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# CONVOLUTION AND THE FOURIER-WIENER TRANSFORM ON ABSTRACT WIENER SPACE

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ABSTRACT. In this paper we define the convolution of functionals on abstract Wiener space and establish the relationship between the Fourier-Wiener transforms of each functional and the Fourier-Wiener transform of their convolution. Also we obtain Parseval's and Plancherel's relation from the above relationship. The main results in a paper of Yeh then follow from our results as corollaries.

**1. Introduction.** In their papers [**1, 2**] Cameron and Martin introduced and established the existence of the Fourier-Wiener transform for certain classes of functionals on classical Wiener space. Further, they established an appropriate version of the formulas of Plancherel and Parseval. J. Yeh [**13**] also defined the convolution of functionals on classical Wiener space and proved the relationship between the Fourier-Wiener transforms of each functional and the Fourier-Wiener transform of their convolution. After that, Cameron and Storvick [**3**] defined an $L_2$ analytic Fourier-Feynman transform on Wiener space, and this concept was extended to $L_p$ by Johnson and Skoug [**8**].

More recently, Y.J. Lee [**11**] extended the Fourier-Wiener transform on classical Wiener space to that on abstract Wiener space and applied this transform to differential equations on infinite dimensional spaces.

In this paper we define the convolution of functionals on abstract Wiener space and examine the relationship between the Fourier-Wiener transforms of each functional and the Fourier-Wiener transform of their convolution. Also we establish Parseval's relation and Plancherel's relation from the above relationship. The main results in [**13**] will then be corollaries of our results.

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**2. Preliminaries.** Let $H$ be a real, separable infinite dimensional Hilbert space with inner product $\langle\ ,\ \rangle$ and norm $||\cdot||$. Let $|||\cdot|||$ be a measurable norm on $H$ with respect to the Gaussian cylinder set measure $m$ on $H$. Let $B$ denote the completion of $H$ with respect to $|||\cdot|||$. Let $i$ denote the natural injection from $H$ into $B$. The adjoint operator $i^*$ of $i$ is one-to-one and maps $B^*$ continuously onto a dense subset of $H^*$. By identifying $H$ with $H^*$ and $B^*$ with $i^*B^*$, we have a triple $B^* \subset H^* \equiv H \subset B$ and $\langle y, x\rangle = (y, x)$ for all $x$ in $H$ and $y$ in $B^*$, where $(\cdot,\cdot)$ denotes the natural dual pair between $B^*$ and $B$. By a well-known result of Gross [**6**], $m \circ i^{-1}$ has a unique countable additive extension $\nu$ to the Borel $\sigma$-algebra $\mathbf{B}(B)$ of $B$. The triple $(H, B, \nu)$ is called an abstract Wiener space and the Hilbert space $H$ is called the generator of $(H, B, \nu)$. For more details, see [**6, 9, 10, 11**].

Let $\{e_n\}$ be a complete orthonormal (CON) system in $H$ with the $e_n$'s in $B^*$. For each $h$ in $H$ and $x$ in $B$, we define

$$(h, x)^\sim = \lim_{n\to\infty} \sum_{k=1}^{n} \langle h, e_k\rangle (e_k, x)$$

if the limit exists and $(h, x)^\sim = 0$ otherwise. It is well known that for each $h$ $(\neq 0)$ in $H$, $(h, \cdot)^\sim$ is a Gaussian random variable on $B$ with mean zero and variance $||h||^2$.

Let $[X]$ denote the complexification of a real Banach space $X$ where $[X] = \{x_1 + ix_2 : x_1, x_2 \in X\}$ and $||x_1 + x_2||_{[\mathrm{x}]} = (||x_1||_{\mathrm{x}}^2 + ||x_2||_{\mathrm{x}}^2)^{1/2}$. For notational convenience, we use $|||\cdot|||$ as the norm for both $B$ and $[B]$.

**Definition 2.1.** Let $F$ be a functional defined on $[B]$. If the following integral exists, then the functional

$$G_F(y) = \int_B F(x + iy)\, d\nu(x) \quad \text{for } y \in [B] \tag{2.1}$$

is called the *Fourier-Wiener transform* of $F$. In this case

$$G_F^{-1}(y) = \int_B F(x - iy)\, d\nu(x)$$

is called the inverse Fourier-Wiener transform of $F$. Note that $G_F^{-1}(y) = G_F(-y)$.

