathematics has no (existent) subject matter. The main title of this lively, engaging, and insightful book thus describes what mathematics would be if nominalism were correct, although the authors have virtually no sympathy for nominalism. The subtitle accurately describes the contents of this study.

The book has three parts. The 92 pages of Part I provide an introduction to contemporary nominalism and lay out a "common framework" for presenting various nominalistic strategies. Part II, at 72 pages, provides some detail of three such projects: a "geometric strategy" based on (and improving) Field's *Science Without Numbers* [8], a "purely modal strategy" modeled after Chihara's *Constructibility and Mathematical Existence* [7], and a "mixed modal strategy" that follows Hellman's *Mathematics Without Numbers* [11]. The first chapter of Part III gives very brief sketches of some other "miscellaneous" nominalistic approaches, and the second chapter provides an even briefer account of how the various strategies relate to the work of nominalists in the philosophical literature. For the most part, the discussion is limited to book length (or equivalent) nominalistic projects. The book closes with a 40-page "Conclusion" although the authors remark that it should be entitled "In Lieu of Conclusion." Despite this modesty, the main sections of the chapter contain sharp and penetrating criticisms of the nominalistic projects and of the whole point of nominalism. It is about as "conclusive" as polite, professional philosophy gets nowadays. For the most part, however, the criticisms are broadly aimed at the very idea of nominalistic reconstrual and do not directly address the detailed work of the nominalist

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camp, which contains some of the best contemporary philosophical minds. The final section of the book, "Envoi: reconstrual without nominalism," contains proposals concerning the value of nominalistic projects to anti-nominalists like themselves and even to those who do not much care either way about the existence of abstract objects.

***2 What is the issue?*** As advertised, the introduction provides an extensive overview of the topic of nominalism. An early item on the agenda is to say just what the fuss is all about—what makes an object "abstract"? Drawing on Lewis ([12] §1.7), the authors consider a number of attempts to delineate the abstract/concrete boundary. One is that abstract objects are obtained by a process of *abstraction* from concrete entities, supposedly producing old-fashioned universals and more contemporary linguistic and psychological types. So abstract = abstracted. A second approach is to think of an abstract object as a one-over-many, focusing on the distinction between an individual and a class or universal. A third approach is the "way of negation," where one says something about what an abstract object is *not*: abstract objects are not located in space or time, and they do not enter into causal relations either with each other or with concrete objects. The authors express skepticism toward each of these approaches and, following Lewis, they point out serious conflicts between the approaches. Items that qualify as concrete on one approach end up abstract on another. Burgess and Rosen settle on what is certainly the most common approach, "the way of example." One just gives a short list—say numbers, sets, universals, and types—and declares those (if such there be) and similar objects to be abstract. By default, objects which are not similar to those on the list are concrete. Burgess and Rosen suggest that the abstract/concrete dichotomy be replaced with a continuum, with highly abstract objects like high-ranking members of the set-theoretic hierarchy at one end, highly concrete objects like sidewalks at the other, and things like species, books (as opposed to copies of books), theories, and symphonies in between. For the purposes of this book, the boundaries of the abstract can be left open, since the main concern is with *mathematical* objects, like numbers, functions, and sets. These things end up on almost everyone's list of paradigm abstract objects (Maddy [13], notwithstanding). For some nominalists, the relevant features of mathematical objects are the ones highlighted by the way of negation: they are not located in space and time, they do not have causal relations with anything, and their existence (if they exist) is not contingent.

