REMINISCENCES ABOUT PROFESSOR ANDRZEJ GRANAS

Lech Górniewicz

Dedicated to the memory of my teacher

Professor Andrzej Granas
(April 5, 1929 – March 5, 2019)

The death of Professor Andrzej Granas on March 5, 2019 in Warsaw brought sorrow and regret to a wide group of mathematicians and his former students. Professor Andrzej Granas was born on April 5, 1929 in Łódź. In 1947, he completed secondary education in the Liceum Ogólnokształcące im. Tadeusza Kościuszki w Łodzi (High School of Tadeusz Kościuszko in Łódź). In 1952, he graduated from mathematical studies at the University of Warsaw, his area of expertise, under the supervision of professor Karol Borsuk, was topology. Between 1952 and 1955, he completed his studies at the Lomonosov Moscow State University and in 1958, on the basis of his dissertation I.1 written under the direction of professor L. Lusternik, he received his Ph.D. (candidate) in mathematical sciences. In 1962, on the basis of the dissertation I.2, he got a degree of a habilitated doctor and in 1965 Andrzej Granas achieved the scientific title of full professor.

Professor Andrzej Granas had positions at both Polish and foreign universities, namely at:

- Instytut Matematyczny PAN w Warszawie (Institute of Mathematics of the Polish Academy of Sciences in Warsaw),
- Uniwersytet Mikołaja Kopernika w Toruniu (Nicolaus Copernicus University in Torun),
- Wyższa Szkoła Pedagogiczna w Gdańsku (Higher Pedagogical School in Gdańsk),
- Uniwersytet Warmińsko-Mazurski w Olsztynie (University of Warmia and Mazury in Olsztyn),
- Chicago State University,
- Collège de France,
- Université de Montréal (The University of Montreal).

He was a visiting professor invited by well-known centres in the USA, England, Germany, France, Russia, China, Japan, and Australia. While working in Gdańsk, he founded the Gdańsk section of the Polish Academy of Sciences, and in Toruń – the Juliusz P. Schauder Center for Nonlinear Studies as well as the journal Topological Methods in Nonlinear Analysis; in Montreal – the Journal of Fixed Point Theory and Applications. Both journals boast world-wide prestige and they are indexed in the ISI Master Journal List.

The lectures delivered by professor Granas were marked by very high quality, both scientifically and educationaly speaking. In Poland, Professor Granas pioneered algebraic topology, fixed point theory and nonlinear analysis. The scientific seminars organized by Professor Andrzej Granas concerned the issues being subject to the then main-stream global scientific investigation; moreover, the said seminars were highly esteemed by the Ph.D. students. Professor Andrzej Granas supervised Ph.D. theses of the following candidates (in the alphabetical order): H. Ben-El-Mechaiekh, C. Bowszyc, J.N. Corvellec, P. Deguire,

N. EL-Khaattabi, G. Fournier, M. Frigon, G. Gauthier, K. Gęba, L. Górniewicz, Z. Guennoun, C. Horvath, T. Kaczyński, W. Krawcewicz, and M. Lassonde. The development of Polish and Canadian mathematics was of special interest to Professor Granas. For this reason, he organized summer schools as well as conferences. Due to a high scientific status of Professor Andrzej Granas, the main lectures during the above-mentioned events were delivered by world specialists in their respective fields. It is worth noting that for all the schools and conferences organized in Montreal, Professor Granas invited a large number of Polish mathematicians. The above-mentioned activities of the Professor considerably contributed to development of topology and of nonlinear analysis in Poland, in particular in the centers of Toruń and Gdańsk.

Professor Andrzej Granas was a world-famous Polish mathematician. He dealt with topology, fixed points theory, nonlinear and convex analysis, as well as differential equations (see: the list of publications below). In all of the above-listed fields, he obtained some major results, well recognized in the world literature on the subjects. Among others, these include what follows: infinite-dimensional cohomologies, the Borsuk–Ulam theorem, the Lefschetz fixed-point theorem for single- and multi-valued mappings, the topological degree; he introduced the concept of topological essentiality and continuation method, specified important applications of topological methods to differential equations, non-linear analysis, minimax problems and to mathematical economics (see the list of publications below). The publication that earned him a world-wide recognition is the monograph I.5 (I.9), which has been hitherto cited 1000 times. The scientific accomplishments of Professor Granas have granted an unshakable and well-deserved reputation in the world literature.