Now we consider some classes of functionals. The first class $\mathcal{E}_0$ consists of functionals of the form

$$F(x) = \Phi[(h_1, x)^\sim, \dots, (h_n, x)^\sim] \tag{2.2}$$

where $\{h_1, \dots, h_n\}$ is a set of linearly independent elements in $B^*$ and $\Phi(z_1, \dots, z_n)$ is an entire function of $n$ complex variables of exponential type

$$|\Phi(z_1, \dots, z_n)| \le A \exp\left\{ B \sum_{k=1}^{n} |z_k| \right\}. \tag{2.3}$$

The second class $\mathcal{E}_a$ consists of functionals $F(x)$ satisfying the following conditions:

(i) $F(x + zy)$ is an entire function of the complex variable $z$ for all $x$ and $y$ in $[B]$, and

(ii) there exist positive constants $A_F$ and $B_F$ depending on $F$ such that

$$|F(x)| \le A_F \exp\{B_F |||x|||\} \quad \text{for all } x \in [B]. \tag{2.4}$$

*Remark* 2.2. When $B = C_0[0, 1]$, the classical Wiener space, $\mathcal{E}_0$ is the space $E_0$ and $\mathcal{E}_a$ contains the space $E_m$ introduced by Cameron and Martin [**2**].

As in the proof of Theorem 1 in [**2**] and by Proposition 3.3 in [**11**] and Lemma 2.3 in [**12**], we introduce the following theorems without proof.

**Theorem 2.3.** *If $F(x)$ belongs to $\mathcal{E}_0$ or $\mathcal{E}_a$, then its Fourier-Wiener transform $G_F(y)$ exists for all $y \in [B]$ and belongs to the same class.*

**Theorem 2.4.** *If $F \in \mathcal{E}_0$ or $\mathcal{E}_a$ and $\alpha, \beta \in \mathbf{C}$, then*

$$\int_B \int_B F(\alpha x + \beta y)\, d\nu(x) d\nu(y) = \int_B F(\sqrt{\alpha^2 + \beta^2} z)\, d\nu(z).$$

**Corollary 2.5.** *For $F \in \mathcal{E}_0$ or $\mathcal{E}_a$, we have $G^{-1}_{G_F}(z) = F(z)$ for $z \in [B]$.*

*Proof.* Use $G_F^{-1}(z) = G_F(-z)$ and Theorem 2.4. □

**3. Convolution in Fourier-Wiener transform.** We begin this section by defining the convolution of two functionals defined on $[B]$.

**Definition 3.1.** The convolution of two functionals $F_1(x)$ and $F_2(x)$ is defined on $[B]$ by

$$(F_1 * F_2)(x) = \int_B F_1\left(\frac{y+x}{\sqrt{2}}\right) F_2\left(\frac{y-x}{\sqrt{2}}\right) d\nu(y) \tag{3.1}$$

if the integral on the right side exists.

**Theorem 3.2.** *If $F_1(x), F_2(x) \in \mathcal{E}_0$, then the convolution $(F_1 * F_2)(x)$ exists for every $x \in [B]$ and belongs to $\mathcal{E}_0$. Moreover, the Fourier-Wiener transform $G_{F_1*F_2}(z)$ of $F_1 * F_2$ exists and satisfies*

$$G_{F_1*F_2}(z) = G_{F_1}\left(\frac{z}{\sqrt{2}}\right) G_{F_2}\left(-\frac{z}{\sqrt{2}}\right) \quad \text{for every } z \in [B]. \tag{3.2}$$

*Proof.* We first prove the theorem for the special case where $\{h_1, \dots, h_n\}$ is an orthonormal set in $B^*$. Let

$$F_k(x) = \Phi_k[(h_1, x)^\sim, \dots, (h_n, x)^\sim]$$

where $\Phi_k(z_1, \dots, z_n)$, $k = 1, 2$, are two entire functions of exponential type of $n$ complex variables. Since each $(h_j, x)^\sim$ is normally distributed with mean zero and variance one, we have

