

ASPLUND SPACES AND DECOMPOSABLE NONSEPARABLE BANACH SPACES

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ABSTRACT. We show that an Asplund space of density character \aleph_1 is weakly compactly generated if and only if it has a projectional resolution of identity for each equivalent norm. We show that every nonseparable Asplund space has a nonseparable subspace which has an equivalent strictly convex norm. We give an example of a non-Asplund space such that every bounded weakly closed subset is an intersection of finite union of balls. We show the existence of an Eberlein compact K such that $(\mathcal{C}(K), \|\cdot\|_\infty)$ has no λ -norming Markushevich basis if $\lambda < 2$.

0. Introduction. In this note we investigate some properties of the nonseparable Banach spaces which admit a “decomposition” into separable subspaces. We show, for instance, that there exists a weakly compactly generated (wcg) Banach space X with no λ -norming Markushevich basis for $\lambda < 2$, and in fact that there exists an Eberlein compact K such that $(\mathcal{C}(K), \|\cdot\|_\infty)$ has this property. This improves some results from [18]. We also answer a question from [8].

Let us recall some notation. Let X be a Banach space of density character $\text{dens}(X) = \mu$. A “decomposition” of X is a well-ordered collection $\{P_\alpha; \omega_0 \leq \alpha \leq \mu\}$ of projections such that $P_\alpha P_\beta = P_\beta P_\alpha = P_\alpha$ if $\alpha \leq \beta$, $P_\mu = \text{Id}_X$, $P_\beta(x) \in \{P_{\alpha+1}(x); \alpha < \beta\}$ for all $x \in X$ and β , and $\text{dens}(P_\alpha(X)) \leq |\alpha|$ for all α . The decomposition $\{P_\alpha; \omega_0 \leq \alpha \leq \mu\}$ is called a *projectional resolution of identity* (PRI) if $\|P_\alpha\| \leq 1$ for all α . It is called a *separable decomposition* if $(P_{\alpha+1} - P_\alpha)(X)$ is separable for all $\alpha < \mu$.

Jayne-Rogers selectors were shown to exist in [13] (see [2, Chapter I.4]). They are multivalued maps from Asplund spaces X to the set $(X^*)^{\mathbb{N}}$ of countable subsets of X^* . We denote them by Δ . A subset $Y \subset X^*$ is called (λ) -norming if there exists $\lambda < \infty$ such that

$$\|x\| \leq \lambda \sup\{|f(x)|; f \in Y, \|f\| \leq 1\}$$

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for all $x \in X$. The infimum of such λ 's, denoted by $r(Y)^{-1}$, is the inverse of the Dixmier characteristic $r(Y)$ of Y .

A Markushevich basis (in short M -basis) of X is a subset $\{(x_\alpha, f_\alpha); \alpha \in A\}$ of $X \times X^*$ such that

$$\overline{\text{span}}^{\|\cdot\|}\{x_\alpha; \alpha \in A\} = X$$

and

$$\bigcap_{\alpha \in A} \ker(f_\alpha) = \{0\}.$$

An M -basis $\{(x_\alpha, f_\alpha); \alpha \in A\}$ is said to be λ -norming if

$$r(\text{span}\{f_\alpha; \alpha \in A\}) \geq \lambda^{-1}.$$

We refer, e.g., to [6, 9, 18, 19, 20] for recent results on M -basis.

A compact set K is called a Corson compact if there exists a set I that is homeomorphic to a subset of

$$\sum(I) = \{x : I \rightarrow \mathbf{R}; |\{i; x(i) \neq 0\}| \leq \aleph_0\}$$

where $\sum(I)$ is equipped with the pointwise topology.

1. Asplund spaces and application of Jayne-Rogers selectors.

Our first statement should be compared to the main result of [3]. Its proof actually uses techniques from [3].

Proposition 1.1. *Let X be an Asplund space with $\text{dens}(X) = \aleph_1$. Then X is weakly compactly generated if and only if $(X, \|\cdot\|)$ has a PRI for each equivalent norm $\|\cdot\|$.*

Proof. Any wcg space has a PRI [1], hence the “only if” part is clear. Let us show the converse.

