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ALGEBRAIC VECTOR BUNDLES ON SL(3, C)

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ABSTRACT. We show that all algebraic vector bundles on $\mathrm{SL}(3, \mathbf{C})$ are topologically trivial.

1. Introduction. There are a large number of analogies and relations between algebra and topology, cf. [**7, 8**]. For example, Serre's conjecture, cf. [**4**–**6**], was arising from analogies between projective modules and vector bundles. In many cases, topological viewpoint inspires us with several problems on algebraic vector bundles. In this paper, we deal with algebraic vector bundles over $\mathrm{SL}(3, \mathbf{C})$.

We start with Grothendieck's theorem.

Theorem 1.1 [Grothendieck **1**]. *Let G be a semi-simple simply connected affine algebraic group over an algebraically closed field. Then $K_0(G) = \mathbf{Z}$.*

From this, all algebraic vector bundles on G are stably free. Here we say that an algebraic vector bundle E is *stably free* if there exists a trivial algebraic vector bundle F such that $E \oplus F$ is also trivial. Let us consider the question whether all algebraic vector bundles over G are free.

In the case $G = \mathrm{SL}_2$, M.P. Murthy has shown the following.

Theorem 1.2 [**8**]. *Let $A = k[x, y, z, w]/(xy - zw - 1)$ be the coordinate ring of SL_2 over any field k. Then all finitely generated projective A-modules are free.*

However, in general the answer to our question is negative. In the case $G = \mathrm{SL}_4$ we have a counterexample.

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Theorem 1.3 (Swan [**8**]). *Let A be the coordinate ring of* SL_4 *over* $\mathbf{C}$*. Then there is a nonfree projective A-module of rank* 2*.*

So how about the case $G = \mathrm{SL}_3$? We introduce the following conjecture.

Conjecture 1.4. *Let A be the coordinate ring of* SL_3 *over any field k. Then all finitely generated projective A-modules are free.*

We say that a commutative ring R is *Hermite* if all finitely generated stably free R-modules are free (see [**3**]). If k is an algebraically closed field, then Conjecture 1.4 follows from the claim that the coordinate ring of SL_3 is Hermite. Unfortunately, we cannot prove the conjecture in this article. However, as an evidence that the conjecture is true we present our main result.

Theorem 1.5 (Theorem 2.1). *All algebraic vector bundles on* $\mathrm{SL}(3, \mathbf{C})$ *are topologically trivial.*

Although we do not know whether all algebraic vector bundles over $\mathrm{SL}(3, \mathbf{C})$ are algebraically trivial, we see that they are topologically trivial. In other words, we can prove that the coordinate ring of $\mathrm{SL}(3, \mathbf{C})$ is "topologically Hermite." Theorem 1.5 is a topological result rather than an algebraic one. We prove our main result by using topological methods in the following sections.

2. Main theorem. Our main theorem is the following:

Theorem 2.1. *All algebraic vector bundles on* $\mathrm{SL}(3, \mathbf{C})$ *are topologically trivial.*

The main theorem can be followed by the next proposition.

Proposition 2.2. *All stably free topological vector bundles on* $\mathrm{SU}(3)$ *are trivial.*

Proof of Theorem 2.1. By Theorem 1.1, $K_0(\mathrm{SL}(3,\mathbf{C})) \cong \mathbf{Z}$. Hence any algebraic vector bundle on $\mathrm{SL}(3,\mathbf{C})$ is stably free. Since $\mathrm{SL}(3,\mathbf{C})$ is isotopic to SU(3), Proposition 2.2 implies Theorem 2.1. □

We only need to prove Proposition 2.2 for our main theorem. For the proof of Proposition 2.2 we prepare the proposition $P(n)$.

$$P(n) \quad : \quad f : SU(3) \to BU(n) \text{ is trivial up to homotopy if } SU(3) \xrightarrow{f} BU(n) \to BU(n+1) \text{ is trivial up to homotopy.}$$

Here $\mathrm{BU}(n) \to \mathrm{BU}(n+1)$ is the morphism associated to $E \mapsto E \oplus \mathbf{C}$.