$$
\begin{aligned}
(F_1 * F_2)(x) &= \int_B F_1\left(\frac{y+x}{\sqrt{2}}\right) F_2\left(\frac{y-x}{\sqrt{2}}\right) d\nu(y) \\
&= \int_B \Phi_1\left[\frac{1}{\sqrt{2}}((h_1,y)^\sim + (h_1,x)^\sim, \dots, (h_n,y)^\sim + (h_n,x)^\sim)\right] \\
&\quad \Phi_2\left[\frac{1}{\sqrt{2}}((h_1,y)^\sim - (h_1,x)^\sim, \dots, (h_n,y)^\sim - (h_n,x)^\sim)\right] d\nu(y) \\
&= (2\pi)^{-n/2} \int_{R^n} \Phi_1\left[\frac{1}{\sqrt{2}}(u_1 + (h_1,x)^\sim, \dots, u_n + (h_n,x)^\sim)\right] \\
&\qquad \Phi_2\left[\frac{1}{\sqrt{2}}(u_1 - (h_1,x)^\sim, \dots, u_n - (h_n,x)^\sim)\right] \\
&\qquad\qquad \exp\left\{-\sum_{k=1}^n \frac{u_k^2}{2}\right\} du_1 \cdots du_n
\end{aligned}
$$

where the last integral exists and belongs to $\mathcal{E}_0$ by Remarks 1–3 in [**13**].

From Remark 3 in [**13**] and Theorem 2.3, it follows that the Fourier-Wiener transform $G_{F_1 * F_2}(z)$ of $F_1 * F_2$ exists for every $z \in [B]$ and is given by

$$
\begin{aligned}
G_{F_1 * F_2}(z) &= \int_B (F_1 * F_2)(x + iz)\, d\nu(x) \\
&= (2\pi)^{-n} \int_{R^{2n}} \Phi_1\left[\frac{1}{\sqrt{2}}(u_1 + v_1 + i(h_1,z)^\sim, \dots, \right. \\
&\qquad\qquad \left. u_n + v_n + i(h_n,z)^\sim)\right] \\
&\quad \Phi_2\left[\frac{1}{\sqrt{2}}(u_1 - v_1 - i(h_1,z)^\sim, \dots, u_n - v_n - i(h_n,z)^\sim)\right] \\
&\qquad\qquad \exp\left\{-\sum_{k=1}^n \frac{u_k^2 + v_k^2}{2}\right\} du_1 \cdots du_n\, dv_1 \cdots dv_n.
\end{aligned}
$$

Next, letting $p_k = (u_k + v_k)/\sqrt{2}$ and $q_k = (u_k - v_k)/\sqrt{2}$ for $k =$

$1, 2, \dots, n$, we obtain

$$\begin{aligned} G_{F_1 * F_2}(z) = (2\pi)^{-n} \int_{R^{2n}} & \Phi_1\Big[p_1 + \frac{i}{\sqrt{2}}(h_1, z)^\sim, \dots, p_n + \frac{i}{\sqrt{2}}(h_n, z)^\sim\Big] \\ & \Phi_2\Big[q_1 - \frac{i}{\sqrt{2}}(h_1, z)^\sim, \dots, q_n - \frac{i}{\sqrt{2}}(h_n, z)^\sim\Big] \\ & \exp\Big\{-\sum_{k=1}^{n} \frac{p_k^2 + q_k^2}{2}\Big\}\, dp_1 \cdots dp_n\, dq_1 \cdots dq_n \\ = \int_B F_1\Big(x + i\frac{z}{\sqrt{2}}\Big)\, d\nu(x) & \int_B F_2\Big(x - i\frac{z}{\sqrt{2}}\Big)\, d\nu(x) \\ = G_{F_1}\Big(\frac{z}{\sqrt{2}}\Big) G_{F_2}\Big(-\frac{z}{\sqrt{2}}\Big). & \end{aligned}$$

In the general case where $\{h_1, \dots, h_n\}$ is a set of linearly independent element in $B^*$, according to the Gram-Schmidt orthonormalization process, we can write $F_k(x)$, $k = 1, 2$, defined by

$$F_k(x) = \Phi_k^*[(e_1, x)^\sim, \dots, (e_n, x)^\sim]$$

where the $\Phi_k^*[z_1, \dots, z_n]$s are entire functions of exponential type and $\{e_1, \dots, e_n\}$ is an orthonormal set in $B^*$. Now the result for the special case applies, and the theorem is proved. □

