ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 29, Number 2, Summer 1999

MEROMORPHIC FUNCTIONAL CALCULUS AND LOCAL SPECTRAL THEORY

TERESA BERMÚDEZ

ABSTRACT. We study the behavior of the local spectrum of a bounded linear operator on the vectors of the range of the meromorphic functional calculus. In particular we analyze some relations of the restriction and coinduced operators of the meromorphic functional calculus. We also obtain the local spectral mapping theorem and conditions for the stability of the single valued extension property (SVEP).

1. Introduction. Let X be a complex Banach space, let $T, S \in L(X)$ be commuting continuous linear operators and let $x \in X$. Denoting by $\sigma(x, T)$ the local spectrum of T at x, we have [**5**, Proposition 1.5] that

$$\sigma(Sx, T) \subset \sigma(x, T). \tag{1}$$

Bartle [**1**] derived the following relations for $n \in \mathbf{N}$ and $S = (\alpha - T)^n$

$$\sigma(Sx, T) \subset \sigma(x, T) \subset \sigma(Sx, T) \cup \{\alpha\}, \tag{2}$$

where α is a complex number. Similar results have been derived in [**3**] for S, an operator given by the local functional calculus (see [**4**] for further details).

In this paper we study this problem when S is given by the meromorphic functional calculus. We observe that this operator is closed and unbounded, in general.

As an application we derive some properties of the meromorphic functional calculus such as: relations of the restriction and coinduced operators of the meromorphic functional calculus, the local spectral mapping theorem (with a different proof from the usual one for the

Received by the editors on February 1, 1997.
Partially supported by Consejería de Educación, Gobierno Autónomo de Canarias, proyecto 967/94 and by DGICYT grant PB 94-0591, Spain.

holomorphic functional calculus) and the stability of the single valued extension property.

2. Preliminaries. Let X be a complex Banach space. We denote by $L(X)$ the class of all (bounded linear) operators on X and by $C(X)$ the class of all closed linear operators with *domain* $D(A)$ and *range* $R(A)$ in X. We say that a closed subspace $Y \subset X$ is an *invariant subspace* under $A \in C(X)$, in symbols, $Y \in \operatorname{Inv}(T)$, if $A(Y \cap D(A)) \subset Y$. An invariant subspace Y produces two operators: the *restriction* $A|Y$ defined in $D(A) \cap Y$ by $A|Yy = Ay$ and the *coinduced* operator A/Y on the quotient space X/Y, defined by and $(A/Y)(x+Y) = Ax + Y$ on

$$D(A/Y) := \{x + Y \in X/Y : (x+Y) \cap D(A) \neq \varnothing\}.$$

Given an operator $A \in C(X)$, a complex number λ belongs to the *resolvent set* $\rho(A)$ of A if there exists $R(\lambda, A) := (\lambda - A)^{-1} \in L(X)$. We denote by $\sigma(A) := \mathbf{C} \setminus \rho(A)$ the *spectrum* of A. The *resolvent map* $R(., A) : \rho(A) \to L(X)$ is analytic.

Moreover, given an arbitrary closed linear operator $A : D(A) \subset X \to X$ and $x \in X$, we say that a complex number λ belongs to the *local resolvent set* of A at x, denoted $\rho(x, A)$, if there exists an analytic function $w : U \subset \mathbf{C} \to D(A)$, defined on a neigborhood U of λ, which satisfies

$$(\mu - A)w(\mu) = x, \tag{3}$$

for every $\mu \in U$. The *local spectrum set* of A at x is $\sigma(x, A) := \mathbf{C} \setminus \rho(x, A)$.

Since w is not necessarily unique, a complementary property is needed to prevent ambiguity. A linear operator A satisfies the single valued extension property, hereafter referred to as SVEP, if for every analytic function $h : U \to D(A)$ defined on an open $U \subset \mathbf{C}$, the condition $(\lambda - A)h(\lambda) \equiv 0$ implies $h \equiv 0$. If A satisfies the SVEP, then for every $x \in X$ there exists a unique analytic function $\hat{x}_A$ defined on $\rho(x, A)$ satisfying (3), which is called the *local resolvent function* of A at x.

