

W.J. Thron on His 70th Birthday

August 17, 1988

Wolfgang Joseph Thron (fondly known to his friends as “Wolf”) was the second of two children born to Ludwig and Annemarie Joseph Thron on August 17, 1918 in Ribnitz, Germany (now Ribnitz—Dangarten, DDR). Wolf’s parents met while both were studying medicine at the University of Erlangen. His birth came in the last year of world War I while his father was still at the front in France. Wolf grew up in an old “doctors’ house” where his father and maternal grandfather jointly worked in an extended small town medical practice.

Recognizing that their son was well developed both physically and mentally, Wolf’s parents started him in school at age five. For the next six years he progressed through Realschule in Ribnitz. From 1930 until 1936 he and his older sister, Liselotte Thron (Caesar), attended the Gymnasium in the major port city of Rostock, living in a Rostock Pension the first two years and commuting by train from Ribnitz the next four. His classical education included Latin, Greek, English, French, history, physics, chemistry, biology and mathematics (through analytic geometry). Although history captured his greatest interest in Gymnasium, he recognized ability in mathematics because it came so easily.

As Wolf and his sister grew older, their parents tried to draw them out of the small town atmosphere by sending them each summer to enlarge their horizons in foreign countries. They wanted them to become acquainted with other peoples’ customs and to learn to respect them. At first they traveled with official exchange organizations. Wolf traveled to Salzburg, Austria in the summer of 1932 where he first hiked in the high mountains. At home there was only water and flat country. The next summer he became acquainted with a country in which a foreign language was spoken, as he stayed outside Stockholm for a month, making frequent trips into the city as well as to Uppsala and the island of Gotland.

The summer of 1934 took him again in a southerly direction with three weeks on the Italian Riviera and then via Pisa to Rome where his passport was lost and his stay delayed until a new one could be

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obtained. This incident prevented a visit to Florence and Wolf returned home slightly shaken by the experience. 1935 and 1937 summers were devoted to sailing, first helping sail a two-mast schooner to England and then to Sweden and Denmark.

The year 1936 brought graduation from Gymnasium and also an end to Wolf's life in Germany. Political events at the time led his parents to send their son to the ETH in Zürich during the summer, where he studied analytical geometry, calculus, physics and history. Another turn of events was to have a significant impact on Wolf's life; his aunt, Helene Joseph, married the mathematician, Herman Weyl, in 1913 and then came to the Institute for Advanced Study at Princeton in 1933. Wanting their son to have a good education away from the political climate of Europe, Wolf's parents sent him to Princeton University in the fall of 1936. There his aunt and uncle could keep an eye on him and lend a helping hand if necessary. Although he lived in university dormitories, Wolf was a frequent visitor in the Weyl home on weekends where he not only received the warmth of family love but also encountered many of the world's leading scientists and mathematicians.

At Princeton Wolf chose mathematics for his major discipline not only for the joy and satisfaction derived from it but also for the ease with which it came to him. It is also possible that the well known "hereditary phenomenon" of father-in-law passing down mathematical ability to son-in-law applied in this case from uncle-in-law to nephew-in-law. His undergraduate studies in mathematics concentrated on analysis with courses from S. Bochner and H.F. Bohnenblust. He recalls from the latter, a Swiss analyst, the remarkable fact that only in the last of four successive courses was a single error detected. Other areas studied were algebra from the topologist, A.W. Tucker, statistics from S.S. Wilks and algebraic number theory from Herman Weyl.

Receiving a B.A. degree from Princeton in 1939 after three years, Wolf was offered a teaching fellowship at Washington State University in Pullman by the Department Chairman, M.S. Knebelman, who had been on the faculty at Princeton. Wolf's stay was only for one year, since in those days Washington State offered very few graduate courses. One of his principal duties was to teach solid geometry to football players for an academic year salary of \$450. The WSU year was his first real American experience, away from the intellectual ivory tower

of Princeton. Here in the American West, Wolf discovered that he liked America and he liked teaching.

In 1940 H.E. Bray, Chairman of the Mathematics Department at Rice Institute, asked O. Veblen at Princeton to recommend promising students for the graduate program. Inquiring of Herman Weyl if his nephew would be interested in a position, Veblen received an affirmative answer and Wolf was offered a teaching fellowship for graduate study. The Department of Mathematics at Rice was essentially a “Department of Analysis.” S. Mandelbrojt (student and son-in-law of Hadamard), a complex analyst, was the star. F.E. Ulrich (a student of Ahlfors) was in complex analysis, H.E. Bray in real analysis and Walter Leighton (a student of Marston Morse at Harvard) in differential equations. Since Leighton was in need of a student, Wolf was assigned to work with him. Leighton had previously written a master’s thesis on continued fractions with H.S. Wall at Northwestern, and he decided to give a seminar on the subject in 1940–41. By the end of the year Wolf felt that he had obtained enough ideas to develop a thesis. The results on parabolic convergence regions for continued fractions were published in two papers, one with Leighton [2] in 1942 and one [4] in 1943. These results significantly improved the original parabola theorem of W.T. Scott and H.S. Wall that appeared in 1940. During the next (almost) five decades Wolf Thron has continued to make major contributions to the convergence theory of continued fractions. He has introduced new methods, tools, concepts, problems and results that have stimulated much of the rebirth of interest in the subject among numerous researchers. Approximately one-third of his more-than-100 research publications have been devoted to various aspects of continued fraction convergence theory. Included are results on convergence regions, uniform convergence, speed of convergence, truncation error estimation, acceleration of convergence, general convergence and convergence of modified continued fractions. His results of 1958 [21] on parabolic convergence regions, proved entirely by constructive methods, form a basic landmark of the subject.