**Proposition 3.3.** *Let* $\{F_{1,n}(x)\}$, $F_1(x)$, $\{F_{2,n}(x)\}$, $F_2(x)$ *be such that*

(i) $\lim_{n\to\infty} F_{k,n}(x) = F_k(x)$ *for every* $x \in [B]$, $k = 1, 2$,

(ii) *the Fourier-Wiener transform exists for every* $F_{k,n}$, $n = 1, 2, \dots$, $k = 1, 2$; *the convolution* $(F_{1,n} * F_{2,n})(x)$ *exists, its Fourier-Wiener transform also exists and satisfies*

$$G_{F_{1,n} * F_{2,n}}(z) = G_{F_{1,n}}\Big(\frac{z}{\sqrt{2}}\Big) G_{F_{2,n}}\Big(-\frac{z}{\sqrt{2}}\Big) \tag{3.3}$$

*for every* $z \in [B]$ *for* $n = 1, 2, \dots$,

(iii) $|F_{k,n}(x)| \le A \exp\{B|||x|||\}$ *for* $n = 1, 2, \dots$, $k = 1, 2$, *where* $A, B > 0$. *Then the Fourier-Wiener transforms of* $F_1$ *and* $F_2$ *exist, the* *convolution* $(F_1 * F_2)(x)$ *exists for all* $x \in [B]$ *and has a Fourier-Wiener transform, and, finally,* (3.2) *holds.*

*Proof.* The equality (3.3) can be written

$$(3.4)\quad \int_B \left( \int_B F_{1,n}\left( \frac{y+x+iz}{\sqrt{2}} \right) F_{2,n}\left( \frac{y-x-iz}{\sqrt{2}} \right) d\nu(y) \right) d\nu(x) = \int_B F_{1,n}\left( x + i\frac{z}{\sqrt{2}} \right) d\nu(x) \int_B F_{2,n}\left( x - i\frac{z}{\sqrt{2}} \right) d\nu(x)$$

for $n = 1, 2, \dots$. Now we observe that for any $n$ complex numbers $z_1, \dots, z_n$,

$$(3.5)\quad \left| \sum_{k=1}^{n} z_k \right| \le n^2 \sum_{k=1}^{n} |z_k|.$$

By (iii) and (3.5), we obtain

$$(3.6)\quad \left| F_{1,n}\left( x + i\frac{z}{\sqrt{2}} \right) \right| \le A \exp\{4B(|||x||| + |||z|||)\}.$$

Since $\int_B \exp\{K|||x||\}\, d\nu(x) < \infty$ for every $K > 0$, the right side of (3.6) is integrable with respect to $x$ over $B$ for fixed $z$. Hence, (i), (2.1) and the dominated convergence theorem give

$$(3.7)\quad \lim_{n\to\infty} \int_B F_{k,n}\left( x + (-1)^{k+1} i \frac{z}{\sqrt{2}} \right) d\nu(x) = G_{F_k}\left( (-1)^{k+1} \frac{z}{\sqrt{2}} \right)$$

for $k = 1, 2$, and for every $z \in [B]$.

Similarly, by (3.5), the left side of (3.4) is finite for every $z \in [B]$ and hence, by (i) and the dominated convergence theorem, and by (2.1) and (3.1),

$$(3.8)\quad \lim_{n\to\infty} \int_B \left( \int_B F_{1,n}\left( \frac{y+x+iz}{\sqrt{2}} \right) F_{2,n}\left( \frac{y-x-iz}{\sqrt{2}} \right) d\nu(y) \right) d\nu(x) = G_{F_1 * F_2}(z)$$

for every $z \in [B]$. By letting $n \to \infty$ on both sides of (3.4), and by (3.7) and (3.8), this proposition is established. □

Let $\mathcal{E}_a^*$ be the class of functionals $F$ in $\mathcal{E}_a$ such that $\lim_{n\to\infty} F(x_n) = F(x)$ holds for all $x$ and $x_n$ in $[B]$ for which $|||x_n - x||| \to 0$ as $n \to \infty$.

