Topological **M**ethods in **N**onlinear **A**nalysis
Volume 52, No. 1, 2018, 5–9
DOI: 10.12775/TMNA.2018.032

MAREK BURNAT — LIFE AND RESEARCH

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Marek Burnat was born May 29, 1929 in Cracow (Poland). He completed high school in Lodz in 1947. In the years 1949–1952, he was studying mathematics at the Faculty of Mathematics, Physics and Chemistry at the University of Warsaw obtaining the degree of Master of Philosophy in mathematics. He completed his doctoral studies at Leningrad State University where he obtained his PhD in 1956 under the supervision of Olga Alexandrovna Ladyzhenskaya. From Leningrad, he arrived not only with his PhD but also with his Russian wife, Larka. It was a good and happy marriage. Unfortunately, Larka passed away earlier than he, and he was left alone.

Anyone who knew Marek Burnat remembers that he had a warm and friendly personality. His work style was primarily a deep contemplation and, from time to time, some calculations. He had a well-developed geometrical imagination, what probably was associated with his hobby: sculpture.

In the years 1955–1964 Marek was employed at the Institute of Mathematics, University of Warsaw, where in 1963, he completed his habilitation. In 1964, he moved to the Institute of Fundamental Technological Research of the Polish Academy of Sciences. During the work there, he was awarded the title of Extraordinary Professor of Engineering Sciences. In 1972, he returned to the University of Warsaw to the Faculty of Mathematics and Mechanics (now the Faculty of Mathematics, Informatics and Mechanics) where, in 1988, he obtained the title of Professor of Mathematics. For many years, he was the head of the Division of Equations of Mathematical Physics and, in 1987, a co-founder of the Institute of Applied Mathematics and Mechanics where he was the director in the years 1987-1993. Professor Burnat was a co-founder in 1991 of the Julius Schauder Center of Nonlinear Studies at the Nicolaus Copernicus University in Toruń and a member of its Scientific Council. He was also an editor of Topological Methods in Nonlinear Analysis published by the Center.

The scientific activity of Marek Burnat was mainly concentrated on several aspects of the theory of partial differential equations. In his research, the following problems received the most attention: mathematical models of quantum mechanics in non-separable Hilbert spaces, Riemann invariants and the geometrical theory of PDE's and numerical methods.

His research in the field of geometrical methods in partial differential equations was closely connected with his eight-year affiliation in the Department of Fluid Dynamics of the Institute of Fundamental Technological Research. This was a time of the considerable development of computers that could be used for computation of fluid flows and aircraft construction. However, the computers were still not powerful enough to treat real 3-dimensional flows. Therefore, every reduction in dimensionality of the problem was very helpful. From Riemann's time, the method of characteristics was well known. It allows one to easily solve two dimensional hyperbolic problems, i.e. one dimensional nonstationary or two dimensional stationary flows of an inviscid compressible fluid. It was also well known that there exists a class of 3-dimensional flows accessible by solutions of some two dimensional problems. These were so called double wave solutions of Euler equations, or solutions with degenerated, two-dimensional hodographs. As noted by Marek these solutions can be used to describe flows which are interesting from the practical point of view, for example, flows past profiles which are developable surfaces. He also noticed that double wave solutions can exist, in general, for non-elliptic quasilinear homogeneous first order systems. The double wave solutions can be searched in two steps. At first one needs to determine the possible hodograph — the two-dimensional surface representing the set of possible values of the solution. Having the hodograph surface, the set of dependent variables is reduced to two, and one can determine the solution by solving the system reduced to two equations, using Riemann invariants. This suggested that the solution represents some nonlinear combination of simple waves. In fact, one can prove that locally every hyperbolic double wave solution can be interpreted as resulting from the interaction of two simple waves.

In addition, one can also reduce the problem to two space dimensions. If n is the number of independent variables, then as one demonstrates, the double wave solution is constant on a certain family of $(n-2)$-dimensional hyperplanes perpendicular to the local wavevectors of simple waves involved in the construction of a double wave. Clearly, these hyperplanes can intersect each other. In such a case, gradient catastrophe appears and we have only local solutions. While working in the Institute of Fundamental Technological Research, Marek has discovered a number of interesting properties of such solutions. One very important property is that for each hodograph of any double wave there exists an infinite number of double wave solutions having the same surface (or a part of it) as the image set. However, not every two-dimensional surface in the space of dependent variables can serve as a hodograph of a double wave. Some integrability conditions must be satisfied.

By introducing the very convenient language of integral elements and, especially, the notion of simple integral elements, he was able to express integrability conditions in terms of these last elements. Simple integral elements are related to characteristic co-vectors of the system; more precisely, they are tangent mappings of simple waves at a given point. These integrability conditions constitute the system of equations for the hodograph surface of a double wave family.

Soon it became obvious for Marek that one can also search for solutions representing interactions of more than two simple waves (say k simple waves) and that the integrability conditions can be easily generalized for this case. In this way, Marek gave us the foundations for the further development made in the thesis of Zbigniew Peradzyński where the Cartan–Kähler theory of overdetermined systems was employed and a number of examples of such solutions for Euler equations of gas-dynamics were shown. Wojciech Zajączkowski, also a former PhD student of Burnat, was applying the newly developed theory to the system of equations of magnetogasdynamics, classifying possible interactions between various families of waves.