Receiving his Ph.D. degree from Rice in 1943 (after three years), Wolf accepted an Instructorship at Harvard University. Here he continued the research on continued fractions, in spite of a teaching load consisting of 12 to 15 hours of lectures per week. After one year the Instructorship was interrupted by a call from Uncle Sam and the years 1944–46 were

spent in the U.S. Army. Infantry basic training at a military base near Spartanburg, South Carolina, was followed by a tour of duty with military government intelligence in Europe. This required knowledge of the German language but not of mathematics. An advantage of being drafted was that he immediately became eligible for U.S. citizenship which he applied for and received shortly after induction.

At the end of the war in 1946 Wolf was offered a tenure-track position at Washington University in St. Louis where Walter Leighton was then chair of the Mathematics Department. During 8 years at Washington University he published his first book, *Introduction to the Theory of Functions of a Complex Variable* (John Wiley and Sons, Inc., 1953), a concise but completely self-contained, rigorous treatment of the subject. His research interests expanded to include topics such as zero-free regions for polynomials, singularities and natural boundaries of analytic functions and functional equations. In 1948 he introduced a special class of continued fractions which Perron later generalized and called “die Thronschen Kettenbrüche” [O. Perron, *Die Lehre von den Kettenbrüchen*, B.G. Teubner, 1957, p. 174]. These continued fractions, now referred to in the literature as general T-fractions, have since been used by Thron, his collaborators and others for the development of two-point Padé tables, strong moment theory, orthogonal Laurent polynomials and Gaussian quadrature. His interests in point set topology also began to surface during this period as evidenced by chapters on Hausdorff and metric spaces in the complex variables text.

Beginning in 1939 Wolf took part in the first of a series of summer workcamps sponsored by the American Friends Service Committee. These “extra curricular” activities were to have a significant impact on his life. The first workcamp was involved with helping resettle miners in rural areas of western Pennsylvania. In 1941 the project was to construct a dam for a small community in New England. Following the war years, in 1948 the work consisted of improving buildings at Storer College (for black students) at Harpers Ferry, West Virginia. It was during a transatlantic ocean voyage (of two weeks in each direction) in 1949 that Wolf and Ann Lukach became acquainted. Having been admitted to medical school at New York University, Ann was on her way to a workcamp in Germany, while Wolf was going to one in Sweden. They met at the orientation sessions and got to know each other during

four weeks at sea. The following summer (1950) both participated in separate workcamps again, Wolf at an inner city project in Philadelphia and Ann in Missouri. In the summer of 1951 they managed to go to a common location. While Wolf taught at the Freie Universität of Berlin, Ann (a third-year medical student) worked in Berlin hospitals. At the end of the school term, they joined each other for several weeks in another workcamp, building homes for Yugoslavian refugees. Ann completed her medical degree in the spring of 1953 and one day before the school commencement, she and Wolf left for Bermuda on their wedding trip.

Wolf Thron began his long career at the University of Colorado in Boulder in fall, 1954, filling a vacancy in mathematics resulting from Albert Edrei's move to Syracuse University. Wolf's tenure at Colorado continued until his retirement in 1985. He served as the Department Chairman from 1972 to 1974. During the Colorado years he had 21 Ph.D. students (see attached list). He wrote two additional books: *Topological Structures* (Holt, Rinehart and Winston, Inc., 1966) and *Continued Fractions: Analytical Theory and Applications* (with his student and long-time collaborator, William B. Jones) in the Encyclopedia of Mathematics and Its Applications 11 (Addison-Wesley Pub. Co., 1980, distributed now by Cambridge University Press). He also published a large number of research papers on classical analysis and general topology. Although continued fractions continued to play a major role in his research, his work on classical analysis included moment theory, orthogonal polynomials, quadrature, special functions and approximation. His research in topology explored many topics: separation axioms, uniformities, metrics, proximities, nearness, order, filters and lattices.