The next issue is the motivation for nominalism, and anti-nominalism for that matter. Why should one refuse to believe in abstract objects, and why should one refuse the refusal? To initiate the discussion, the authors describe a stereotypical nominalist and a stereotypical anti-nominalist. The former focuses on the epistemic difficulties with abstract objects. She claims that it is a mystery how human beings, as physical organisms in a physical universe, can have knowledge of the eternal, detached, acausal mathematical realm. The stereotypical nominalist argues that since there are no causal connections between mathematical entities and ourselves—the human knowers—then her opponent, the anti-nominalist, cannot account for mathematical knowledge without postulating some mystical abilities to grasp the mathematical universe. Burgess and Rosen point out that a crucial link in this argument is the "causal theory of knowledge," a general thesis that we cannot know anything about some objects unless we have a causal connection with at least samples of the objects. Burgess and Rosen point out that in the philosophical literature, the causal theory was developed to solve some epistemological puzzles unrelated to mathematical knowledge and, in some cases, the formulation of the theory presupposes mathematical knowledge. The epistemological goal was to show how what untutored common sense takes to be knowledge really is knowledge. The stereotypical nominalist imposes a general causal constraint in order to engender skepticism toward what untutored common sense takes to be knowledge (of mathematics). But neither the literature on causal theories nor on nominalism contains an argument in favor of a general causal constraint for knowledge, explaining and defending just where and what type of causal relations are required for knowledge. Burgess and Rosen go through several possible arguments in favor of a strong, general causal constraint, and they find each such argument wanting. In place of the relevant argument, the stereotypical nominalist, along with many of her real allies, shifts the burden to the anti-nominalist to provide an acceptable epistemology for mathematical objects.

What of the stereotypical anti-nominalist? He is a naturalized epistemologist, rejecting first philosophy and holding that science gives us our best line on knowledge. If mathematics is used in science, then mathematics is true and mathematical entities exist. Burgess and Rosen sum up the stereotype as follows.

> We [anti-nominalists] come to philosophy believers in a large variety of mathematical and scientific theories—not to mention many deliverances of common sense—that are up to their ears in suppositions about entities nothing like concrete bodies we can see or touch, from numbers to functions to sets, from species to genera to phyla, from shapes to books to languages, from games to corporations to universities. (p. 34)

The stereotypical anti-nominalist thus shifts the burden over to the nominalist. And this anti-nominalist will not accept appeals to philosophical intuition or to some "generalizations from what holds for the entities with which we are most familiar to what must hold for any entity whatsoever." No sir. The stereotypical anti-nominalist will only accept *scientific* reasons against the existence of abstract objects.

So at the level of stereotypes, each side claims the high ground and puts the burden on the opposition, and each side suggests that the burden of proof is so incredibly tough that it cannot be met—an interesting standoff.

Note that sometimes explanations—even causal explanations—of physical phenomena involve mathematical facts. For example, an explanation of why rain forms into drops might invoke surface tension and the fact that a sphere is the largest volume that can be enclosed with a given surface. An explanation of why a package of 173 tiles will not cover a rectangular area (unless it is one tile wide) might mention the fact that 173 is a prime number. If we are to know the explanation, then we must know the constituent mathematical facts. Whatever the stereotypical characters hold, the real nominalists considered in the book take *this* burden seriously.

One popular and influential argument for the existence of mathematical objects is the Quine-Putnam indispensability consideration (the standard formulation of which is in Putnam [15]). Since science is full of mathematics, and since we are to accept the science as true, then we must accept the mathematics as true as well, or at least the mathematics that finds application in science. In effect, this mathematics is part of natural science, our best account of reality. If taken at face value, the truth of mathematics requires the existence of mathematical objects. So mathematical objects exist.

The indispensability argument thus ties mathematical knowledge to the empirical success of science. Thus, the argument runs against a traditional (anti-nominalist) view that mathematical objects exist of necessity and mathematical truths are knowable a priori. Tennant ([19], p. 309), for example, claims that considerations about the role of mathematics in science are "not . . . strictly relevant to the philosophical problem of the existence of numbers." Against the traditional view, the Quine-Putnam indispensability argument is of a piece with the naturalist's thesis that science gives us our best line on what does and what does not exist.

