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HARMONIC BESOV SPACES ON THE UNIT BALL IN $\mathbf{R}^n$

MIROLJUB JEVTIĆ AND MIROSLAV PAVLOVIĆ

ABSTRACT. We define and characterize the harmonic Besov space B^p, $1 \le p \le \infty$, on the unit ball B in $\mathbf{R}^n$. We prove that the Besov spaces B^p, $1 \le p \le \infty$, are natural quotient spaces of certain L^p spaces. The dual of B^p, $1 \le p < \infty$, can be identified with B^q, $1/p + 1/q = 1$, and the dual of the little harmonic Bloch space B_0 is B^1.

1. Introduction. Let $d\nu$ be the volume measure on the unit ball $B = B_n$ in R^n normalized so that B has volume equal to one. For any real $\alpha > 0$ we consider the measure $d\nu_\alpha(x) = c_\alpha(1-|x|^2)^{\alpha-1}\, d\nu(x)$ where the constant c_α is chosen so that $d\nu_\alpha$ has total mass 1. An integration in polar coordinates shows that $c_\alpha = (2/n)[B(n/2, \alpha)]^{-1}$. See [**1**]. Also, we let $d\tau(x) = (1-|x|^2)^{-n}\, d\nu(x)$.

For f harmonic on B, $f \in h(B)$, and any positive integer m, we write $|\partial^m f(x)| = \sum_{|\alpha|=m} |\partial^\alpha f(x)|$, where $\partial^\alpha f(x) = (\partial^{|\alpha|} f/\partial x^\alpha)(x)$, α a multi-index.

For $1 \le p \le \infty$, the harmonic Besov space $B^p = B^p(B)$ consists of harmonic functions f on B such that the function $(1-|x|^2)^k |\partial^k f(x)|$ belongs to $L^p(B, d\tau)$ for some positive integer $k > (n-1)/p$. We note that the definition is independent of k (see Theorem 3.2).

Let B_0 be the subspace of B^∞ consisting of functions $f \in h(B)$ with

$$(1-|x|^2)^k |\partial^k f(x)| \longrightarrow 0, \quad \text{as } x \to S, \quad \text{for some } k > 0,$$

where $S = \partial B$ is the (full) topological boundary of B in R^n.

For $\alpha > 0$ and $0 < p < \infty$, we let $l^{p,\alpha-1}$ denote the closed subspace of $L^{p,\alpha-1} = L^p(B, d\nu_\alpha)$ consisting of harmonic functions in $L^{p,\alpha-1}$.

The purpose of the present paper is to study the Besov spaces B^p.

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In Section 2 we list some of the known properties of the Bergman kernel K_α of the orthogonal projection P_α of the space $L^{2,\alpha-1}$ onto $l^{2,\alpha-1}$ that will be of great importance in the rest of the paper.

In Section 3 we characterize the Besov spaces B^p in terms of certain differential and integral operators that involve the Bergman kernel K_α.

In Section 4 we show that P_α maps $L^p(B, d\tau)$ onto B^p and $C_0(B)$, the space of continuous functions on $\overline{B}$ that vanish on the boundary ∂B, onto the little Bloch space B_0.

Section 5 deals with duality. The results are: $(B^p)^\star = B^q$, $1 \le p < \infty$ and $1/p + 1/q = 1$; $B_0^\star = B^1$.

It should be noted that the analogous results for analytic functions are known. See, for example, [**8**] and [**10**] and the references therein.

2. The Bergman kernel. It is well known that the projection operator P_α from $L^{2,\alpha-1}$ onto $l^{2,\alpha-1}$ is an integral operator

$$P_\alpha f(x) = \int_B K_\alpha(x,y) f(y)\, d\nu_\alpha(y), \quad f \in L^{2,\alpha-1}. \tag{2.1}$$

In [**1**, p. 154], an explicit formula for the reproducing kernel $K_1(x,y)$ is given. It is shown that $K_1(x,y) = \sum_{j=0}^\infty A_j^1 Z_j(x,y)$, where $A_j^1 = B(n/2,1)/B(n/2+j,1)$, $j = 0,1,2,\dots$, and $Z_j(x,y)$ are extended zonal harmonics. The same argument shows that $K_\alpha(x,y) = \sum_{j=0}^\infty A_j^\alpha Z_j(x,y)$, where $A_j^\alpha = B(n/2,\alpha)/B(n/2+j,\alpha)$, $\alpha > 0$, $j = 0,1,2,\dots$.