We denote also by $\mathbf{C}_\infty$ the one-point compactification of the complex field $\mathbf{C}$. Given $A \in C(X)$ and $x \in X$, then $\infty \in \rho_\infty(x, A) := \mathbf{C}_\infty \setminus \sigma_\infty(x, A)$, if there exist an open neighborhood U_∞ and an analytic function $u : U_\infty \to D(A)$ such that $(\mu - A)u(\mu) = x$ for $\mu \in U_\infty \cap \mathbf{C}$.

For every subset $H \subset \mathbf{C}$, $X(A,H) = \{x \in X : \sigma(x,A) \subset H\}$ is a linear manifold of X. If $X(A,F)$ is closed for all closed sets F, we say that A has property (C). If $T \in L(X)$ satisfies property (C), then T has the SVEP, as proved in [**11**, Theorem 2.3].

For $T \in L(X)$, the *holomorphic functional calculus* is defined as follows [**12**]. Let f be an analytic function defined on an open set $\triangle(f)$ containing $\sigma(T)$. The operator $f(T) \in L(X)$ is defined by the "Cauchy formula"

$$f(T) := \frac{1}{2\pi i} \int_\Gamma f(\lambda) R(\lambda, T)\, d\lambda,$$

where Γ is the boundary of a Cauchy domain D such that $\sigma(T) \subset D \subset \overline{D} \subset \triangle(f)$.

The definition of the holomorphic functional calculus was extended to meromorphic functions by Gindler [**8**]. Let $T \in L(X)$, let f be a meromorphic function on an open set $\Omega_T(f)$ containing $\sigma(T)$ and let $\alpha_1, \dots, \alpha_k$ be the poles of f in $\sigma(T)$, with multiplicities $n_1, \dots, n_k$, respectively. We assume that the poles of f are not eigenvalues of T, and consider the polynomial $p(\lambda) = \prod_{i=1}^k (\alpha_i - \lambda)^{n_i}$.

The function $g(\lambda) := f(\lambda)p(\lambda)$ is analytic on a neighborhood of $\sigma(T)$. So we can define the operator $f\{T\}$ of the *meromorphic functional calculus* by

$$f\{T\} := g(T)p(T)^{-1},$$

obtaining a closed operator $f\{T\}$. Obviously, the meromorphic calculus is an extension of the holomorphic calculus.

We denote, by $M(T)$, the class of the admissible functions of the meromorphic functional calculus for T.

3. Main results. Before analyzing relations between $f\{T\}$, $f\{T|Y\}$ and $f\{T/Y\}$, we need the definition of some classes of closed invariant subspaces.

Recall that, given $A \in C(X)$, an invariant subspace Y of A is said to be a *ν-space* of A if $\sigma(A|Y) \subset \sigma(A)$ [**6**, Definition 4.1]. And Y is said to be an *A-absorbent* space if, for any $y \in Y$ and all $\lambda \in \sigma(A|Y)$, the equation $(\lambda - A)x = y$ has all solutions $x \in Y$ [**6**, Definition 4.17].

It is clear that if Y is an A-absorbent space then it is a ν-space [**6**, Proposition 4.18].

Lemma 1. *Let $T \in L(X)$. If Y is a T-absorbent space, then $Y \in \operatorname{Inv}(p(T)) \cap Inv(p(T)^{-1})$ and*

$$p(T)^{-1}/Y = (p(T)/Y)^{-1}$$

where p is a polynomial such that its zeros are not eigenvalues of T.

Proof. It is clear that if Y is a T-absorbent space then Y is invariant of $p(T)$ and $p(T)^{-1}$. Moreover,

$$(p(T)^{-1}/Y)(p(T)x + Y) = x + Y = (p(T)/Y)^{-1}(p(T)x + Y). \qquad \square$$

Theorem 1. *Let $T \in L(X)$ and $f \in M(T)$. If Y is a T-absorbent space, then the following properties hold:*