Field [8] concedes that the Quine-Putnam argument is a powerful consideration. He goes further and claims that it is the *only* weighty argument for the existence of mathematical objects. To be precise, he holds that other arguments have weight only if that one succeeds. So he, like the other nominalists canvassed in the present book, sets out to undermine the indispensability argument by showing how to recapture at least part of science without invoking the existence of mathematical entities. The idea is that science *could* proceed without mathematics, even if scientists find it inconvenient to work that way. If one of the nominalistic programs succeeds, then the presence of mathematics within science does not support the existence of mathematical objects since the science could be done (albeit not as conveniently) without the mathematics. Mathematics would be dispensable in principle.

Burgess and Rosen note that many anti-nominalists concede that undermining the indispensability argument is sufficient to establish nominalism. That is, parties to the debate accept the biconditional: we are warranted in belief in mathematical objects if *and only if* mathematics is indispensable for science. This is to concede that (1) it is the anti-nominalist that has the initial burden of proof and (2) to hold, with Field, that the indispensability argument is really the only one to take seriously (Tennant [19] notwithstanding). Undermining that one argument puts the issue back at the default state, where the anti-nominalist has the unbearable burden. Burgess and Rosen suggest that the anti-nominalist is not warranted in agreeing to the biconditional, in part because the issue of burden of proof should not be decided so quickly.

***3 Major strategies*** Part II of the book provides details of three nominalistic attempts to show that mathematics is, after all, not indispensable for science. In each case, Burgess and Rosen attempt to provide enough detail for the properly prepared reader to see how the program goes, without getting lost in the technical details. They carry out this delicate balancing act well. The book's Introduction claims that the bulk of the treatment requires only knowledge of "introductory-level logic," just enough to be able to "read formulas written in symbolic notation." The few sections which delve into more technical logical matters are considered "optional semi-technical appendices." However, in several places, the main development proceeds rather quickly. I would not recommend the technical aspects of Part II to anyone with less than a full year of graduate level logic and perhaps some post-calculus mathematics as well.

In each of the three cases considered, Burgess and Rosen pay careful attention to the nominalistic items involved in the reconstruction and to the logical resources used therein. This provides the diligent reader the wherewithal to see just what is being traded for what and what is being presupposed by each side to the debate.

The program surveyed in Chapter II.A attempts to eliminate mathematical objects in favor of geometric objects like points, lines, and perhaps regions of space or space-time. In contemporary foundations of mathematics, of course, geometrical objects are mathematical. In fact, they are taken to be set-theoretic constructions on real numbers. Points are triples or quadruples of reals; lengths and other magnitudes are real numbers via an arbitrary unit. For the most part, geometry is taken to be an algebraic theory that applies to any structure that meets certain axioms. It is as "pure" or abstract as mathematics gets, pursuing the properties of structures independent of their applications. It is perhaps for this reason that some mathematicians and scientists claim that even if *numbers* have been eliminated from the theories developed in Field [8], the *mathematics* is still there, in the geometry (see Shapiro [18], pp. 75–77). However, these mathematicians and scientists are not much interested in philosophical matters of ontology. The geometric nominalist reverses the contemporary trend, taking geometry to be the theory of *physical space* (or space-time). Following techniques dating at least to Descartes, our nominalist replaces real numbers with ratios of geometric magnitudes like lengths, areas, and volumes. Surrogates to derivatives, integrals, and other mathematical constructs are developed, or at least sketched. The proposed nominalistic science would be a physics based on the synthetic geometry of Euclid instead of the now more common analytic geometry.

Field's program has two parts. The first is to develop a synthetic version of Newtonian gravitational theory, as an example of the nominalistic technique. The presentation in Field [8] invokes mereology, which for present purposes is a second-order logic. However, in later work Field [9] comes close to an exclusive use of first-order presentations, sometimes with non-first-order quantifiers such "there exist finitely many." Burgess and Rosen show how to accomplish the reconstrual without going beyond first-order logic.

The second part of Field's program is to show that adding standard mathematics to the nominalistic theory does not allow one to establish any new results in the synthetic, nominalistic language. In other words, the nominalistic theory with mathematics and bridge principles is a conservative extension of the nominalistic theory. Thus, the mathematics may be terribly convenient for the scientist, but it is dispensable in principle. A nominalist is free to use mathematics to his heart's content, with the assurance that any concrete results could have been obtained without invoking mathematics—assuming, of course, that the conservativeness result is obtained by nominalistically sound means.