The following two estimates for the Bergman kernel were obtained in [**5**].

Lemma 2.1. *If $\alpha > 0$, $|x| < 1$ and $|y| = 1$, then*

$$|K_\alpha(x,y)| \le C|x-y|^{-n+1-a}.$$

Lemma 2.2. *If α is a multi-index, $s > 0$, $x \in B$ and $y = r\xi$, where $0 \le r < 1$ and $\xi \in S$, then*

$$|\partial_x^\alpha K_s(x,y)| \le \frac{C}{|rx-\xi|^{n-1+s+|\alpha|}}.$$

Here C is a constant that depends only on n, s and α.

Lemma 2.3. *Let $m > n-1$. There exists a constant $C > 0$ such that*

$$\int_S \frac{d\sigma(y)}{|x-y|^m} \le \frac{C}{(1-|x|)^{m-n+1}}, \quad \text{for all } x \in B.$$

As usual, σ is the normalized surface measure on S.

Proof. Without loss of generality, we may assume $x = re_1$, $e_1 = (1, 0, \dots, 0)$, $0 < r < 1$. Then

$$\begin{aligned}\int_S |x-y|^{-m}\, d\sigma(y) &= \int_S (r^2 - 2ry_1 + 1)^{-m/2}\, d\sigma(y) \\ &= C_n \int_{-1}^{1} (r^2 - 2rt + 1)^{-m/2} (1-t^2)^{(n-3)/2}\, dt,\end{aligned}$$

by Corollary A6 [**1**, p. 216]. A change of variable $1-t = (1-r)^2\xi$ gives

$$\begin{aligned}\int_S |x-y|^{-m}\, d\sigma(y) &\le C \int_0^1 (1 - 2rt + r^2)^{-m/2} (1-t)^{(n-3)/2}\, dt \\ &\le C \int_0^{(1-r)^{-2}} [(1-r)^2 + 2r(1-r)^2\xi]^{-m/2} \\ &\quad \cdot (1-r)^{n-1} \xi^{(n-3)/2}\, d\xi \\ &\le C(1-r)^{-m+n-1} \int_0^\infty \frac{\xi^{(n-3)/2} d\xi}{(1+2r\xi)^{m/2}} \\ &\le C(1-r)^{-m+n-1}.\end{aligned}$$

(The last integral converges since $(m/2) - (n-3)/2 > 1$.)

Using integration in polar coordinates, Lemma 2.1 and Lemma 2.3 we obtain

Lemma 2.4. *Let $m, \gamma > 0$ and $(n+m-1)p > n-1+\gamma$. Then*

$$\begin{aligned}&\int_B (1-|y|^2)^{\gamma-1} |K_m(x,y)|^p\, d\nu(y) \\ &\qquad \le C(1-|x|)^{\gamma+n-1-p(n+m-1)}, \quad x \in B.\end{aligned}$$

3. A characterization of the Besov space B^p. Now we characterize the Besov space B^p in terms of certain fractional differential and integral operators on B whose kernel involves the Bergman kernel K_α.

Let $s > 0$ and $m \geq 0$. We define a linear operator R_s^m on $L^1(B, d\nu_s)$ by

$$R_s^m u(x) = c_s \int_B K_{s+m}(x,y)u(y)(1-|y|^2)^{s-1}\, d\nu(y), \qquad u \in L^1(B, d\nu_s).$$

We note that the formula (2.1) extends the domain of P_s to $L^1(B, d\nu_s)$ and P_s is the identity map on $l^{1,s-1}$. We write $P_s u = R_s^0 u$, $u \in L^1(B, d\nu_s)$. We also write $E_{m,s}u(x) = (1-|x|^2)^m R_s^m u(x)$ for $u \in L^1(B, d\nu_s)$.