If we prove that the proposition $P(n)$ is true for each $n \geq 1$, then we see that any topological stably free vector bundle on SU(3) is trivial. In the sequel, we prove $P(n)$ for each $n \geq 1$.

Proposition 2.3. $P(1)$ *is true.*

Proof. Since $H^2(\mathrm{SL}(3,\mathbf{C}),\mathbf{Z}) \cong 0$, $[\mathrm{SL}(3,\mathbf{C}), \mathrm{BU}(1)] = \{*\}$. This completes the proof. □

Proposition 2.4. $P(n)$ *is true for each* $n \geq 4$.

Proof. Let us consider the fibre sequence $\mathrm{S}^{2n+1} \to \mathrm{BU}(n) \to \mathrm{BU}(n+1)$. From the assumption, f factors through S^{2n+1}. Because $\mathrm{SU}(3) = e^0 \cup e^3 \cup e^5 \cup e^8$, the morphism $\mathrm{SU}(3) \to \mathrm{S}^{2n+1}$ is trivial up to homotopy for $n \geq 4$. Hence f is also trivial. □

Proposition 2.5. $P(3)$ *is true.*

Proof. Let us consider the fibre sequence $\mathrm{S}^7 \to \mathrm{BU}(3) \to \mathrm{BU}(4)$. By the assumption, f factors through S^7. Since $\mathrm{SU}(3) = e^0 \cup e^3 \cup e^5 \cup e^8$, the morphism $\mathrm{SU}(3) \to \mathrm{S}^7$ induces $\mathrm{S}^8 \to \mathrm{S}^7$ and the next homotopy commutative diagram:

$$\begin{array}{ccc} S^8 & \longrightarrow & S^7 \\ \uparrow & & \downarrow \\ SU(3) & \xrightarrow{f} & BU(3). \end{array}$$

Because $\pi_8(BU(3)) = \pi_7(U(3)) = 0$, the morphism $S^8 \to S^7 \to BU(3)$ is trivial up to homotopy. Hence f is trivial. □

By the above propositions, we only have to prove $P(2)$, which will be shown in the next section.

3. Proof of $P(2)$. For the proof of $P(2)$, we only need to show the next proposition $P'(2)$ since $[SU(3), BU(n)] = [SU(3), BSU(n)]$ for $n \geq 1$.

$$\begin{array}{lll} P'(2) & : & f : SU(3) \to BSU(2) \text{ is trivial up to homotopy if} \\ & & SU(3) \xrightarrow{f} BSU(2) \to BSU(3) \text{ is trivial up to homotopy.} \end{array}$$

Here $BSU(2) \to BSU(3)$ is the morphism associated to $E \mapsto E \oplus \mathbf{C}$.

Before starting the proof of $P'(2)$, we make several preparations. Considering the fibre sequence $S^5 \to BSU(2) \to BSU(3)$, we see that f factors through S^5. Since $SU(3) = e^0 \cup e^3 \cup e^5 \cup e^8$, the morphism $SU(3) \to S^5$ induces $SU(3)/S^3 = S^5 \cup e^8 \to S^5$. Set $X = S^5 \cup e^8$. Then we have the following homotopy commutative diagram:

$$\begin{array}{ccc} X = S^5 \cup e^8 & \longrightarrow & S^5 \\ \uparrow & & \downarrow \\ SU(3) & \xrightarrow{f} & BSU(2). \end{array} \tag{1}$$

Lemma 3.1. $X \simeq S^5 \vee S^8$.

Proof. We have a fibre bundle $SU(2) \xrightarrow{i} SU(3) \xrightarrow{\pi} S^5$ and i may be identified with the inclusion in the bottom cell. Hence the projection π factors through the quotient map $SU(3) \to X = SU(3)/S^3$:

$$\begin{array}{ccc} SU(3) & \longrightarrow & X \\ & {\scriptstyle \pi}\searrow \quad \swarrow{\scriptstyle \pi'} & \\ & S^5. & \end{array}$$

Then π' is a retraction of X to S^5. This implies that $X \simeq S^5 \vee S^8$. □

Let us consider the following cofibre sequence

$$S^3 \to SU(3) \to X \simeq S^5 \vee S^8 \xrightarrow{\varphi} S^4. \tag{2}$$

We determine the morphism $\varphi = (\varphi_1, \varphi_2) : X \simeq S^5 \vee S^8 \to S^4$. Here we denote $\varphi|_{S^5}$ and $\varphi|_{S^8}$ by φ_1 and φ_2, respectively.