(i) *$f \in M(T|Y)$ and $f\{T\}|Y = f\{T|Y\}$.*

(ii) *$f \in M(T/Y)$ and $f\{T\}/Y = f\{T/Y\}$.*

Proof. (i) It is clear that $f \in M(T|Y)$, since if α is a pole of f on $\sigma(T|Y)$, then α is a pole of f on $\sigma(T)$. Let $f(\lambda) = g(\lambda)p(\lambda)^{-1}$, where g is an analytic function on $\sigma(T)$ and p is the polynomial of the poles on $\sigma(T)$ defined as $p(\lambda) = \prod_{j=1}^{k}(\alpha_j - \lambda)^{n_j}$. And let $f(\lambda) = h(\lambda)q(\lambda)^{-1}$, where h is an analytic function on $\sigma(T|Y)$ and q is the polynomial of the poles on $\sigma(T|Y)$ defined as $q(\lambda) = \prod_{i=1}^{m}(\alpha_{k_i} - \lambda)^{n_{k_i}}$ with $m \le k$. It is clear by [**9**] that

$$\begin{aligned} D(f\{T\}) \cap Y &= \bigcup_{j=1}^{k} R(\alpha_j - T)^{n_j} \cap Y \\ &= \bigcup_{i=1}^{m} R(\alpha_{k_i} - T|Y)^{n_{k_i}} = D(f\{T|Y\}). \end{aligned}$$

Let $y \in D(f\{T|Y\})$. By using [**2**, Theorem 2.9] and basic properties of the meromorphic functional calculus we obtain that

$$f\{T|Y\}y = h(T|Y)q(T|Y)^{-1}y = g(T)p(T)^{-1}y = f\{T\}y.$$

This concludes the proof of (i).

(ii) Suppose that there exists an eigenvalue α of $T|Y$, which is a pole of f. Hence there exists $x+Y \in X/Y$ such that

$$(\alpha - T/Y)(x+Y) = 0.$$

Then $(\alpha - T)x \in Y$. By considering that Y is a T-absorbent space, then $x \in Y$, i.e., $f \in M(T/Y)$. Applying Lemma 1 and [**2**, Theorem 4.4], we have

$$f\{T/Y\} = p(T/Y)^{-1}g(T/Y) = (p(T)^{-1}g(T))/Y. \qquad \square$$

In order to show the connection between the meromorphic functional calculus and the local spectrum, the following results will be useful.

The content of the following proposition is the same as that of [**3**, Corollary 1] but we give a different proof.

Proposition 1. *Assume* $T \in L(X)$ *has the* SVEP *and* p *is a polynomial such that the zeros are not eigenvalues of* T. *Then*

$$\sigma(p(T)x, T) = \sigma(x,T), \tag{4}$$

for all $x \in X$.

Proof. In order to prove equation (4) it is enough to prove it for $p(T) = \alpha - T$ where α is not an eigenvalue of T. By using the following inclusion from [**1**]

$$\sigma((\alpha - T)x, T) \subset \sigma(x,T) \subset \sigma((\alpha - T)x, T) \cup \{\alpha\},$$

it is sufficient to prove that if $\alpha \in \sigma(x,T)$, then $\alpha \in \sigma((\alpha - T)x, T)$. Suppose that $\alpha \in \rho(y,T)$ where $y = (\alpha - T)x$, then by [**6**, Theorem 2.2] there exist a number $R > 0$ and a sequence $\{y_k\}_{k=0}^{\infty} \subset X$ such that $(\alpha - T)y_0 = y$, $(\alpha - T)y_k = y_{k-1}$ for $k \geq 1$ and $\|y_k\| \leq R^k$ for $k \geq 0$ and, moreover, $\hat{y}_T(\lambda) = \sum_{k=0}^{\infty} y_k(\alpha - \lambda)^k$. Hence, the analytic function

$$u(\lambda) := \sum_{k=0}^{\infty} y_{k+1}(\alpha - \lambda)^k$$

satisfies the following equality

$$(\lambda - T)u(\lambda) = x$$

in a neighborhood of α. Then $\alpha \in \rho(x, T)$. □

Proposition 2. *Assume $T \in L(X)$ has the* SVEP *and $f \in M(T)$. Then the following properties hold*:

(i) *If $S \in L(X)$ commutes with T, then S commutes with $f\{T\}$.*

(ii) *If $x \in D(f\{T\})$, then $\sigma(f\{T\}x, T) \subset \sigma(x, T)$.*

(iii) *If $x \in D(f\{T\})$, then*

$$\sigma(x, T) = \sigma(f\{T\}x, T) \cup Z_x(f, T),$$

where $Z_x(f, T)$ denotes the set of all zeros of f in $\sigma(x, T)$.

Proof. (i) Obvious from the definition of the meromorphic functional calculus.