What played a significant role in the Professor's life was classical music, in which he took a great interest in, simultaneously being a great pianist himself. Apart from mathematics and music, a major interest cherished by Andrzej Granas was playing chess. Finally, he also regarded tennis and playing bridge as a worthy sort of entertainment.

Professor Andrzej Granas was a scholar with broad interests, a great teacher who was very kind to his students. He made a very significant contribution to the world of mathematics. He deservedly earned his good reputation in both Polish and Canadian mathematics. The memory of him shall always be cherished by innumerable students and by a large number of mathematicians.

Professor Andrzej Granas loved Poland and confirmed it with his words and initiatives all his live.

Publications

I. Monographs, lecture notes and surveys

1. *Zastosowania twierdzenia o antypodach w topologii przestrzeni Banacha* (*The application of Borsuk–Ulam theorem in the topology of Banach's spaces*), a doctoral dissertation, Uniwersytet Łomonosowa w Moskwie, 1958 (in Russian).
2. *The theory of compact vector fields and some of its applicactions to topology of functional spaces*, Dissertationes Math. **30** (1962).
3. *Topics in Fixed Point Theory*, Sem. J. Leray, 1969/1970.
4. *Points fixes pour les applications compactnes*: *espaces de Lefschetz et la théorie de l'indice*, Press Univ. Montreal, 1980.
5. *Fixed Point Theory* I, Mathematical Monographs, Vol. 61, Polish. Sci. Publ., Warsaw, 1982, (with James Dugundji).
6. *Nonlinear boundary value problems for ordinary differential equations*, Dissertationes Math. **244** (1985) (with R.B. Guenther and J.W. Lee).
7. *Sur quelques méthodes topologiques en analyse convexe*, Press Univ. Montreal, 1990.
8. *Topological methods in differential equations and inclusions*, Kluwer Academic Publishers, Series C: Mathematical and Physical Sciences, Vol. 472, 1995 (A. Granas and M. Frigon, eds.).
9. *Fixed Point Theory*, Springer Monographs in Mathematics, Springer–Verlag, New York, 2003 (with James Dugundji).

II. Articles

(A) Results related to Brouwer, Borsuk and Banach, Schauder theorems

1. *Some theorems of the sweeping in Banach spaces*, Bull. Acad. Polon. Sci. **7** (1959), (with K. Gęba and A. Jankowski).
2. *Weakly contractive maps and elementary domain invariance theorem*, Bull. Greek Math. Soc. **19** (1978), (with J. Dugundji).
3. *A proof of Borsuk antipodal theorem for Fredholm maps*, J. Math. Anal. Appl. **96** (1983), (with K. Gęba).
4. *A proof of Borsuk antipodal theorem*, J. Math. Anal. Appl. **96** (1983), (with K. Gęba).
5. *On the Leray–Schauder alternative*, Topol. Methods Nonlinear Anal. **2** (1993).
6. *Continuation methods for contractive maps*, Topol. Methods Nonlinear Anal. **3** (1994).
7. *Résultats de type Leray–Schauder pour des contractions sur des espaces de Fréchet*, Ann. Sci. Math. Quebec **22** (1998).

8. *The order theoretic Cantor Theorem*, Taiwanese J. Math. **4** (2000), (with C. Horwath).