**Theorem 3.4.** *Let $(H, B, \nu)$ be the abstract Wiener space such that, for any $x \in B$, $\sum_{n=1}^{\infty}(\alpha_n, x)^{\sim}\alpha_n$ converges in norm to $x$ where $\{\alpha_n\}$ is a CON set in $H$ which lies in $B^*$. If $F_1(x), F_2(x) \in \mathcal{E}_a^*$, then the convolution $(F_1 * F_2)(x)$ exists for every $x \in [B]$. Moreover, its Fourier-Wiener transform $G_{F_1*F_2}(z)$ exists and satisfies* (3.2).

*Proof.* Let

$$F_{k,n}(x) = F_k\left[\sum_{k=1}^{n}(\alpha_k, x)^{\sim}\alpha_k\right] \tag{3.9}$$

and let

$$x_n = \sum_{k=1}^{n}(\alpha_k, x)^{\sim}\alpha_k \tag{3.10}$$

for $n = 1, 2, \dots$ and $k = 1, 2$. To prove this theorem, it suffices to show that $\{F_{1,n}(x)\}, F_1(x), \{F_{2,n}(x)\}$ and $F_2(x)$ satisfy the conditions of Proposition 3.3. From the definition of $\mathcal{E}_a^*$, it follows that

$$\lim_{n\to\infty} F_{k,n}(x) = F_k(x)$$

for every $x \in [B]$ and $k = 1, 2$.

Let $\Phi_{k,n}[z_1, \dots, z_n]$ be defined by

$$\Phi_{k,n}[z_1, \dots, z_n] = F_k\left[\sum_{j=1}^{n} z_j\alpha_j\right] \tag{3.11}$$

for $n = 1, 2, \dots$ and $k = 1, 2$. By the definition of $\mathcal{E}_a^*$ and Hartog's regularity theorem in [**7**], $\Phi_{k,n}$ is an entire function of the $n$ complex variables $\{z_1, \dots, z_n\}$ and also that it is of exponential type follows from

$$|\Phi_{k,n}[z_1, \dots, z_n]| \le A_k \exp\left\{ B_k \left|\left|\left| \sum_{j=1}^{n} z_j \alpha_j \right|\right|\right| \right\} \\ \le A_k \exp\left\{ B_k \sum_{j=1}^{n} |z_j| \right\}$$

for $n = 1, 2, \dots$ and $k = 1, 2$. On the other hand, from (3.9) and (3.11), it follows that

$$F_{k,n}(x) = \Phi_{k,n}[(\alpha_1, x)^\sim, \dots, (\alpha_n, x)^\sim]$$

for $n = 1, 2, \dots$ and $k = 1, 2$. Therefore, each $F_{k,n}(x)$ belongs to $\mathcal{E}_0$, and hence its Fourier-Wiener transform exists. Moreover, by Theorem 3.2, the convolution $(F_{1,n} * F_{2,n})(x)$ exists and satisfies (3.3) for every $z \in [B]$ for every $n = 1, 2, \dots$.

Finally, let $A = \max\{A_{F_1}, A_{F_2}\}$ and $B = \max\{B_{F_1}, B_{F_2}\}$. Since $\sum_{k=1}^{n} (\alpha_k, x)^\sim \alpha_k$ converges in norm to $x$,

$$|F_{k,n}(x)| \le A \exp\left\{ B \left|\left|\left| \sum_{j=1}^{n} (\alpha_j, x)^\sim \alpha_j \right|\right|\right| \right\} \le A^* \exp\{B^* |||x|||\}$$

for some positive constants $A^*, B^*$. By the conclusion of Proposition 3.3, this theorem is proved. □

*Remark* 3.5. Let $(H, B, \nu) = (C_0'[0,1], C_0[0,1], m)$ be the classical Wiener space where $C_0'[0,1] = \{x \in C_0[0,1] : x(t) = \int_0^t f(s)\, ds, f \in L_2[0,1]\}$ with inner product $\langle x_1, x_2 \rangle = \int_0^1 (dx_1/dt)(t)(dx_2/dt)(t)\, dt$. Let $\{h_n(s)\}$ be the Haar functions which are a CON set on $[0,1]$. Then $\{\alpha_n(t) = \int_0^t h_n(s)\, ds\}$ is an orthonormal basis of $C_0'[0,1]$ and, for any $x \in C_0[0,1]$,

$$x(t) = \sum_{n=1}^{\infty} \int_0^1 h_n(s)\, dx(s) \alpha_n(t) = \sum_{n=1}^{\infty} (\alpha_n, x)^\sim \alpha_n(t)$$

in the uniform topology on $C_0[0,1]$ (see [**4**]). Thus, $(C_0'[0,1], C_0[0,1], m)$ satisfies the hypothesis of Theorem 3.4.