Of particular value is Section 1 of this chapter, which puts the techniques underlying this nominalistic program in a historical context (see also Burgess [6]). The nominalist must argue that geometrical items, such as points of space-time, are *concrete*, and not abstract entities. Following the "way of example", this depends on how close geometric entities are to paradigm abstract objects like numbers and sets. Are points and regions located in space and time? Well, points and regions *are* locations in space and time. What of causal relations? Field argues that in contemporary field theories, properties of space-time points figure in explanations of physical phenomena. Thus, Field comes down with Newton, on the side of so-called substantival views of physics that regard space-time itself as physically real, and against Leibniz's "relationalism" which dispenses with space-time in favor of (mathematically defined) relations among possible physical objects. Contemporary physics does seem to favor substantivalism, but there is not much more than a hint that the nominalistic reconstruction can be carried out for those theories. Burgess and Rosen point out the irony:

> The main dilemma for geometric nominalism is this: the case for accepting geometric entities as concrete draws on realistic, contemporary, twentieth-century physics; but the most elegant elimination of numerical entities in favour of such geometric entities can be carried out only for unrealistic, classical, nineteenth century physics. It remains an open question how attractive a nominalistic alternative to up-to-date physics can be developed. (p. 98)

Some obstacles to extending Field's geometric treatment to contemporary physics are raised in Malament [14] (a rare omission from the extensive bibliography of the present book).

Chapter II.B considers a nominalistic program that invokes a modal notion, such as "there possibly exists" or "one can construct", but otherwise sticks to ordinary, first-order logic. Thus, it is called a "purely modal" strategy. The bulk of the chapter is devoted to a careful discussion of the resources of the modal language via an insightful comparison of modal with temporal discourse, noting what ontological commitments are invoked in each case. Another gem of this book is a careful analysis of what sort of modal logic is needed for this program.

The idea behind the program is to replace statements like "the mass of this rock is $r$ grams" (where $r$ is a real number) with something like "there could be a token for a numeral that marks how massive this rock is". Purely mathematical statements get reinterpreted as complex statements about what sizes get marked by various possible numerals. For example, a sentence like $\forall x \exists y (x < y)$ would get rendered something like: for every possible real-number-numeral-token, there could be another such token that marks a larger quantity.

The ideology for this program thus involves not just modal logic, but also semantical notions like denotation, satisfaction, and truth. Perhaps the nominalist can maintain that ordinary scientific language, as it stands, has numerals (or possible numeral-tokens) for every natural number and every rational number, but one cannot seriously maintain that ordinary scientific language has enough numerals to denote every real number. There are too many of them. Yet reals, not to mention functions on the real numbers, are standard items in mathematical physics. So if something in the neighborhood of real analysis is to be reconstructed, the pure modal program must invoke possible *extensions* of ordinary scientific language to include new singular terms. Our nominalist also requires a notion of satisfaction for formulas in these extended languages. Thus, the semantic notions needed for this nominalism are not internal to any fixed language, and so they are what Quine calls "transcendent." Nominalists who pursue this pure modal strategy cannot rest content with the language-bound "immanent" conception of truth (and satisfaction) favored by Quine. Strange as it may sound, this brand of nominalism is incompatible with deflationism (and so is not attractive to Field, a leading deflationist). Robust semantic notions are part of the price to be paid for eliminating abstract objects via the purely modal route.

