ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 22, Number 1, Winter 1992

AN EXTENSION OF AN INEQUALITY FOR NONDECREASING SEQUENCES

JOSIP E. PEČARIĆ

A. Meir [**1**] proved the following result:

Theorem A. *Let* $0 = a_0 \leq a_1 \leq \cdots \leq a_n$. *Suppose* $a_i - a_{i-1} \leq p_i$ $(i = 1, 2, \dots, n)$ *and* $(*)$ $p_1 \leq p_2 \leq \cdots \leq p_n$. *If* $r \geq 1$ *and* $s + 1 \geq 2(r+1)$, *then*

$$(1) \qquad \left((s+1)\sum_{i=1}^{n-1} a_i^s \frac{p_i + p_{i+1}}{2}\right)^{\frac{1}{s+1}} \leq \left((r+1)\sum_{i=1}^{n-1} a_i^r \frac{p_i + p_{i+1}}{2}\right)^{\frac{1}{r+1}}.$$

G.V. Milovanović and I.Ž. Milovanović [**2**] proved a stronger inequality. Here we shall show that, using the idea of their proof, we can prove another extension of Meir's inequality.

Theorem 1. *The conclusion of Theorem* A *remains valid if* $(*)$ *is replaced by* $\binom{*}{*}$ $p_i \leq p_n$ $(i = 1, 2, \dots, n-1)$.

Proof. Let us denote

$$q_j = \sum_{i=1}^{j} a_i^r \frac{p_i + p_{i+1}}{2}, \qquad c_j = q_j - p_{j+1} a_j^r / 2 = q_{j-1} + p_j a_j^r / 2,$$

and

$$k = (s+1)/(r+1).$$

The following inequalities are proved in [**2**]:

$$(2) \qquad a_j^{r+1} \leq (r+1) c_j, \quad \text{i.e.,} \quad a_j^{s-r} \leq (r+1)^{k-1} c_j^{k-1},$$

Received by the editors on October 6, 1988, and in revised form on November 28, 1988.

Copyright ©1992 Rocky Mountain Mathematics Consortium

and

$$(3)\qquad k\frac{p_j+p_{j+1}}{2}a_j^r c_j^{k-1}+k(k-1)\frac{p_{j+1}^2-p_j^2}{8}a_j^{2r}c_j^{k-2}\le q_j^k-q_{j-1}^k.$$

After summing inequalities (3) for $j=1,\dots,n-1$, we get

$$(4)\quad k\sum_{j=1}^{n-1}a_j^r c_j^{k-1}\frac{p_j+p_{j+1}}{2}+\frac{1}{8}k(k-1)\sum_{j=1}^{n-1}(p_{j+1}^2-p_j^2)a_j^{2r}c_j^{k-2}\le q_{n-1}^k.$$

Since $q_{j-1}\le c_j\le q_j$ $(j=1,\dots,n)$, we have that $\{a_i^{2r}c_i^{k-2}\}$ is a nondecreasing sequence, so

$$\begin{aligned}\sum_{j=1}^{n-1}&(p_{j+1}^2-p_j^2)a_j^{2r}c_j^{k-2}\\ &=a_1^{2r}c_1^{k-2}(p_n^2-p_1^2)+\sum_{i=2}^{n-1}(p_n^2-p_i^2)(a_i^{2r}c_i^{k-2}-a_{i-1}^{2r}c_{i-1}^{k-2})\\ &\ge 0.\end{aligned}$$

Therefore, (4) becomes

$$k\sum_{j=1}^{n-1}a_j^r c_j^{k-1}\frac{p_j+p_{j+1}}{2}\le q_{n-1}^k,$$

wherefrom, by using (2), we get

$$(s+1)\sum_{i=1}^{n-1}a_i^s\frac{p_i+p_{i+1}}{2}\le((k+1)q_{n-1})^k,$$

which is equivalent to (1).

REFERENCES

1. A. Meir, *An inequality for nondecreasing sequences*, Rocky Mountain J. Math. **11** (1981), 577–579.

2. G.V. Milovanović and I.Ž. Milovanović, *A generalization of a result of A. Meir for non-decreasing sequences*, Ibid. **16** (1986), 237–239.

Faculty of Technology, University of Zagreb, 41000 Zagreb, Croatia