Theorem 3.1. *Suppose $m > \max\{0, -\alpha\}$, α real, $s > \max\{0, \alpha\}$ and $1 \leq p \leq \infty$. Then the operator $E_{m,s}$ is bounded on $L^p(B, d\mu_\alpha)$ where $d\mu_\alpha(x) = (1-|x|^2)^{\alpha-1}\, d\nu(x)$.*

Proof. The case $p = 1$ follows from Lemma 2.4 and Fubini's theorem. Also the case $p = \infty$ is a direct consequence of Lemma 2.4.

Next we consider the case $1 < p < \infty$. As usual, we shall need to use Schur's theorem (see [**9**]).

Let $p^{-1} + q^{-1} = 1$, let ε be any positive number satisfying $0 < \varepsilon < \min\{m/q, (s-\alpha)/p\}$, and let $H(x) = (1-|x|^2)^\varepsilon$. Using Lemma 2.4 again, we obtain

$$\int_B (1-|x|^2)^m (1-|y|^2)^{s-\alpha} |K_{s+m}(x,y)| H(y)^q \, d\mu_\alpha(y) \leq CH(x)^q$$

and

$$\int_B (1-|x|^2)^m (1-|y|^2)^{s-\alpha} |K_{s+m}(x,y)| H(x)^p \, d\mu_\alpha(x) \leq CH(y)^p$$

for some constant $C > 0$ and all $x, y \in B$. This completes the proof of Theorem 3.1 in view of Schur's theorem.

Theorem 3.2. *Let $1 \leq p \leq \infty$ and $s > 0$. If $f \in h(B)$, then the following statements are equivalent*:

(i) *There exists a positive integer* $m > (n-1)/p$ *such that* $(1-|x|^2)^m R_s^m f(x) \in L^p(\tau)$,

(ii) *There exists a positive integer* $m > (n-1)/p$ *such that* $(1-|x|^2)^m |\partial^m f(x)| \in L^p(\tau)$,

(iii) *For all positive integers* $k > (n-1)/p$, $(1-|x|^2)^k R_s^k f(x) \in L^p(\tau)$,

(iv) *For all positive integers* $k > (n-1)/p$, $(1-|x|^2)^k |\partial^k f(x)| \in L^p(\tau)$.

Proof. Let $(1-|x|^2)^m |\partial^m f(x)| \in L^p(\tau)$. Then $(1-|x|^2)^m |\partial^m f(x)| = O(1)$ and therefore $f \in L^p(B, d\nu_s)$ for any $p > 0$ and $s > 0$. Let $f(x) = \sum_{j=0}^{\infty} f_j(x)$, $x \in B$, be a homogeneous expansion of f. Then we have

$$\begin{aligned} R_s^m f(x) &= c_s \int_B (1-|y|^2)^{s-1} K_{s+m}(x,y) f(y)\, d\nu(y) \\ &= c_s n \int_0^1 t^{n-1}(1-t^2)^{s-1}\, dt \\ &\quad \cdot \int_S \bigg(\sum_j A_j^{s+m} Z_j(x, ty) \bigg) \bigg(\sum_j f_j(ty) \bigg)\, d\sigma(y) \\ &= \sum_j A_j^{s+m} c_s \frac{n}{2} B(n/2 + j, s) f_j(x). \end{aligned}$$

Using this and the equality

$$A_j^{s+m} = \frac{n/2 + j + s + m - 1}{n/2 + s + m - 1} A_j^{s+m-1}$$

we find that

$$R_s^m f(x) = \bigg(\Big(I + \frac{1}{n/2 + s + m - 1} R \Big) R_s^{m-1} \bigg)(f)(x).$$

Here, as usual, $R = \sum_{j=1}^{n} x_j (\partial/\partial x_j)$ denotes the radial derivative. Note that $R_s^0 f(x) = P_s f(x) = f(x)$. Thus

$$|R_s^m f(x)| \le C \sum_{|a| \le m} |\partial^\alpha f(x)|.$$

It is easy to see that if $|\partial^m f(x)|(1-|x|^2)^m \in L^p(\tau)$, then $(1-|x|^2)^m|\partial^\alpha f(x)| \in L^p(\tau)$ for any multi-index α for which $|\alpha| \le m$. Thus $(1-|x|^2)^m R_s^m f(x) \in L^p(\tau)$.