Let $|\cdot|$ be a given norm on X . By assumption, $(X, |\cdot|)$ has a PRI $\{P_\alpha, 0 \leq \alpha \leq \omega_1\}$. Clearly, $(P_{\alpha+1} - P_\alpha)(X)$ is separable for all $\alpha < \omega_1$, hence we can apply [5, Proposition 1.2] which states in particular the existence of an equivalent norm $\|\cdot\|$ on X such that

$$(1) \quad B_{X^*} = B_{X^*}(\|\cdot\|) = \overline{\text{conv}}^{\|\cdot\|}(\mathcal{E})$$

where \mathcal{E} denotes the set of all points in B_{X^*} which are w^* -strongly exposed in B_{X^*} . Clearly, we have for all $f \in X^*$

$$f = w^* - \lim_{\alpha \rightarrow \omega_1} P_\alpha^*(f).$$

It follows that for all $f \in \mathcal{E}$, we have

$$\lim_{\alpha \rightarrow \omega_1} \|f - P_\alpha^*(f)\| = 0,$$

and thus there exists $\alpha < \omega_1$ such that $f = P_\alpha^*(f)$. Now (1) implies that

$$(2) \quad X^* = \bigcup_{\alpha < \omega_1} P_\alpha^*(X^*).$$

We may now conclude the proof as in [3]. Let $\Delta = X \rightarrow (X^*)^{\mathbb{N}}$ be a Jayne-Rogers selector. We let $\alpha_0 = \beta_0 = 0$. The set $\Delta(P_{\alpha_0}(X))$ is norm-separable, hence by (2) there is $\alpha_0 < \alpha_1 < \omega_1$ such that

$$\Delta(P_{\alpha_0}(X)) \subset P_{\alpha_1}^*(X^*).$$

We find similarly $\alpha_1 < \alpha_2 < \omega_1$ such that

$$\Delta(P_{\alpha_1}(X)) \subset P_{\alpha_2}^*(X^*).$$

Continuing in this way, we obtain an increasing sequence (α_n) of countable ordinals. Letting $\beta_1 = \sup\{\alpha_n\}$, we have (see [2, Lemma VI.3.1])

$$\Delta(P_{\beta_1}(X)) \subset P_{\beta_1}^*(X^*),$$

and in fact ([4]; see [2, Lemma VI.3.2])

$$(3) \quad \overline{\text{span}}^{\|\cdot\|}(\Delta(P_{\beta_1}(X))) = P_{\beta_1}^*(X^*).$$

We now construct by induction an increasing sequence of ordinals $\{\beta_n; n \geq 1\}$ such that (3) is satisfied with β_n . If we now let $\gamma = \sup\{\beta_n\}$, we have (again by [2, Lemma VI.3.1 and 3.2])

$$P_\gamma^*(X^*) = \overline{\text{span}}^{\|\cdot\|} \left(\bigcup \{ \Delta(P_{\beta_n}(X)); n \geq 1 \} \right)$$

and therefore

$$(4) \quad P_\gamma^*(X^*) = \overline{\bigcup P_{\beta_n}^*(X^*)}^{\|\cdot\|}.$$

We may now let $\gamma = \beta_{\omega_0}$ and proceed by transfinite induction to construct a “shrinking” PRI on X , that is, a PRI $\{P_\beta; \beta \leq \omega_1\}$ such that (4) is satisfied for any sequence $\{\beta_n\}$ increasing to γ . It is easy to conclude that X is wcg, as in [2, Corollary VI.4.4]. \square

Note that the assumption “ X Asplund” is necessary in Proposition 1.1 since, for instance, there exist wcd spaces of density \aleph_1 which are not wcg [15] and wcd spaces have PRI’s in each norm (see [2, Theorem VI.2.5]).