Lemma 3.2. $\varphi_1 : S^5 \to S^4$ *is not a trivial element in* $\pi_5(S^4) \cong \mathbf{Z}/2\mathbf{Z}$.

Proof. This follows from the fact that the attaching map of e^5 to S^3 in $SU(3) = S^3 \cup e^5 \cup e^8$ is nontrivial. □

Before determining $\varphi_2 : S^8 \to S^4$, we recall the homotopy group $\pi_8(S^4)$. (For example, see [**2**].) The homotopy group $\pi_8(S^4)$ is isomorphic to $\mathbf{Z}/2\mathbf{Z} \oplus \mathbf{Z}/2\mathbf{Z}$. Its generators α and β are given by $\alpha = \Sigma^2\eta \circ \Sigma q$ and $\beta = q \circ \Sigma^5\eta$, where $q : S^7 \to S^4$ is the Hopf map. Putting $\gamma = \alpha + \beta$, we have $\pi_8(S^4) = \{0, \alpha, \beta, \gamma\}$.

The class of $\varphi_2 : S^8 \to S^4$ in $\pi_8(S^4)$ depends on how to regard X as $S^5 \vee S^8$. Indeed, the following lemmas imply that the class can be only determined modulo $\langle \alpha, 0 \rangle$.

Lemma 3.3. $[S^5 \vee S^8, S^5 \vee S^8] \cong \pi_5(S^5) \oplus \pi_8(S^5) \oplus \pi_8(S^8)$.

Proof. We have $[S^5 \vee S^8, S^5 \vee S^8] \cong \pi_5(S^5) \oplus \pi_8(S^5 \vee S^8)$. From the homotopy exact sequence of the pair $(S^5 \vee S^8, S^5)$ and the splitting $S^5 \vee S^8 \to S^5$, there is a split exact sequence

$$0 \longrightarrow \pi_8(S^5) \longrightarrow \pi_8(S^5 \vee S^8) \longrightarrow \pi_8(S^5 \vee S^8, S^5) \longrightarrow 0.$$

By the Hurewicz theorem, $\pi_8(\mathrm{S}^5 \vee \mathrm{S}^8, \mathrm{S}^5) \overset{\cong}{\to} \pi_8(\mathrm{S}^8)$. This completes the proof. □

By Lemma 3.3, the group of self-homotopy equivalences of $\mathrm{S}^5 \vee \mathrm{S}^8$ is

$$\left\{ \begin{pmatrix} \varepsilon_1 & \kappa \\ 0 & \varepsilon_2 \end{pmatrix} \middle| \begin{array}{c} \varepsilon_1 = \pm 1 \in \pi_5(\mathrm{S}^5), \quad \varepsilon_2 = \pm 1 \in \pi_8(\mathrm{S}^8), \\ \kappa \in \pi_8(\mathrm{S}^5) \end{array} \right\}.$$

Note that $\pi_8(\mathrm{S}^5) \cong \mathbf{Z}/24\mathbf{Z}$ and the generator may be Σq. Then the following composition

$$\mathrm{S}^8 \hookrightarrow \mathrm{S}^5 \vee \mathrm{S}^8 \xrightarrow{A} \mathrm{S}^5 \vee \mathrm{S}^8 \xrightarrow{\varphi} \mathrm{S}^4, \qquad A = \begin{bmatrix} \varepsilon_1 & \kappa \\ 0 & \varepsilon_2 \end{bmatrix},$$

is homotopic to $\varphi_2 + m \cdot \alpha$, where $\kappa = m \cdot \Sigma q$. Hence we see that $\varphi_2 \in \pi_8(\mathrm{S}^4)$ is determined modulo $\langle \alpha, 0 \rangle$. Note that $\Sigma SU(3)$ is the cofibre of φ. Hence, if $\varphi_2 = 0$ or α, then $\Sigma SU(3) \simeq \Sigma^2 P^2(\mathbf{C}) \vee S^9$.