At the same time, Marek noticed that equations of an ideal plasticity can be treated by the same method. Simple wave solutions had already appeared to be very useful in solving some practical problems. This was developed in the thesis of Marek's PhD student, Franciszek Labisch. Later on, it became clear that two-dimensional plastic flows described by a system of nonlinear PDE's are in fact double waves, so one can start from the determination of their hodographs and then, by using the 2 dimensional method of characteristics, determine the flow itself. Studies in this direction resulted in the PhD thesis of Jerzy Czyż.

Unfortunately, the results of Marek Burnat are little known. The internet did not yet exist during his career, and his papers were published in not widely read journals like The Bulletin of the Polish Academy of Sciences. In addition, at that time people were fascinated with new results in the theory of nonlinear partial differential equations — the soliton equations. Consequently, the dispersion-less equation seemed to be less interesting. It was many years later, that this topic again rose to the surface, mainly due to a group of Russian mathematicians (E. Ferapontov, M. Pavlov, S.P. Tsarev and their collaborators). It is interesting that the rediscovery of Burnat and his follower's results came from people from the Novikov school working in solitons and integrable systems. When studying so called "commuting flows" (S.P. Tsarev), the integrability conditions and the role of Riemann invariants in the description of nonlinear waves superposition were rediscovered 20 years after Marek Burnat although in a less general form. Hence the integrability conditions are now known as Tsarev integrability conditions. On the other hand, the above mentioned group went much further, solving numerous examples and establishing the relation of Riemann invariants with the integrable systems and soliton equations.

In the beginning of 70s, after the work of Lax and Zakharov the theory of soliton equations was rapidly developing. At the beginning, there was no apparent connection between those two theories, i.e. interaction of waves by Riemann invariants and soliton equations. Only later, it appeared that the dispersion-less limits of soliton equations led to equations which can be treated by the method of Riemann invariants and, at the same time, are also completely integrable.

The method of Riemann invariants, if applicable, permits one to search for solutions representing nonlinear superpositions of simple wave solutions. Simple waves in the dispersion-less equations play the same role as is played by solitary waves in the soliton equation. If the interaction of simple waves can be described by Riemann invariants, then using the physical language of interacting particles, one may say that one has to do with a kind of "elastic interaction" of two waves, one wave can pass through the other without producing anything new. The interaction can change only the wave profiles and their directions of propagation.

Numerical problems in the research activity of Marek Burnat largely originated from his employment in Polish Academy of Sciences. His interests were in numerical solutions of systems of equations of continuum mechanics, in particular, problems of gas dynamics and the theory of plasticity. An essential difficulty in solving these systems of equations numerically is in the change of the type of equations inside the region of integration. Designing an efficient algorithm for solving these problems was a challenging task. Marek Burnat used his deep knowledge of the geometrical structure of solutions to propose a numerical scheme that would converge to a true solution. His ideas were then tested on several problems by his PhD students. A partial result of these investigations evolved into dissertations by M. Bratos (flow from the Laval nozzle) and J. Czyż (plastic deformations). This research was then abandoned after Marek Burnat returned to the Institute of Mathematics at the University of Warsaw.

The research area in which Marek Burnat worked for the longest time was the mathematical foundations of quantum mechanics. To a certain extent we can simplify quantum mechanics to the theory of electron movements. When electrons are in bound states like in atoms or chemical molecules, there is an elegant mathematical model: electron states are described by square integrable functions in the space of electron position and momentum (electron wave function). The time evolution of an electron state is described by the Schroedinger equation. But this model cannot be used to describe electrons in crystals where electron wave functions are not square integrable. Burnat offered a new, original model embedding electron wave functions into a space of functions integrable in the sense of Besicovitch. Although Besicovitch spaces existed for a long time, the achievement of Marek Burnat was to observe that electron wave functions in crystals can be treated as elements of these spaces. The most important achievement, however, was the formulation of the Schroedinger equation in Besicovitch's spaces. The first result in this direction was published by Burnat in 1964 in Studia Mathematica. Then Burnat wrote a paper on the formulation of quantum mechanics in Besicovitch's spaces (the paper was published as a preprint in 1973). After his return to the University, Burnat began a seminar on quantum mechanics in Besicovitch's spaces, which he was running until his retirement. Many students attending his seminar have written their dissertations under Burnat's supervision (Andrzej Palczewski, Jan Herczyński, Andrzej Krupa, Bogdan Zawisza, Marcin Moszyński).

After his retirement, Marek Burnat was still conducting research, which engaged him in modeling turbulent flows of chemically active mixtures. His idea was to construct a model of turbulence independent of the Navier–Stokes equations. He was working on this problem with Krzysztof Moszyński until the day he died.

Marek Burnat was a mathematician whose research was always deeply rooted in physical reality. In this aspect he was one of only a few truly applied mathematicians in Poland. The need for the connection of mathematical research with physical reality is a concept he instilled in many of his students.

As a person, Burnat was very modest when speaking of his achievements. He treated them as new found pieces of a puzzle of the universe. He was a warm and friendly person in contacts with his students, collaborators and friends. His long-time hobby was sculpture in wood. Looking at a result, he used to claim that the sculpture was already in a piece of wood and his work was merely to recover it from hiding.

Marek Burnat passed away in Warsaw on December 19, 2015.