Since R. Haydon’s fundamental work [10, 11], it is known that there is an Asplund space X with $\text{dens } X = \aleph_1$, on which no “good” renorming can be completed. Our next statement will imply that this cannot take place hereditarily.

Proposition 1.2. *Let X be an Asplund space. The following are equivalent.*

- 1) *There exists a countable subset D of X^* which separates X .*
- 2) *Every weakly compact subset W of X is weakly metrizable.*

Proof. 1) \Rightarrow 2). By compactness, the topology $\sigma(X, D)$ of pointwise convergence on D agrees on W with the weak topology, and $\sigma(X, D)$ is metrizable since D is countable.

2) \Rightarrow 1). We assume now that no countable subset of X^* separates X .

We will construct by transfinite induction a weakly compact non-metrizable subset W of X . We proceed as follows: pick any $x_1 \in X$ with $\|x_1\| = 1$. If the x_α ’s are constructed for all $\alpha < \beta < \omega_1$, set

$$D_\beta = \overline{\text{span}}^{\|\cdot\|} \{ \Delta(\overline{\text{span}}^{\|\cdot\|} \{x_\alpha; \alpha < \beta\}) \}$$

where Δ denotes as before a Jayne-Rogers selector. Clearly D_β is norm-separable, hence according to our assumption we may pick $x_\beta \in X$ with $\|x_\beta\| = 1$ and $f(x_\beta) = 0$ for all $f \in D_\beta$.

We claim that the set $W = \{x_\alpha; \alpha < \omega_1\} \cup \{0\}$ is weakly compact in X . To prove this, it clearly suffices to show that if $\{\alpha_n; n \geq 1\}$ is a strictly increasing sequence of countable ordinals then $\{x_{\alpha_n}\}$ weakly converges to 0. Let $\beta = \sup(\alpha_n)$. We let for all γ , $X_\gamma = \overline{\text{span}}\{x_\alpha; \alpha < \gamma\}$. By ([4]; see [2, Lemma VI.3.2]), we have

$$X_\beta^* = i_\beta^*(\overline{\text{span}}\{\Delta(X_\beta)\})$$

where i_β^* denotes the canonical quotient map from X^* onto X_β^* . For any $f \in X_\beta^*$ and any $\varepsilon > 0$ there exists $\alpha'_1 < \alpha'_2 < \dots < \alpha'_k < \beta$ and $r_1, \dots, r_k \in \mathbf{R}$ such that

$$\left\| f - i_\beta^* \left(\sum_{i=1}^k r_i y_i \right) \right\| < \varepsilon$$

with $y_i \in \Delta(X_{\alpha'_i})$. If $\alpha_n > \alpha'_k$ we now have by construction of the x_α 's that $|f(x_{\alpha_n})| < \varepsilon$, and this shows our claim.

The set (W, w) is homeomorphic to the one-point compactification of a discrete set of cardinality \aleph_1 . Hence it is not metrizable, and this concludes the proof. \square

Note that one cannot dispense with the assumption “ X Asplund” in Proposition 1.2, since for any set $\Gamma, l_1(\Gamma)$ has the Schur property and therefore any weakly compact subset of $l_1(\Gamma)$ is norm-compact and thus is metrizable.

Corollary 1.3. *Let X be a nonseparable Asplund space. Then X contains a closed nonseparable subspace Y , which has an equivalent strictly convex norm.*

Proof. If there exists $D = \{f_n; n \geq 1\}$ which separates X , we let

$$\| \|x\| \|^2 = \|x\|^2 + \sum_{n=1}^{\infty} 2^{-n} \|f_n\|^{-2} (f_n(x))^2$$

and $\| \| \cdot \| \|$ is an equivalent strictly convex norm on X . If such a set D fails to exist, X contains by Proposition 1.2 a weakly compact norm

metrizable subset W , and thus X contains a wcg nonseparable subspace Y . Since any wcg space has a strictly convex norm (and even an lur norm; see [2, Theorem VII.2.1]) the conclusion follows. \square

It is not clear to me whether the assumption “ X Asplund” is necessary in Corollary 1.3. Note that Corollary 1.3 implies that an Asplund space with no strictly convex norm (such as Haydon’s example in [10]) contains a nonseparable WCG space (which in particular admits a Fréchet-differentiable norm).

