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A REMARK FOR "OSCILLATIONS OF SECOND-ORDER NONLINEAR PARTIAL DIFFERENCE EQUATIONS"

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ABSTRACT. In this note we shall show some counter examples for the main results of the recent paper *Oscillations of second-order nonlinear partial difference equations*, (Rocky Mountain J. Math **34** (2004), 699–711).

1. Results. In [**1**], the authors consider the following nonlinear partial difference equation

$$T(\Delta_1,\Delta_2)[C_{m,n}T(\Delta_1,\Delta_2)(y_{m,n})]+p_{m,n}(y_{m+1,n}+y_{m,n+1})^{\nu}=0, \tag{1}$$

where $m,n\in N_0=\{0,1,2,\dots\}$, $T(\Delta_1,\Delta_2)=\Delta_1+\Delta_2+I$, $\Delta_1 y_{m,n}=y_{m+1,n}-y_{m,n}$, $\Delta_2 y_{m,n}=y_{m,n+1}-y_{m,n}$, $Iy_{m,n}=y_{m,n}$, $\{C_{m,n}\}$ and $\{p_{m,n}\}$ are real sequences, $m,n\in N_0$, ν is a quotient of odd positive integers with $\nu>1$ and $0<\nu<1$.

By a solution of (1), we mean a real double sequence $\{y_{m,n}\}$ satisfying (1) for $m,n\in N_0$. A solution $\{y_{m,n}\}$ of (1) is called nonoscillatory if it is eventually positive or eventually negative; otherwise, it is said to be oscillatory.

Assume that $C_{i,j}>0$ for all $i,j\in N_0$ and

$$\sum_{i=i_0}^{\infty}\sum_{j=j_0}^{\infty}\frac{1}{C_{i,j}}<\infty, \tag{2}$$

where $i_0>0$ and $j_0>0$.

There are two main results in [**1**].

Theorem 1 [**1**]. *Assume that $C_{i,j}>0$ and (2) hold. Further, assume that $\nu>1$, $p_{i,j}>0$, $i,j>0$ and*

$$\sum_{i=i_0}^{\infty}\sum_{j=j_0}^{\infty}\left(\frac{1}{2}\right)^{i+j}p_{i,j}\rho_{i+1,j}^{\nu}=+\infty, \tag{3}$$

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where $\rho_{m,n} = \sum_{i=m}^{\infty}\sum_{j=n}^{\infty} 1/C_{i,j}$. *Then all solutions of* (1) *are oscillatory.*

We see the proof of Theorem 1 [**1**, page 703]. "Since $C_{m,n}T(\Delta_1, \Delta_2) \times (y_{m,n}) \geq 0$ for $m \geq M, n \geq N$, that is,

$$y_{m+1,n} + y_{m,n+1} \geq y_{m,n},$$

there exists a positive number c such that $y_{m,n} > c > 0$ for $m \geq M$, $n \geq N$. Thus, there exist $M_1 \geq M, N_1 \geq N$ such that

$$y_{m+1,n} + y_{m,n+1} \geq y_{m+1,n} \geq \rho_{m+1,n} \quad \text{for } m \geq M_1,\ n \geq N_1, \tag{4}$$

since $\rho_{m,n} \to 0$ as $m, n \to \infty$."

The following counter example will show that the above proposition in [**1**] is not true.

Counter example 1. Let $y_{m,n} = 1/(mn)$, $m \geq 1$, $n \geq 1$. It is easy to see that $y_{m,n} > 0$ and $y_{m+1,n} + y_{m,n+1} \geq y_{m,n}$. But there is no positive constant c such that $y_{m,n} > c > 0$ for $m \geq M$, $n \geq N$. Therefore, the mentioned proposition in [**1**, page 703] is wrong.

We find that not only does the proof of Theorem 1 contain a mistake, but also the conclusion of Theorem 1 is wrong.

Counter example 2. Consider the partial difference equation

$$T(\Delta_1, \Delta_2)(2^{m+n}T(\Delta_1, \Delta_2)(y_{m,n})) + 2^{m+n-3}3^{2(m+n)+1} \times (y_{m+1,n} + y_{m,n+1})^3 = 0, \tag{5}$$

where $C_{m,n} = 2^{m+n}$, $\rho_{i,j} = 1/(2^{i+j-2})$, $p_{m+n} = 2^{m+n-3}3^{2(m+n)+1}$ and $\nu = 3$. We see that

$$\sum_{i=i_0}^{\infty}\sum_{j=j_0}^{\infty}\left(\frac{1}{2}\right)^{i+j} p_{i,j}\rho_{i+1,j}^{\nu} = \sum_{i=i_0}^{\infty}\sum_{j=j_0}^{\infty}\left(\frac{1}{2}\right)^{i+j} 2^{i+j-3}3^{2(i+j)+1}\left(\frac{1}{2^{i+j-1}}\right)^3 = \infty.$$

That is, all assumptions of Theorem 1 are satisfied. But it is easy to check that equation (5) has a positive solution $\{y_{m,n}\} = \{(1/3)^{m+n}\}$. Therefore, Theorem 1 in [**1**] is wrong.

Theorem 2 [**1**]. *Assume that* $C_{i,j} > 0$ *for all* $i \geq 0$, $j \geq 0$ *and* (2) *hold. Further, assume that* $0 < \nu < 1$, $p_{i,j} > 0$, $i, j > 0$ *and*

$$\sum_{i=i_0}^{\infty} \sum_{j=j_0}^{\infty} \left(\frac{1}{2}\right)^{i+j} p_{i,j} \rho_{i+1,j} = +\infty. \tag{6}$$

Then all solutions of (1) *are oscillatory.*

In the proof of Theorem 2, authors also use (4). As Counter example 1, this is wrong. Therefore, Theorem 2 has not been proved yet.

On the other hand, Example 2 [**1**] does not satisfy Condition (6). In fact, in the case of Example 2 [**1**], we have

$$\sum_{i=i_0}^{\infty} \sum_{j=j_0}^{\infty} \left(\frac{1}{2}\right)^{i+j} p_{i,j} \rho_{i+1,j} < +\infty,$$

which does not satisfy (6). Therefore, Example 2 [**1**] is not effective.

In the above we point out that the main results in [**1**] are wrong.

REFERENCES

1. Shu Tang Liu and Guanrong Chen, *Oscillations of second-order nonlinear partial difference equations*, Rocky Mountain J. Math. **34** (2004), 699–711.

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