The third of the major nominalistic programs is a “mixed modal strategy,” adopting what I call a “modal eliminative structuralism” (Shapiro [18], Chapters 3 and 4). Instead of reference to the real numbers as *sui generis* objects, the structuralist speaks of the “real number structure,” the form common to any complete, ordered field. Real numbers—places in the real number structure—are offices, rather than office holders. A structure is a one-over-many, like a traditional universal: structure is to structured as universal is to particular. One approach to structuralism is that the real number structure exists independent of any systems that may exemplify it (see Shapiro [18], Chapter 3). I call this *ante rem* structuralism, after the analogous view concerning universals. This orientation, of course, is not available to a nominalist, since it reifies structures. *Ante rem* structures are just as “abstract” as real numbers, perhaps more. A second orientation is to think of structures as somehow constituted out of systems that exemplify the structure. Statements in real analysis are construed as statements about what holds in any instance of the real number structure. For example, $\forall x \exists y (x < y)$ comes “in any complete ordered field $O$, for every element of $O$, there is a greater element of $O$”. This eliminative, *in re* structuralism is articulated and pursued in the Benacerraf’s classic [2]. It is a structuralism without structures.

According to the eliminative structuralism, real analysis is vacuous if there are no complete ordered fields. If there are no such fields, then $\forall x \exists y (x < y)$ comes out true, but so does $\neg \forall x \exists y (x < y)$, since that sentence, too, would hold in every complete ordered field. Every complete ordered field is the size of the continuum. Clearly, a nominalist cannot postulate the existence of continuum-many *abstract* objects without conceding utter defeat. So how can the structuralist-nominalist save real analysis from vacuity? One option, perhaps, would be to adopt the aforementioned geometric strategy and argue for the existence of a continuum of *concrete* space-time points, or perhaps follow Maddy [13] and argue that there are a lot of concrete sets. A third option is to reformulate eliminative structuralism in modal terms. Accordingly, in real analysis, $\forall x \exists y (x < y)$ comes to “in any *possible* complete ordered field $O$, for every element of $O$, there is a greater element of $O$”. Thus, modal structuralists need not establish the existence of a complete ordered field. They avoid vacuity by maintaining that a complete ordered field is *possible*.

What is the nature of this modality? Most nominalists who pursue these strategies prefer to speak of *logical possibilities* and *logical necessities*. However, on contemporary treatments, these modal notions are understood in terms of *abstract* objects via model theory or proof theory. A sentence $\Phi$ is logically possible if there is a model that satisfies $\Phi$, or if there is no deduction that refutes $\Phi$. To belabor the obvious, nominalists do not have the option of explicating modal notions in such blatant mathematical terms. Instead, they take the modalities as primitive, not to be understood in terms of anything more basic or at least not in terms of abstract objects like models and proofs. There is thus a question concerning the extent to which our nominalists are entitled to the hard won results of mathematical logic.

The eliminative structuralist also requires the wherewithal to characterize the relevant structures up to isomorphism. To formulate the counterparts of statements like $\forall x \exists y (x < y)$, the eliminative structuralist must be able to say just what a complete ordered field is. The completeness and Löwenheim-Skolem theorems indicate that no first-order theory can do this (see Shapiro [16], Chapter 4). The "mixed" part of the nominalistic "mixed modal strategy" refers to logical resources that go beyond first-order logic. Two common options are to invoke mereology, which may be called the "complete logic of the part-whole relation," and to invoke the Boolos ([4], [5]) interpretation of monadic, second-order quantifiers as plural quantifiers (which Burgess and Rosen call "plethynicology"). The book provides a good sketch of the philosophical issue concerning the extent to which these notions are (or should be) acceptable to a nominalist.

As noted above, Burgess and Rosen's stereotypical nominalist pooh-poohs abstract objects on the ground that human beings, as physical entities, cannot have knowledge of causally inert abstract objects. The nominalist says that her opponent cannot fulfil the burden of giving a naturalistic epistemology for mathematics. Notice, however, that the geometric nominalist has the burden of providing an epistemology for the highly abstract structure of space-time, and the modal nominalists have the burden of explaining our knowledge of the modal truths invoked in the program. How do human beings, as physical organisms in a physical universe, have such detailed knowledge of what is possible concerning such abstruse constructions as infinitary numerals and complete ordered fields? The mixed modal nominalist—the modal eliminative structuralist—has the further burden of explaining how human beings have knowledge of the higher-order logical truths invoked in the program. Might not these reconstructive nominalists have epistemological problems very similar to those attributed to the anti-nominalist (see Shapiro [17])?