We conclude this section with an observation which answers [8, Question E,2]. We refer to [8] for basic results about the ball topology.

Proposition 1.4. *There is an equivalent norm on the non-Asplund space $X = l_1(\mathbf{N}) \oplus l_2(c)$ such that the ball topology coincides on bounded subsets of X with the weak topology; that is, such that any weakly closed bounded set is an intersection of finite unions of balls. In particular, X^* contains no proper norming subspace.*

Proof. The last statement is in fact a special case of [5, Corollary 2.8]. Indeed, since $\text{dens}(X^*) = \text{dens}(X) = c$, we may apply [5] as in the proof of Proposition 1.1 to obtain an equivalent norm $\|\cdot\|$ on X such that

$$(1) \quad B_{X^*}(\|\cdot\|) = \overline{\text{conv}}^{\|\cdot\|}(\mathcal{E})$$

and clearly (1) implies that X^* contains no proper norming subspace. Now observe that all $f \in \mathcal{E}$ are points of w^* -to-norm continuity of the identity map on B_{X^*} . Then [7, Theorem 2.6] and (1) show that every $g \in X^*$ is ball-continuous on the ball of X , and the conclusion follows. \square

It is still unknown whether a space such that every closed convex bounded set is an intersection of balls is an Asplund space. Proposition 1.4 and some results from [21] support the conjecture that the above problem has a negative answer.

Remark 1.5. If X satisfies the conclusion of Proposition 1.4, then there is no $z \in X^{**} \setminus \{0\}$ such that $\|z - x\| = \|z + x\|$ for all $x \in X$.

Indeed, the space $\ker(z)$ would then be a norming subspace of X^* . Proposition 1.4 is therefore related to an example produced in [15]; see also [12, p. 489].

2. Norming Markushevich bases in WCG spaces. The following statement is the main result of this note.

Theorem 2.1. *There exists a Banach space X which is a direct sum $X = S \oplus R$, with S separable and R reflexive, and which has no λ -norming Markushevich basis for $\lambda < 2$.*

Proof. 1) We let $Z = l_1(\mathbf{N}) \oplus l_2(c)$. The space Z has a separable decomposition and clearly we have $\text{dens}(Z) = \text{dens}(Z^*) = c$. It follows now from [5, Corollary 2.8] and the computations made in [7, Proof of Theorem 9] that for all $n \geq 1$, there exists an equivalent norm $\|\cdot\|_n$ on Z such that

$$r(Y) \leq \frac{1}{2} + \frac{1}{n}$$

for all closed proper subspaces Y of $(Z^*, \|\cdot\|_n^*)$. It follows that if $\{(z_\gamma, z_\gamma^*); \gamma \in \Gamma\}$ is a λ -norming M -basis of $(Z, \|\cdot\|_n)$ with $\lambda < (1/2 + 1/n)^{-1}$, then $\overline{\text{span}}^{\|\cdot\|_n}(z_\gamma^*) = Z^*$. We may and do assume that $\|z_\gamma^*\| \leq 1$ for all γ . The operator $T : Z \rightarrow l_\infty(\Gamma)$ defined by $T(z) = (z_\gamma^*(z))$ takes its values into $c_0(\Gamma)$, and $T^*(l_1(\Gamma))$ contains $\text{span}\{z_\gamma^*; \gamma \in \Gamma\}$ and is therefore norm-dense. But then T^{**} is one-to-one and thus by [2, Corollary VI.5.4] Z is Asplund. But since Z contains $l_1(\mathbf{N})$, this is a contradiction. Hence $(Z, \|\cdot\|_n)$ has no λ -norming M -basis for $\lambda < (1/2 + 1/n)^{-1}$.