Recall that there are inclusions $S^3 \subset \Sigma P^2(\mathbf{C}) \subset SU(3)$. The product map $SU(3) \times SU(3) \to SU(3)$ induces a map $h : S^3 \times \Sigma P^2(\mathbf{C}) \hookrightarrow SU(3) \times SU(3) \to SU(3)$. Then the following diagram commutes:

$$\begin{array}{ccc} S^3 \times S^3 & \xrightarrow{\quad m \quad} & S^3 \\ \downarrow & & \downarrow \\ S^3 \times \Sigma P^2(\mathbf{C}) & \xrightarrow{\quad h \quad} & SU(3), \end{array}$$

where m is the product map of S^3 and the vertical arrows are inclusions.

Lemma 3.4. *The class of* $\varphi_2 : \mathrm{S}^8 \to \mathrm{S}^4$ *in* $\pi_8(\mathrm{S}^4)$ *is* β *modulo* $\langle \alpha, 0 \rangle$.

Proof. There is a homotopy equivalence $\Sigma(S^3 \times S^3) \simeq S^4 \vee S^4 \vee S^7$ and the map $S^7 \hookrightarrow S^4 \vee S^4 \vee S^7 \simeq \Sigma(S^3 \times S^3) \overset{\Sigma m}{\to} \Sigma(S^3) = S^4$ is the Hopf map q. There is also a homotopy equivalence $\Sigma(S^3 \times \Sigma P^2(\mathbf{C})) \simeq S^4 \vee \Sigma^2 P^2(\mathbf{C}) \vee \Sigma^5 P^2(\mathbf{C})$ and the map $\Sigma^5 P^2(\mathbf{C}) \hookrightarrow S^4 \vee \Sigma^2 P^2(\mathbf{C}) \vee \Sigma^5 P^2(\mathbf{C}) \simeq \Sigma(S^3 \times \Sigma P^2(\mathbf{C})) \overset{\Sigma h}{\to} \Sigma SU(3)$ makes the following diagram homotopy commute:

$$\begin{array}{ccc} S^7 & \xrightarrow{\quad q \quad} & S^4 \\ \downarrow & & \downarrow \\ \Sigma^5 P^2(\mathbf{C}) & \longrightarrow & \Sigma SU(3), \end{array}$$

where the vertical arrows are the inclusions in the bottom cell.

If $\varphi_2 = 0$ or α, then $\Sigma SU(3) \simeq \Sigma^2 P^2(\mathbf{C}) \vee S^9$. Let ϕ be the map $\Sigma^5 P^2(\mathbf{C}) \to \Sigma SU(3) \simeq \Sigma^2 P^2(\mathbf{C}) \vee S^9 \to \Sigma^2 P^2(\mathbf{C})$, where the last map is the projection. Then ϕ makes the following diagram homotopy commute:

$$\begin{array}{ccc} S^7 & \xrightarrow{\quad q \quad} & S^4 \\ \downarrow & & \downarrow \\ \Sigma^5 P^2(\mathbf{C}) & \xrightarrow{\quad \phi \quad} & \Sigma^2 P^2(\mathbf{C}). \end{array}$$

Let C be the cofibre of ϕ. By the above homotopy commutative diagram, we see that the cohomology of C is given as follows:

$$\widetilde{H}^*(C; \mathbf{Z}) = \mathbf{Z}\{x_4, x_6, x_8, x_{10}\}, \quad |x_i| = i$$

and

$$Sq^2 \overline{x}_4 = \overline{x}_6,\ Sq^4 \overline{x}_4 = \overline{x}_8,\ Sq^2 \overline{x}_8 = \overline{x}_{10},$$

where $\overline{x}_i \in \widetilde{H}^i(C; \mathbf{Z}/2)$ is the mod 2 reduction of x_i. The unstable condition implies that $Sq^4 \overline{x}_4 = \overline{x}_4^2$. By the Cartan formula, $Sq^2(\overline{x}_4^2) = 2\overline{x}_4 \overline{x}_6 = 0$. On the other hand, $Sq^2 Sq^4 \overline{x}_4 = Sq^2 \overline{x}_8 = \overline{x}_{10} \neq 0$. This is a contradiction. This completes the proof. □