***4 What to conclude?*** The next question concerns what is claimed on behalf of each reconstruction with respect to ordinary mathematical physics. Assuming that we have an acceptable physics in which everything is nominalistically kosher, what do we conclude? What is the ontologically clean theory to be used *for*? Burgess and Rosen propose two orientations for reconstructive nominalism. The first, *revolutionary* approach is to claim that the nominalistic theory is superior to, and so should *replace* standard mathematical physics. The second, *hermeneutic* approach is to claim that the reconstructed theory provides the underlying *meaning* of the original theory. So the nominalist claims that despite appearances, properly understood mathematical physics—as it stands—does not invoke mathematical objects.

There is a further division within revolutionary nominalism. A *first-philosophy approach* would be to claim that the nominalistic science should be preferred over the received mathematical science on a priori or metaphysical grounds that stand prior to the criteria used by scientists in selecting their theories. From this perspective, the nominalist claims that philosophical analysis reveals that abstract objects are pernicious, and so he admonishes scientific colleagues to conform to his scruples, on pain of some fault or other. Burgess and Rosen mention the first-philosophy orientation, but do not discuss it at any length and presumably do not take it very seriously. Given the focus on Quine-Putnam indispensability, contemporary nominalists also do not follow the first-philosophy route. As noted above, the "stereotypical" anti-nominalist will not even hear of this approach.

The *naturalistic* revolutionary nominalist argues that the reconstructed theories are superior to ordinary science on ordinary scientific grounds. They claim that there are good *scientific* reasons to prefer theories that eschew abstract objects, once those theories are available. Showing professional modesty, Burgess and Rosen remind the reader that philosophers are not the ones to adjudicate questions of scientific merit. In several places, they helpfully recommend that the issue be decided by having the reconstructive nominalists submit their work to a mainstream physics journal. We philosophers would then anxiously await the judgment of the scientific community, via the extent that the reconstructed theories get adopted and used by working physicists.

I presume that this is not a serious proposal. Suppose that a paper outlining Field [8], Hellman [11], or the like, or perhaps a paper that recapitulates some up-to-date physical theory, were submitted to a frontline physics research journal. The most diehard nominalist would admit that the best response would be a polite suggestion to try a philosophy outlet for this philosophical project. The serious point that underlies Burgess and Rosen's flippant suggestion is that scientists, *qua* scientists, are not much interested in eliminating reference to mathematical objects. And who but practicing scientists (including editors of professional scientific journals) are to determine what counts as *scientific* merit? And for a naturalist, what else counts as *merit*?

In a modest, tentative way Burgess and Rosen provide a list of criteria of scientific theory-choice that most observers have noted and accepted:

(i) correctness and accuracy of predictions,
(ii) precision, range, and breadth of predictions,
(iii) internal rigor and consistency,
(iv) minimality or economy of assumptions in various respects,
(v) consistency and coherence with familiar, established theories (or failing this, minimality of change),
(vi) perspicuity of the basic notions and assumptions, and
(vii) fruitfulness, or capacity for extension.

Burgess and Rosen conclude:

> . . . the reconstructive nominalist seems to be giving far more weight to factor (iv), economy, or more precisely, to a specific variety of thereof, economy of abstract ontology, than do working scientists. And the reconstructive nominalist seems to be giving far less weight to factors (v) and (vi), familiarity and perspicuity. (p. 210)

They include an illuminating discussion of the role of Occam's razor in the history of science and philosophy.