During most of the Boulder years, Wolf has been instrumental in research seminars in topology and/or continued fractions. The continued fraction seminar has been active in most of the years from 1958 until the present. In addition to graduate students, regular participants in these seminars have included William B. Jones, Arne Magnus and Burnett Meyer as well as the periodic visitors from Trondheim, Norway: Olav Njåstad and Haakon Waadeland. Wolf's long and close association with Arne Magnus began when Arne was a graduate student at Washington University and continued when Arne joined the faculty at the University of Colorado and subsequently moved to nearby Colorado

State University. The Boulder seminar has greatly benefitted all the participants, and Wolf's contributions to this have been extensive.

A unique activity that significantly affected the professional career and family of Wolf Thron has been the practice of spending every fourth year at a foreign university (from 1957 through 1987). 1957–58 was at the University of Munich in close proximity to Oscar Perron (with support from the AFOSR). In 1962–63 and 1974–75 the Throns traveled to Panjab University in Chandigarh, India, where Wolf did research and teaching with a Fulbright grant in the first year. The Ford Foundation sent the Throns to Mindanao State University in the Phillipines in 1966–67. There Wolf participated in a development program and returned to Colorado with a bright Ph.D. student. Two others of his Ph.D. students accompanied the Throns to the University of Erlangen in 1970–71 during a Sabbatical-Visiting Professorship. The last three foreign tours have been at the University of Trondheim, Norway, with support from the Norwegian Marshall Fund, the Norwegian Research Council for Science and the Humanities (NAVF)(1978–79, 1982–83 and January–July, 1987). There Wolf continued a long and productive collaboration with Lisa Jacobsen, Olav Njåstad and Haakon Waadeland. During two of the Norwegian years, Ann provided medical services in Trondheim hospitals and medical centers. Two additional summers were spent at Institutes for College Teachers in India (Jodhpur in 1967 and Aligarh in 1969) under the auspices of the United States AID NSF and the Indian UGC. Even though it is not possible to trace precisely the roots of such extensive involvement in traveling and living in foreign countries, one can conjecture that two earlier experiences had a strong influence. The first was when the Thron parents in Ribnitz sent their young son on summer trips to enlarge his horizons with people of foreign cultures. Later came the summer workcamps attended by both Wolf and Ann.

All five of the Throns' children were born during the years in Colorado and abroad: Jonathan in 1954, Penny (Penelope) in 1956, Peter in 1959, Karin in 1961 and Raji (Rajinder) in 1962. The only one not arriving in Boulder was Raji who made his appearance in Chandigarh. One special part of the children's education has been the German language, spoken in the home. Another has been the international, service-oriented experience in foreign countries and cultures.

Those who know Wolf admire and respect his extensive involvement in woodcarving, photography and art history. Carving skills were inherited from and/or inspired by his father who did both oil painting and woodcarving, mostly of people or animals. Wolf's carvings are abstract with graceful surfaces that flow in interesting patterns, bringing out the inherent beauty of the wood. His extensive knowledge of art history has been combined with travel and photography to produce many delightful slide shows and mini-lectures on art from around the world.

In 1980 W.J. Thron was elected to *The Royal Norwegian Society of Sciences and Letters (Det Kongelige Norske Videnskabers Selskab)* for his outstanding creative research in mathematics and for his great inspiration for others to do creative work. Upon his retirement in 1985 he was awarded the University of Colorado Medal for outstanding contributions to the University, for his distinguished career as a research scholar and teacher and for his national and international reputation as a mathematician. In recognition of his outstanding leadership role in developing continued fractions and related topics during half a century, an international conference was held at the University of Colorado in June, 1988. These proceedings of that conference are dedicated to Professor Emeritus W.J. Thron on the occasion of his 70th birthday, August 17, 1988.

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Ph.D. Students of W.J. Thron

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- 1962 C.H. Cook, Topological spaces of holomorphic functions, 65 pp.
 C.E. Aull, Separation axioms, 68 pp.
 J.D. DePree, On sequences of linear fractional transformations, 54 pp.
 K.L. Hillam, Some convergence criteria for continued fractions, 45 pp.
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- 1964 David Drake, On the representations of an abstract lattice as the family of closed sets of a topological space, 63 pp.
 J.L. Hursch, Jr., The ordering of uniformities, 81 pp.
- 1965 N.S. Morez, Generators for the semi-topogenous orders of Caászár, 68 pp.
- 1966 N.P. Callas, Singular points of certain functions represented by C-fractions, 47 pp.
 F.W. Stevenson, Uniform spaces with linearly ordered bases, 75 pp.
 T.E. Doohar, Proximity relations on an abstract lattice, 66 pp.
 Ellen E. Reed, Uniform structures, 84 pp.
- 1969 T.H. Jefferson, Jr., Some additional properties of T-fractions, 81 pp.
- 1970 R.E. Larson, On the lattice of topologies, 82 pp.
 Susan J. Zimmerman, Order-induced topological properties, 177 pp.
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