We now set

$$X = \left(\sum \oplus (Z, \|\cdot\|_n) \right)_2.$$

The following lemma is a straightforward consequence of [20, Proposition 4.6], and [19, Proposition 2.6] is a stronger statement. Yet we outline the proof for completeness.

Lemma 2.2. *Let V be a Banach space and Z be a subspace of V such that (B_{Z^*}, w^*) is a Corson compact. If V has a λ -norming M -basis,*

then so does Z .

Proof. Let $\{(v_i, f_i), i \in I\}$ be a λ -norming M -basis of $(V, \|\cdot\|)$. If we let

$$\|v\| = \sup\{|f(v)|; f \in \text{span}\{f_i\}, \|f\| \leq 1\}$$

then $\|\cdot\|$ is an equivalent norm on V such that for all $v \in V$,

$$(5) \quad \lambda^{-1}\|v\| \leq \|v\| \leq \|v\|.$$

We equip V , and its subspace Z as well, with $\|\cdot\|$. We let $g_i = f_{i|Z}$ and

$$Y = \overline{\text{span}}^{\|\cdot\|}(\{g_i; i \in I\}).$$

The space Y is a one-norming subspace of $(Z^*, \|\cdot\|^*)$ and since $\{(v_i, f_i)\}$ is an M -basis, we have for all $z \in Z$,

$$(6) \quad |\{i \in I; g_i(z) \neq 0\}| \leq \aleph_0.$$

Since (B_{Z^*, w^*}) is a Corson compact, Z has an M -basis [16, Proposition 4.1] $\{(z_\gamma, h_\gamma); \gamma \in \Gamma\}$. Any Corson compact is angelic, and it follows easily from the Banach-Dieudonné theorem that, for all $h \in Z^*$,

$$(7) \quad |\{\gamma \in \Gamma; h(z_\gamma) \neq 0\}| \leq \aleph_0.$$

This applies in particular to $h = g_i$ for any $i \in I$. Now (6) and (7), together with the fact that $\text{span}\{g_i; i \in I\}$ is $(\|\cdot\|)$ -1-norming, imply that $(Z, \|\cdot\|)$ has a one-norming M -basis. This latter fact is shown in [20, Theorem 2.3] by the techniques used in [2, Chapter VI] and in particular in the proof of [2, Lemma VI.7.5].

Hence $(Z, \|\cdot\|)$ has a one-norming M -basis $\{(u_\alpha, t_\alpha); \alpha \in A\}$. It follows that $(Z, \|\cdot\|)$ has a λ -norming M -basis. Indeed, for all $x \in X$ with $\|x\| = 1$ and all $\varepsilon > 0$, we have by (5) $\|x\| \geq \lambda^{-1}$ and thus there exists $f \in \text{span}\{t_\alpha; \alpha \in A\}$ with $\|f\|^* \leq 1$ and $f(x) > \lambda^{-1} - \varepsilon$. Since $\|f\|^* \leq \|f\|^*$, the conclusion follows. \square

Since X is wcg, B_{Z^*} is Eberlein and thus Corson compact for all $Z \subset X$. Hence, by Lemma 2.2, X contains no λ -norming M -basis if $\lambda < 2$. We now recall

Fact 2.3. *Let $X = S \oplus R$, with S separable and R reflexive. Then X is isometric to a direct sum $X = S_0 \oplus R$, with S_0 separable, R_0 reflexive and S_0 is one-complemented in X .*

Proof. The space X is clearly wcg, and thus (see [2, Lemma VI.2.4]) S is contained into a one-complemented separable subspace S_0 of X . Let $\pi = X \rightarrow S_0$ be a norm-one projection. We have

$$R_0 = \ker \pi \simeq X/S_0,$$

but there is a canonical quotient map from $X/S \simeq R$ onto X/S_0 , and thus R_0 is reflexive. \square