(B) Nonlinear equations

1. *On a certain class of non-linear mappings in Banach spaces*, Bull. Acad. Polon. Sci. **5** (1957).
2. *On continuous mappings of open sets in Banach spaces*, Bull. Acad. Polon. Sci. **6** (1958), (in Russian).
3. *Homotopy extension theorem in Banach spaces and some of its applications to the theory of nonlinear equations*, Bull. Acad. Polon. Sci. **7** (1959).
4. *Sur le multiplication cohomotopique dans les espace de Banach*, C.R. Acad. Sci. Paris **254** (1962).
5. *Quelques théorèmes concernant la stabilité des solutions de certaines équations non linéaires*, C.R. Acad. Sci. Paris Sér. **260** (1965).
6. *Homotopie et équations non linéaires dans les espaces de Banach*, C.R. Acad. Sci. Paris Sér. **300** (1985), (with K. Gęba, T. Kaczyński and W. Krawcewicz).

(C) Topological degree and fixed point index theory

1. *The Lefschetz fixed point theorem for some general class of maps*, (with G. Gauthier).
2. *Algebraic topology in normed spaces*, I–V, Bull. Acad. Polon. Sci. **13** (1965); **15** (1967); **17** (1969), (with K. Gęba).
3. *Fixed point theorem for approximative* ANR-*s*, Bull. Acad. Polon. Sci. **16** (1968).
4. *Some theorems in fixed point theory: the Leray–Schauder index and the Lefschetz number*, Bull. Acad. Polon. Sci. **17** (1969).
5. *On cohomology theory in normed spaces*, Ann. Math. Study **69** (1972), (with K. Gęba).
6. *Generalizing the Hopf–Lefschetz fixed point theorem for non compact* ANR-*s*, Ann. Math. Stud. **69** (1972).
7. *The Leray–Schauder index and the fixed point theory for arbitrary* ANR-*s*, Bull. Acad. Sci. France **100** (1972).
8. *The Lefschetz fixed point theorem for non metrizable spaces*, J. Math. Pures Appl. **52** (1973), (with G. Fournier).
9. *Infinite cohomology theory*, J. Math. Pures Appl. **52** (1973), (with K. Gęba).
10. *Sur la méthode de continuité de Poincaré*, C.R. Acad. Sci. Paris Sér. A-B **282** (1976).

11. *On some generalization of the Leray–Schauder theory*, Proceedings of the International Conference on Geometric Topology (Warsaw, 1978), PWN, Warsaw, 1980.

(D) Convex analysis

1. KKM *maps and variational inequalities*, Ann. Scuola Norm. Sup. Pisa Cl. Sci. **5** (1978), (with J. Dugundji).
2. KKM *maps and their applications to nonlinear problems*, The Scottish Book (R.D. Mauldin, ed.), Birkhäuser, Massachusetts, 1981.
3. *A proof of Ryll–Nardzewski fixed point theorem*, J. Math. Anal. Appl. **97** (1983), (with J. Dugundji).
4. *Remark on a theorem of Ky Fan concerning systems of inequalities*, Bull. Inst. Math. Acad. Sinica **11** (1983), (with F.C. Liu).
5. *Théorèmes du minimax*, C.R. Acad. Sci. Paris Sér. I Math. **298** (1984), (with F.C. Liu).
6. *Sur une certaine alternative non-linéaire en analyse convexe*, Studia Math. **83** (1986), (with P. Deguire).
7. *Coincidences of set-valued maps and minimax equalities*, J. Math. Pures Appl. **65** (1986), (with F.C. Liu).
8. *Sur un principe géométrique en analyse convexe*, Studia Math. **101** (1991), (with M. Lassonde).
9. *Some elementary general principles of convex analysis*, Topol. Methods Nonlinear Anal. **5** (1995), (with M. Lassonde).