(ii) By applying [**9**] we obtain that, if $x \in D(f\{T\}) = R(p(T))$, then $x = p(T)y$ for some $y \in X$. Then by [**5**, Proposition 1.5] we obtain that

$$\sigma(f\{T\}x, T) = \sigma(g(T)p(T)^{-1}x, T) = \sigma(g(T)y, T) \subset \sigma(y, T),$$

since $g(T) \in L(X)$ and commutes with T. Moreover by Proposition 1 we have that

$$\sigma(x, T) = \sigma(p(T)y, T) = \sigma(y, T).$$

Hence $\sigma(f\{T\}x, T) \subset \sigma(x, T)$, for all $x \in D(f\{T\})$.

(iii) By using [**3**, Proposition 6] and Proposition 1 we have that

$$\sigma(x, T) \subset \sigma(g(T)x, T) \cup Z_x(f, T) = \sigma(g(T)p(T)^{-1}x, T) \cup Z_x(f, T).$$

The other inclusion is obvious. □

Remark 1. (a) Notice that part (ii) and (iii) of the above proposition are similar versions of equations (1) and (2) respectively for the meromorphic functional calculus.

(b) If we assume in part (ii) that f is an analytic function on $\triangle(f)$ which is identically zero in no component of $\triangle(f) \cap \sigma(T)$, then

$$\sigma(x,T) = \sigma(f\{T\}x,T) \cup \{\alpha \in \sigma(x,T) \cap \sigma_p(T) : f(\alpha) = 0\},$$

where $\sigma_p(T)$ denotes the set of all eigenvalues of T.

Applying the ideas of [**13**], it is possible to prove the following theorem for $T \in L(X)$ satisfying the SVEP. However, we give a different proof for $T \in L(X)$ satisfying property (C) by using Proposition 2.

Theorem 2. (Local spectral mapping theorem). *Assume* $T \in L(X)$ *has property* (C), *and let* $f \in M(T)$. *Then, for every* $x \in X$,

$$\sigma_\infty(x, f\{T\}) = f(\sigma(x,T)).$$

Proof. The proof of this theorem consists of three steps.

Step 1. $f(\sigma(x,T)) \subset \sigma_\infty(x, f\{T\})$. Let us consider $\lambda_0 \in \Omega_T(f)$ with $f(\lambda_0) \in \rho_\infty(x, f\{T\})$, and let G be an open neighborhood of λ_0 such that $f(G) \subset \rho_\infty(x, f\{T\})$. Then, denoting by u, the resolvent function of $f\{T\}$ at x, we have

$$(f(\lambda) - f\{T\})u(f(\lambda)) = x,$$

for all $\lambda \in G$. Let $\mu \in G$. Let us define the function $g_\mu : \Omega_T(f) \to \mathbf{C}$ as

$$g_\mu(\lambda) := \begin{cases} (f(\lambda) - f(\mu))/(\lambda - \mu) & \text{if } \lambda \neq \mu \\ f'(\lambda) & \text{if } \lambda = \mu. \end{cases}$$

Then $g_\mu(\lambda) = h_\mu(\lambda) q_\mu(\lambda)^{-1}$, where h_μ is analytic on $\sigma(T)$ and q_μ is the polynomial of the poles of g_μ, that are also poles of f. Then $(\mu - \lambda)g_\mu(\lambda) = f(\mu) - f(\lambda)$ for $\mu \in U$, $\lambda \in \triangle(f)$ and $\lambda \neq \mu$. Hence

$$(\mu - T)g_\mu\{T\}u(f(\mu)) = (f(\mu) - f\{T\})u(f(\mu)) = x,$$

and

$$(\mu - T)p(T)g_\mu(T)\hat{x}_T(f(\mu)) = p(T)x,$$

where $p(T)g_\mu(T)\hat{x}_T(f(\mu))$ is analytic in a neighborhood of λ_0, hence $\lambda_0 \in \rho(p(T)x, T)$. By applying Proposition 1 we obtain that $\lambda_0 \in \rho(x, T)$.