**Corollary 3.6.** *Under the hypothesis of Theorem* 3.4, *if* $F_1(x)$ *and* $F_2(x)$ *belong to* $\mathcal{E}_0$ *or* $\mathcal{E}_a^*$, *then Parseval's relation*

$$\int_B F_1\left(\frac{x}{\sqrt{2}}\right)F_2\left(-\frac{x}{\sqrt{2}}\right)d\nu(x) = \int_B G_{F_1}\left(\frac{x}{\sqrt{2}}\right)G_{F_2}\left(\frac{x}{\sqrt{2}}\right)d\nu(x) \tag{3.12}$$

*holds. Moreover, the formula* (3.12) *induces Plancherel's relation of the form*

$$\int_B \left|F\left(\frac{x}{\sqrt{2}}\right)\right|^2 d\nu(x) = \int_B \left|G_F\left(\frac{x}{\sqrt{2}}\right)\right|^2 d\nu(x). \tag{3.13}$$

*Proof*. From Theorem 3.2, Theorem 3.4 and Theorem 2.4, it follows that (3.12) holds. Also by (3.12) and letting $G_{F_2}(z) = F_3(z)$, we obtain

$$\int_B G_{F_1}\left(\frac{z}{\sqrt{2}}\right)F_3\left(\frac{z}{\sqrt{2}}\right)d\nu(z) = \int_B F_1\left(\frac{z}{\sqrt{2}}\right)G_{F_2}\left(\frac{z}{\sqrt{2}}\right)d\nu(z)$$

and hence we have our formula (3.13) by using $G_F^{-1}(z) = G_F(-z)$, $\overline{G_F(z)} = G_F^{-1}(z)$ and Corollary 2.5, which are the same trick found in [**11**, Corollary 3.12]. □

*Remark* 3.7. Main results, Theorem 1 and Theorem 2, in [**13**], are the special case of our Theorem 3.2 and Theorem 3.4, respectively. Moreover, we can apply our results to many examples of abstract Wiener spaces.

## REFERENCES

**1.** R.H. Cameron, *Some examples of Fourier-Wiener transforms of analytic functionals*, Duke Math. J. **12** (1945), 485–488.

**2.** R.H. Cameron and W.T. Martin, *Fourier-Wiener transforms of analytic functionals*, Duke Math. J. **12** (1945), 489–507.

**3.** R.H. Cameron and D.A. Storvick, *An $L_2$-analytic Fourier-Feynman transform*, Michigan Math. J. **23** (1976), 1–30.

**4.** H.C. Finlayson, *Measurability of norm proved by Haar functions*, Proc. Amer. Math. Soc. **53** (1975), 334–336.

**5.** B.A. Fuks, *Introduction to the theory of analytic functions of several complex variables*, Amer. Math. Soc., Providence, RI, 1963.

**6.** L. Gross, *Abstract Wiener space*, Proc. Fifth Berkeley Sympos. Math. Stat. Prob. **2** (1965), 31–42.

**7.** F. Hartogs, *Zur Theorie der Analytischen Funktionen Mehrerer Unabhangiger Veranderlichen*, Math. Ann. **62** (1906), 1–88.

**8.** G.W. Johnson and D.L. Skoug, *An $L_p$ analytic Fourier-Feynman transform*, Michigan Math. J. **26** (1979), 103–127.

**9.** J. Kuelbs, *Abstract Wiener spaces and applications to analysis*, Pacific J. Math. **31** (1969), 433–450.

**10.** H.H. Kuo, *Gaussian measures in Banach spaces*, Lecture Notes in Math. **463**, Springer-Verlag, Berlin, Heidelberg, New York (1975).

**11.** Y.J. Lee, *Applications of the Fourier-Wiener transform to differential equations on infinite dimensional spaces* I, Trans. Amer. Math. Soc. **262** (1980), 259–283.

**12.** ———, *Integral transforms of analytic functions on abstract Wiener spaces*, J. Funct. Anal. **47** (1982), 153–164.

**13.** J. Yeh, *Convolution in Fourier-Wiener transform*, Pacific J. Math. **15** (1965), 731–738.

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