By Fact 2.3, we have for all n ,

$$(Z, \|\cdot\|_n) = S_n \oplus R_n$$

with S_n separable and $(\|\cdot\|_n)$ 1-complemented and R_n reflexive. It follows that

$$X \simeq \left(\sum \oplus S_n \right)_2 \oplus \left(\sum \oplus R_n \right)_2$$

is of the prescribed form. This concludes the proof of Theorem 2.1. \square

It is still unknown whether there exists a wcg space with no norming M -basis. However, the present approach will not suffice for answering this question. Indeed, we have

Proposition 2.4. *Let X be a Banach space which is a direct sum $X = S \oplus R$, where S is separable and R is reflexive. Then X has a four-norming Markushevich basis.*

Proof. By Fact 2.3 we may assume without loss of generality that S is one-complemented in X , with a projection π of kernel R .

Since S is separable, it has a one-norming Markushevich basis $\{(x_n, f_n); n \geq 1\}$ (see [14, p. 44]). The space R is reflexive and thus

it has a Markushevich basis $\{(y_\alpha, g_\alpha); \alpha \in A\}$ which is of course one-norming since $\overline{\text{span}}^{\|\cdot\|}(g_\alpha) = R^*$, by reflexivity. We set

$$\mathcal{B} = \{\{x_n; n \geq 1\} \cup \{y_\alpha; \alpha \in A\}\}$$

and

$$\mathcal{B}^* = \{\{\pi^*(f_n); n \geq 1\} \cup \{(I - \pi^*)(g_\alpha); \alpha \in A\}\}.$$

We claim that $(\mathcal{B}, \mathcal{B}^*)$ is a four-norming M -basis. Indeed, pick $x = x_1 + x_2$ with $x_1 \in S$, $x_2 \in R$. We have

$$\sup\{\|x_0\|, \|x_2\|\} \geq \frac{\|x\|}{2}.$$

Let us assume that $\|x_1\| \geq \|x\|/2$. Pick any $\varepsilon > 0$. By the above, there exists $f \in \text{span}(f_n)$ such that $\|f\| \leq 1$ and $\langle f, x_1 \rangle > \|x\|/2 - \varepsilon$.

Since $x_1 = \pi(x)$, we have $\pi^*(f)(x) = f(x_1) > \|x\|/2 - \varepsilon$ and $\|\pi^*f\| \leq 1$.

Thus, in this case, x is $(1/2)$ -normed by $\text{span}(\mathcal{B}^*)$. If now $\|x_2\| \geq \|x\|/2$, we can proceed along the same lines and find $g \in \text{span}(g_\alpha)$ with $\|g\| \leq 1$ and $\langle g, x_2 \rangle > \|x\|/2 - \varepsilon$.

If we set $h = (I - \pi^*)(g)$, we have $\langle h, x \rangle > \|x\|/2 - \varepsilon$ and $h \in \text{span}(\mathcal{B}^*)$, but this time we only have $\|h\| \leq 2$ since $\|I - \pi\| \leq 2$. Therefore, in that case, x is $(1/4)$ -normed by $\text{span}(\mathcal{B}^*)$. This concludes the proof. \square

I don't know how to fill the gap between $\lambda = 2$ and $\lambda = 4$ (see Theorem 2.1 and Proposition 2.4). We conclude with

Corollary 2.5. *There exists an Eberlein compact K such that $(\mathcal{C}(K), \|\cdot\|_\infty)$ has no λ -norming Markushevich basis if $\lambda < 2$.*

Proof. Let $(X, \|\cdot\|)$ be the space provided by Theorem 2.1. Since X is wcg, $K = (B_{X^*}, w^*)$ is Eberlein compact.

Since X is isometric to a subspace of $(\mathcal{C}(K), \|\cdot\|_\infty)$, it follows from Lemma 2.2 that $(\mathcal{C}(K), \|\cdot\|_\infty)$ has no λ -norming M -basis if $\lambda < 2$.

\square

Note that it follows from Lemma 2.2 that $(C(K), \|\cdot\|_\infty)$ is not isometrically contained into a space which has λ -norming M -basis with $\lambda < 2$.

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