Step 2. $\sigma(x, f\{T\}) = f(\sigma(x, T))$ for $x \in X$ such that $\sigma(x, T) \subset \triangle(f)$. Denoting by $Y := X(T, F)$, where $F := \sigma(x, T)$. Considering Step 1, it is enough to prove that

$$\sigma(x, f\{T|Y\}) \subset f(\sigma(x, T)),$$

since by part (ii) of Proposition 2 we have that $\sigma(f\{T\}y, T) \subset \sigma(y, T) \subset F$, i.e., Y is an invariant subspace of $f\{T\}$, hence $\sigma(x, f\{T\}) \subset \sigma(x, f\{T\}|Y)$. By applying part (i) of Theorem 1 and considering that Y is a T-absorbent space, we have

$$\sigma(x, f\{T|Y\}) = \sigma(x, f\{T\}|Y) = \sigma(x, f\{T\}).$$

Moreover $\sigma(T|Y) \subset \sigma(x, T) \subset \triangle(f)$, hence f is analytic on a neighborhood of $\sigma(T|Y)$. Using the local spectral mapping theorem for the holomorphic functional calculus we obtain that

$$\sigma(x, f\{T\}|Y) = \sigma(x, f(T|Y)) = f(\sigma(x, T|Y)) = f(\sigma(x, T)).$$

Step 3. $\infty \in \sigma(x, f\{T\})$ if and only if there exists at least one point $\alpha \in \sigma(x, T)$ such that $f(\alpha) = \infty$. It is enough to give the proof for f with only one pole α.

$\Rightarrow$. This is clear by Step 2.

$\Leftarrow$. Suppose that $\infty \in \rho(x, f\{T\})$. By Step 1, we have that $\infty \notin f(\sigma(x, T))$, hence $\alpha \notin \sigma(x, T)$. □

Corollary 1. *Assume $T \in L(X)$ has the* SVEP, *$f \in M(T)$ and Y is a T-absorbent space. Then the following assertions hold*:

(i) $\sigma(y, f\{T\}|Y) = f(\sigma(y, T))$.

(ii) $\sigma(x + Y, f\{T\}/Y) = f(\sigma(x + Y, T/Y)) \subset f(\sigma(x, T))$.

Proposition 3. *Let $T \in L(X)$, and let F be a closed subset of* **C**. *If $f \in M(T)$, then*

$$X(f\{T\}, F) = X(T, f^{-1}(F)).$$

Proof. If $x \in X(f\{T\}, F)$, then $\sigma(x, f\{T\}) = f(\sigma(x,T)) \subset F$, by Theorem 2. Then $\sigma(x,T) \subset f^{-1}(F)$. The opposite inclusion is established in a similar fashion. □

Using the ideas from [**7**] and [**11**], we derive the following theorem which shows that the SVEP is stable under meromorphic functional calculus.

Theorem 3. (Stability of the SVEP). *If $T \in L(X)$ satisfies the* SVEP, *then for each $f \in M(T)$, the operator $f\{T\}$ satisfies the* SVEP. *Conversely, if $f \in M(T)$ is constant in no component of its domain and $f\{T\}$ satisfies the* SVEP, *then T satisfies the* SVEP.

Proof. Let us assume that $f\{T\}$ satisfies the SVEP and T does not. Then there is an analytic function $w : D \to X$ such that

$$(\lambda - T)w(\lambda) = 0 \quad \text{for all } \lambda \in D \quad \text{with } w(\lambda) \neq 0.$$

For $\lambda \in D$ (fixed) and $\xi \in \triangle(f)$, we define the following function

$$g_\lambda(\xi) := \begin{cases} (f(\lambda) - f(\xi))/(\lambda - \xi) & \xi \neq \lambda \\ f'(\lambda) & \xi = \lambda. \end{cases}$$

It is clear that g_λ is a meromorphic function on $\triangle(f)$ and analytic on D. Using the properties of the meromorphic functional calculus we obtain that

$$f(\lambda) - f\{T\} = (\lambda - T)g_\lambda\{T\},$$

then

$$(f(\lambda) - f\{T\})w(\lambda) = (\lambda - T)g_\lambda\{T\}w(\lambda).$$

Since $(\lambda - T)g_\lambda\{T\}w(\lambda) = g_\lambda\{T\}(\lambda - T)w(\lambda)$, then

$$(f(\lambda) - f\{T\})w(\lambda) = g_\lambda\{T\}(\lambda - T)w(\lambda) = 0.$$

Hence as $f \neq 0$, there exists $\lambda_0 \in D$ such that $f'(\lambda_0) \neq 0$. Then there is a sufficiently small r, such that f^{-1} exists on $f(D_0)$ where D_0 is given by

$$D_0 := \{\lambda \in \mathbf{C} \; |\lambda - \lambda_0| < r\}.$$

We denote $h(\mu) := w(f^{-1}(\mu))$ on $f(D_0)$. Then

$$(\mu - f\{T\})h(\mu) = 0;$$

consequently, $h(\mu) = 0$ for $\mu \in f(D_0)$; hence, $w(\lambda) = 0$ for $\lambda \in D_0$.