Lemma 3.5. *Let* $j : \mathrm{S}^4 \to \mathrm{BSU}(2)$ *be the inclusion in the bottom cell. The morphism* $j_* : [\mathrm{S}^8, \mathrm{S}^4] \to [\mathrm{S}^8, \mathrm{BSU}(2)] = \mathbf{Z}/2\mathbf{Z}$ *maps* α *and* β *to* $[1]$ *and* $[0]$*, respectively.*

Proof. First we show that $j_*(\alpha) = [1]$. Note that the morphism $\mathrm{S}^5 \to \mathrm{BSU}(2)$ in the fibre sequence $\mathrm{S}^5 \to \mathrm{BSU}(2) \to \mathrm{BSU}(3)$ is equal to $\mathrm{S}^5 \xrightarrow{\Sigma^2\eta} \mathrm{S}^4 \xrightarrow{j} \mathrm{BSU}(2)$. Since $\pi_8(\mathrm{S}^5) \to \pi_8(\mathrm{BSU}(2))$ is surjective, the generator Σq is mapped to $[1]$. Hence $j_*(\alpha) = j \circ \Sigma^2\eta \circ \Sigma q = [1]$.

Next, we show that $j_*(\beta) = [0]$. The morphism $\mathrm{S}^7 \xrightarrow{q} \mathrm{S}^4 \xrightarrow{j} \mathrm{BSU}(2)$ is trivial up to homotopy. Hence $\mathrm{S}^8 \xrightarrow{\Sigma^5\eta} \mathrm{S}^7 \xrightarrow{q} \mathrm{S}^4 \xrightarrow{j} \mathrm{BSU}(2)$ is trivial. Therefore $j_*(\beta) = j \circ q \circ \Sigma^5\eta = [0]$. □

The map $\varphi : \mathrm{S}^5 \vee \mathrm{S}^8 \to \mathrm{S}^4$ induces a map $\varphi^* : [\mathrm{S}^4, \mathrm{BSU}(2)] \to [\mathrm{S}^5 \vee \mathrm{S}^8, \mathrm{BSU}(2)]$. Note that φ^* is not a homomorphism of abelian groups.

Lemma 3.6. *The map* $\varphi^* : [\mathrm{S}^4, \mathrm{BSU}(2)] \to [\mathrm{S}^5 \vee \mathrm{S}^8, \mathrm{BSU}(2)]$ *is surjective.*

Proof. It is sufficient to show that $\varphi^* = (\varphi_1^*, \varphi_2^*) : \pi_4(\mathrm{BSU}(2)) \to \pi_5(\mathrm{BSU}(2)) \oplus \pi_8(\mathrm{BSU}(3))$ is surjective. The inclusion $j : \mathrm{S}^4 \to \mathrm{BSU}(2)$ in the bottom cell gives a generator of $\pi_4(\mathrm{BSU}(2))$. Since $\varphi_1 = \Sigma^2\eta$, the map φ_1^* is a surjective homomorphism. More precisely,

$$\varphi_1^*(mj) = \begin{cases} j \circ \Sigma^2\eta \neq 0 & \text{if } m \text{ is odd} \\ 0 & \text{if } m \text{ is even.} \end{cases} \tag{3}$$