So much for the revolutionary orientation. What of the hermeneutic option, wherein the nominalist claims that his reconstruction provides the underlying *meaning* of actual scientific theories? The nominalist allows the scientist to go on using his familiar language, with its apparent reference to mathematical objects, for, she says, all that language really means is . . . —and here she fills in the details of the reconstrual. Let $P$ be a scientific statement that makes reference to abstract objects, such as some real numbers, and let $P'$ be a nominalistic reconstrual of $P$. The hermeneutic nominalist claims that $P$ and $P'$ have the same meaning, and so despite appearances, $P$ does not really make any reference to real numbers. Extending a point made in Alston [1], Burgess and Rosen point out that if $P$ and $P'$ do have the same meaning, then one is justified in the opposite conclusion: despite appearances, $P'$ *does* make reference to abstract objects, since it has the same meaning as $P$. After all, synonymy is a symmetric relation—if $P$ "really means" $P'$, then $P'$ "really means" $P$—and synonymous expressions share their ontological commitments. Wright [20] and Hale [10] make a similar claim in support of the existence of abstract objects (concerning certain "abstraction principles").

The hermeneutic nominalist thus goes beyond a claim of synonymy. He proposes an asymmetry between ordinary statements like $P$ and their nominalistic translations $P'$. He argues that the nominalistic $P'$ provides the deep structure or logical form of $P$, and not vice versa. This, however, is the sort of (empirical) claim that should be referred to experts, such as linguists. Since Burgess and Rosen are not experts in linguistics, they are modestly indefinite, but by reflecting on the methodology of scientific linguistics they provide considerations against these exegetical claims.

So Burgess and Rosen argue that neither the revolutionary approach nor the hermeneutic approach have much of a chance of success, so long as success is understood in scientific terms. And what other terms are available to the naturalistically-minded philosopher? I suspect that nominalists will accuse Burgess and Rosen of proposing a false dilemma, claiming that the revolutionary approach and the hermeneutic approach do not exhaust the options for the reconstructive program. I do not have a third orientation to propose on behalf of nominalism, and so we will leave the dialectic with a challenge for the nominalist to articulate just what he or she claims on behalf of the detailed reconstructive system.

***5 Why all this bother?*** Despite the authors' modesty, they mount substantial arguments that reconstructive nominalism has no real chance on any of the orientations that they take seriously. So the reader might wonder about all the effort that went into elaborating the reconstructive nominalistic programs. What is the point of providing so much detail on philosophical programs that, for all intents and purposes, are doomed from the outset? Along these lines, it is common for critics of Field [8], Hellman [11], and Chihara [7] to express admiration for the intellectual achievements in those books, while strenuously arguing against the philosophical conclusions. If nominalism is so badly false, then what is there to admire? Why should we care how far a scientist can go without mathematics? Are we like the perverse public in Kafka's "Hunger Artist" that comes from miles around to see how long a person can go without food?

A start on an answer to (all but the last of) these questions is provided in the final section of this book, which concerns the value of the reconstructive programs to anti- and non-nominalists. There were several places in the book where I found myself wanting a more extensive treatment and had to rest content with the authors' plea concerning space limitations. But nowhere as much as in the seven pages of this final section.

On a superficial level, the reconstruals represent hypotheses about how current scientific theories, replete with reference to abstract objects, could have been arrived at, and why we, or at least the non-nominalists among us, find them attractive. Whether one is a nominalist or not, it would be nice to have an explanation of how we (perhaps erroneously) came to accept theories that (seem to) entail the existence of abstract objects. At least part of the explanation is found when we see just how awkward and complex every existent nominalist reconstrual is. As Burgess and Rosen put it:

> . . . by their very awkwardness and inconvenience the nominalistic strategies make a real contribution to explaining why . . . linguistic transformations [to theories invoking abstracta] were practically unavoidable if science was to develop: they demonstrate as nothing else does just how much more convenient and perspicuous a numerical or otherwise abstract formulation can be. (p. 239)

In other words, the best efforts of dedicated, intelligent nominalists help demonstrate why current non-nominalist theories are preferable on established scientific grounds. I doubt whether nominalists themselves appreciate this consequence of their efforts.