(E) Fixed points of set-valued mappings

1. *Sur la notion du degré topologique pour une certaine classe de transformations multivalentes dans les espaces de Banach*, Bull. Acad. Polon. Sci. **7** (1959).
2. *Theorem on antipodes and theorems on fixed points for a certain class of multi-valued maps in Banach spaces*, Bull. Acad. Polon. Sci. **7** (1959).
3. *Some theorems on multi-valued maps of subsets of the Euclidean spaces*, Bull. Acad. Polon. Sci. **7** (1959), (with J. Jaworowski).
4. *Fixed point theorems for multi-valued mappings of the absolute neighbourhood retracts*, J. Math. Pures Appl. **49** (1970), (with L. Górniewicz).
5. *Some general theorems in coincidence theory*, J. Math. Pures Appl. **60** (1981).
6. *Points fixes et coïncidences pour les applications multivoques (applications de Ky Fan)*, C.R. Acad. Sci. Paris Sér. I Math. **295** (1982), (with H. Ben-El Mechaiekh and H.P. Deguire).
7. *Points fixes et coïncidences pour les fonctions multivoques (applications de type φ et φ*)*, C.R. Acad. Sci. Paris Sér. I Math. **295** (1982), (with H. Ben-El Mechaiekh and H.P. Deguire).

8. *On a theorem of C. Bowszyc concerning the relative version of the Lefschetz fixed point theorem*, Bull. Inst. Math. Acad. Sinica **13** (1985), (with L. Górniewicz).

9. *Points fixes et coïncidences pour les fonctions multivoques (applications de type M^* et M)*, C.R. Acad. Sci. Paris Sér. I Math. **305** (1987), (with H. Ben-El Mechaiekh and H.P. Deguire).

10. *Sur la méthode de l'homotopie dans la théorie des points fixes pour les applications multivoques. I. Transversalité topologique*, C.R. Acad. Sci. Paris Sér. I Math. **307** (1988), (with L. Górniewicz and W. Kryszewski).

11. *Sur la méthode de l'homotopie dans la théorie des points fixes pour les applications multivoques. II. L'indice dans les ANR-s compacts*, C.R. Acad. Sci. Paris Sér. **308** (1989), (with L. Górniewicz and W. Kryszewski).

12. *The Lefschetz fixed point theorem for multi-valued mappings*, International Conference on Fixed Point Theory and Applications, 5–9 juin 1989, Marseille–Luminy (J.-B. Baillon and M. Thera, eds.) 1989, (with L. Górniewicz).

13. *Topology of morphisms and fixed point problems for set-valued maps*, Fixed point theory and applications (Marseille, 1989), Pitman Res. Notes Math. Ser., vol. 252, Longman Sci. Tech., Harlow, 1991, (with L. Górniewicz and W. Kryszewski).

14. *Résultats du type de Leray–Schauder pour des contractions multivoques*, Topol. Methods Nonlinear Anal. **4** (1994).

15. *An elementary proof of nonlinear alternative*, Ann. Sci. Math. Quebec **19** (1995), (with M. Frigon and Z. Guennoun).

(F) Applications to nonlinear differential equations and inclusions

1. *Remarks on positive periodic solutions for second order differential equations*, Bull. Acad. Polon. Sci. **26** (1978), (with R.B. Guenther, J.W. Lee).

2. *On a theorem of S. Bernstein*, Pacific J. Math. **74** (1978), (with A.R. Guenther and J.W. Lee).

3. *The shooting method for the numerical solution of a class of nonlinear boundary value problems*, SIAM Num. Anal. **16** (1979), (with R.B. Guenther and J.W. Lee).

4. *Nonlinear boundary value problem for some class of ordinary differential equations*, Rocky Mountain J. Math. **10** (1980), (with R.B. Guenther and J.W. Lee).

5. *Applications of topological transversality to differential equations*, I, II, Pacific J. Math. **89** (1980); **92** (1983), (with R.B. Guenther and J.W. Lee).

6. *Boundary value problems on infinite intervals and semiconductor devices*, J. Math. Anal. Appl. **116** (1986), (with R.B. Guenther, J.W. Lee and D. O'Regan).

7. *Quelques résultats dans la théorie de Bernstein–Carathéodory de l'équation* $y'' = f(t, y, y')$, C.R. Acad. Sci. Paris Sér. I Math. **306** (1988), (with Z. Guennoun).

8. *Some general existence principles in the Caratheodory theory of nonlinear differential equations*, J. Math. Pures Appl. **70** (1991), (with A.R. Guenther and J.W. Lee).