Conversely, let $h : \triangle(h) \to D(f\{T\}) \subset X$ be an analytic function such that

$$(\mu - f\{T\})h(\mu) = 0.$$

Let us prove that $h(\mu) \in X(f\{T\}, \{\mu\}) \cap X(f\{T\}, \mathbf{C} \setminus \triangle(h))$. If $\lambda \neq \mu$, then $h(\mu)/(\lambda - \mu)$ is an analytic function that satisfies

$$(\lambda - f\{T\})\left(\frac{h(\mu)}{\lambda - \mu}\right) = (\lambda - \mu)\left(\frac{h(\mu)}{\lambda - \mu}\right) = h(\mu).$$

Hence $\sigma(h(\mu), f\{T\}) \subset \{\mu\}$ for $\mu \in \triangle(f)$. Moreover $\sigma(h(\mu), f\{T\}) \subset \mathbf{C} \setminus \triangle(f)$, for all $\mu \in \triangle(f)$, since $\lambda \in \triangle(h)$. Then $(\lambda - f\{T\})h(\lambda) = 0$, hence

$$(\lambda - T)\left(\frac{h(\mu) - h(\lambda)}{\mu - \lambda}\right) = h(\mu),$$

and we obtain $\sigma(h(\mu), T) \subset \mathbf{C} \setminus \triangle(h)$, for all $\mu \in \triangle(h)$. Then $h(\mu) \in X(f\{T\}, \mathbf{C} \setminus \triangle(h)) \cap X(f\{T\}, \{\mu\})$ and using Proposition 3 and [**11**, Proposition 2.5], we obtain that $h(\mu) \in X(T, f^{-1}\{\mu\} \cap f^{-1}(\mathbf{C} \setminus \triangle(h))) = X(T, \varnothing) = \{0\}$, that is, $h(\mu) \equiv 0$. This concludes the proof. □

REFERENCES

1. R.G. Bartle, *Spectral decomposition of operators in Banach spaces*, Proc. London Math. Soc. **20** (1970), 438–450.

2. R.G. Bartle and C.A. Kariotis, *Some localizations of the spectral mapping theorem*, Duke Math. J. **40** (1973), 651–660.

3. T. Bermúdez, M. González and A. Martinón, *Stability of the local spectrum*, Proc. Amer. Math. Soc. **125** (1997), 417–425.

4. T. Bermúdez, M. González and A. Martinón, *Properties and applications of the local functional calculus*, preprint.

5. I. Erdelyi and R. Lange, *Spectral decompositions on Banach spaces*, Springer, New York, 1977.

6. I. Erdelyi and Wang Shengwang, *A local spectral theory for closed operators*, Cambridge Univ. Press, Cambridge, 1985.

7. I. Colojoara and C. Foias, *The Riesz-Dunford functional calculus with descomposable operators*, Rev. Roum. Math. Pures Appl. **12** (1967), 627–641.

8. H. Gindler, *An operational calculus for meromorphic functions*, Nagoya Math. J. **26** (1966), 31-38.

9. M. González and V.M. Onieva, *On the meromorphic and Schechter-Shapiro operational calculi*, J. Math. Anal. Appl. **116** (1986), 363–377.

10. B. Nagy, *On an operational calculus for meromorphic functions*, Acta Math. Acad. Hungar. **33** (1979), 379–390.

11. M. Radjabalipour, *Decomposable operator*, Bull. Iranian Math. Soc. **9** (1978), 1L-49L.

12. A.C. Taylor and D.C. Lay, *Introduction to functional analysis*, 2nd Edition, John Wiley & Sons, New York, 1980.

13. F.-H. Vasilescu, *Spectral mapping theorem for the local spectrum*, Czech. Math. J. **30** (1980), 28–35.

Departamento de Análisis Matemático, Universidad de La Laguna, 38271 La Laguna (Tenerife), Spain
E-mail address: `tbermude@ull.es`