Burgess and Rosen then sketch a more fundamental, and more intriguing, contribution to naturalized epistemology extracted from nominalistic reconstruals. Clearly, our best scientific theories are due in part to the nature of the nonhuman world and to the nature of human knowers. An ancient issue in philosophy concerns the role of each of these two factors in our theories. As Burgess and Rosen put it: "to what extent does the way we are, rather than the way the world of numerical and material and living entities is, shape our mathematical and physical and biological theories of the world?" (p. 240). Early modern rationalists came down on the "world" side of this dichotomy, holding that good theories say just how the world is, providing a God's eye view of reality. Since then, however, it is more common to hold that "our theories of life and matter and number are to a significant degree shaped by our character, and in particular by our history and our society and our culture."

In contemporary times, "postmodern" or "deconstructionist" anthropologists have gone to the opposite extreme, holding that every aspect of our scientific theories, and what those theories speak of—number and matter and life—are political constructs. *Everything* is on the human side of the dichotomy. Burgess and Rosen pause to provide a sharp attack on such views.

Among contemporary philosophers, a widely held approach, championed by Quine, Putnam, and Davidson, is to reject the dichotomy, arguing that there is no way to separate the "human" and the "world" contributions to our theorizing. There is no God's eye view of the world to be had. It is not as if we have an "objective" description of the world that we can compare to our scientific theories, in order to factor out the human component. Burgess and Rosen all-too-briefly sketch a way in which the nominalistic reconstruals can help get around this difficulty and shed some light on the respective human and world contributions to our theories. The idea is to focus attention on theories different from our own science:

> . . . one could hope to obtain some insight by producing a theory of the world that, though it no more than any other theory 'reflects' reality without the imposition of any 'conceptual scheme', does impose a different 'conceptual scheme' from that imposed by our actual scientific theories. Using the theory might be inconvenient or even unfeasible for us; but provided it would in principle be possible for intelligences unlike us and carrying different biological and social and psychological baggage from ours, comparison of the theory with our actual scientific theories would help give a sense of what and how much our character has contributed to shaping the latter. (pp. 242–43)

In the last two pages of the book, Burgess and Rosen propose that the batch of nominalist reconstruals play this role of providing alternative theories of the world, perhaps suitable for intelligences unlike ours. This justifies the level of detail provided in the book, for we do need to assess how well it is possible "in principle" to use the alternative theories.

This consideration seems related to a distinction Field ([8], e.g., p. 27) makes between "intrinsic" and "extrinsic" explanations and theories. In geometry, for example, an intrinsic theory or explanation makes reference only to the structure of space-time. In this case, an intrinsic theory is synthetic. Although Field would not put it this way, an extrinsic theory or explanation makes reference to structures, like that of the complex numbers, that are imposed on the intrinsic structures, in order to shed light on them. Field suggests that even the anti-nominalist should be interested in intrinsic explanations, when they are available. From the other side, the nominalist may not recognize how illuminating an extrinsic explanation can be.

Suppose, then, that there were some nominalistic reconstruals that succeed by their own lights. They score a tie with contemporary, up-to-date science on the criteria of correctness and accuracy of predictions; precision, range, and breadth of predictions; and internal rigor and consistency. Burgess and Rosen's conclusion would be that an intelligence much unlike ours could successfully approach and theorize about the world without invoking abstract objects. Thus, numbers and sets come from the "human" and not the "world" side of the mix that we call "science." An untutored philosopher might be tempted to conclude that this shows that numbers and sets do not really exist—they are not part of the ultimate furniture of the world. As naturalists, Burgess and Rosen might remind this philosopher that there is no difference between aspiring for an accurate catalogue of the ultimate furniture of the universe and aspiring for a God's eye view. Both aspirations are rejected as infeasible or perhaps unintelligible. The only criteria for existence we have comes from science—*our science*, the only science *we* have—and *that* science is riddled with reference to abstract objects. So be it, but if the nominalistic theory does score the indicated tie, the nominalist cannot help feeling a moral victory.

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Stewart Shapiro
*Department of Philosophy*
*The Ohio State University at Newark*
*Newark, Ohio 43055*
*email: shapiro+@osu.edu*