Next, let us consider the map $\varphi_2^* : \pi_4(\mathrm{BSU}(2)) \cong \mathbf{Z} \to \pi_8(\mathrm{BSU}(2)) \cong \mathbf{Z}/2\mathbf{Z}$. Let ι be a generator of $\pi_4(\mathrm{S}^4)$ and $\xi : \mathrm{S}^6 \to \mathrm{S}^3$ the characteristic map associated to the $\mathrm{Sp}(1)$-bundle $\mathrm{Sp}(1) \to \mathrm{Sp}(2) \to \mathrm{S}^7$. Then we have

$$2q = [\iota, \iota] + \varepsilon\Sigma\xi \tag{4}$$

in $\pi_7(\mathrm{S}^4)$, where $[*, *]$ is the Whitehead product and $\varepsilon = \pm 1$, see [**2**]. By using (4), we obtain

$$\begin{aligned} 2(m\iota \circ q) &= m\iota \circ (2q) = m\iota \circ [\iota, \iota] + m\iota \circ \varepsilon\Sigma\,\xi \\ &= [m\iota, m\iota] + \varepsilon m(\Sigma\,\xi) \\ &= m^2[\iota, \iota] + \varepsilon m(\Sigma\,\xi) \\ &= m^2(2q - \varepsilon\Sigma\,\xi) + \varepsilon m(\Sigma\,\xi) \\ &= 2m^2 q + \varepsilon(m - m^2)\Sigma\,\xi \end{aligned}$$

in $\pi_7(\mathrm{S}^4) = \mathbf{Z} \oplus \mathbf{Z}/12\mathbf{Z}$. Note that the free part and the torsion part of $\pi_7(\mathrm{S}^4)$ are generated by q and $\Sigma\,\xi$, respectively. The calculation above implies that

$$m\iota \circ q = m^2 q + \varepsilon \,\frac{m-m^2}{2}\, \Sigma\,\xi \bmod 6 \cdot \mathbf{Z}/12\mathbf{Z}.$$

Hence

$$\begin{aligned} m\iota \circ \beta = m\iota \circ q \circ \Sigma^5\eta &= m^2 q \circ \Sigma^5 \eta + \varepsilon \,\frac{m-m^2}{2}\, \Sigma\,\xi \circ \Sigma^5\eta \\ &= m^2\beta + \varepsilon\,\frac{m-m^2}{2}\,\alpha \end{aligned}$$

in $\pi_8(\mathrm{S}^4) = \mathbf{Z}/2\mathbf{Z} \oplus \mathbf{Z}/2\mathbf{Z}$. Note that $\alpha = \Sigma^2\eta \circ \Sigma q \simeq \Sigma\,\xi \circ \Sigma^5\eta$.

By Lemma 3.4, $\varphi_2 \simeq \beta$ or $\varphi_2 \simeq \alpha + \beta$. If $\varphi_2 \simeq \beta$, then

$$\begin{aligned} \varphi_2^*(mj) &= mj \circ \beta = j \circ m\iota \circ \beta \\ &= j \circ m^2\beta + j \circ \varepsilon\,\frac{m-m^2}{2}\,\alpha \\ &= \varepsilon\,\frac{m-m^2}{2}\,(j \circ \alpha). \end{aligned} \tag{5}$$

Here we use Lemma 3.5. If $\varphi_2 \simeq \alpha + \beta$, then

$$\begin{aligned} \varphi_2^*(mj) &= mj \circ (\alpha+\beta) = j \circ m\iota \circ \alpha + j \circ m\iota \circ \beta \\ &= j \circ m^2\beta + j \circ \left(\varepsilon\,\frac{m-m^2}{2} + m\right)\alpha \\ &= \left(\varepsilon\,\frac{m-m^2}{2} + m\right)(j \circ \alpha). \end{aligned} \tag{6}$$

The results (3), (5) and (6) imply that $\varphi^* : \mathbf{Z} \to \mathbf{Z}/2\mathbf{Z} \oplus \mathbf{Z}/2\mathbf{Z}$ is surjective. This completes the proof. □

Proposition 3.7. $P(2)$ *is true.*

Proof. Recall the diagram (1). By Lemma 3.6, any morphism $X \to \mathrm{BSU}(2)$ factors through S^4. Considering the cofibre sequence $\mathrm{SU}(3) \to X \to \mathrm{S}^4$, we see that f is trivial. □

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