Conversely, assume that $(1-|x|^2)^m R_s^m f(x) \in L^p(\tau)$. Using Fubini's theorem, we get

$$f(x) = c_{s+m}\int_B (1-|y|^2)^{s+m-1} R_s^m f(y) K_s(x,y)\, d\nu(y).$$

Using Lemma 2.2, we find that

$$|\partial^\alpha f(x)| \le C\int_B (1-|\rho|^2)^{s+m-1}|R_s^m f(\rho\xi)|\frac{d\nu(\rho\xi)}{|\rho x-\xi|^{n-1+s+|\alpha|}}.$$

From this, as in Theorem 3.1, we find that $(1-|x|^2)^m|\partial^\alpha f(x)| \in L^p(\tau)$, $|\alpha| \le m$ (note that by Lemma 2.3 we have $\int_S |r\eta-\xi|^{-s}\, d\sigma(\eta) \le C(1-r)^{-s+n-1}$, where $0 \le r < 1$, $\xi \in S$ and $s > n-1$).

To finish the proof of Theorem 3.2, it is sufficient to prove the equivalence (ii) $\Leftrightarrow$ (iv). This is standard. For more general results, see [**6**]. See also [**2**] and [**4**].

Remark 3.3. Carefully examining the proof of Theorem 3.2 above, we actually see that the following are equivalent norms on B^p for the appropriate p's:

(3.1)
$$\left(\int_B (1-|x|^2)^{mp}|\partial^m f(x)|^p\, d\tau(x)\right)^{1/p} + \sum_{|\alpha|<m}|\partial^\alpha f(0)|,$$

(3.2)
$$\left(\int_B (1-|x|^2)^{mp}|R_s^m f(x)|^p\, d\tau(x)\right)^{1/p} + |f(0)|.$$

In the sequel, by $\|f\|_{B^p}$, we will mean any of the expressions (3.1) and (3.2). In the case $p=\infty$ we have

$$\|f\|_{B^\infty} \cong |f(0)| + \sup_{x\in B}(1-|x|^2)^m|R_s^m f(x)|.$$

Corollary 3.4. *Let $m > (n-1)/p$ be a positive integer, $1 \leq p \leq \infty$, and $s, t > 0$. If $f \in h(B)$, then $(1-|x|^2)^m R_s^m f(x) \in L^p(\tau)$ if and only if $(1-|x|^2)^m R_t^m f(x) \in L^p(\tau)$.*

A similar argument shows that the following is true.

Theorem 3.5. *Let k and m be positive integers and $s > 0$. If $f \in h(B)$, then the following statements are equivalent.*

(i) $(1-|x|^2)^m R_s^m f(x) \to 0$, $|x| \to 1$,

(ii) $(1-|x|^2)^m |\partial^m f(x)| \to 0$, $|x| \to 1$,

(iii) $(1-|x|^2)^k R_s^k f(x) \to 0$, $|x| \to 1$,

(iv) $(1-|x|^2)^k |\partial^k f(x)| \to 0$, $|x| \to 1$.

Corollary 3.6. *Let m be a positive integer and $s, t > 0$. If $f \in h(B)$, then $(1-|x|^2)^m R_s^m f(x) \to 0$, $|x| \to 1$ if and only if $(1-|x|^2)^m R_t^m f(x) \to 0$, $|x| \to 1$.*

4. Harmonic Besov spaces. In this section we prove that the harmonic Besov spaces B^p are natural quotient spaces of certain L^p spaces.

Theorem 4.1. *For $1 \leq p \leq \infty$, the Bergman projection P_s, $s > 0$, maps $L^p(B, d\tau)$ boundedly onto the harmonic Besov space B^p.*

Proof. It is easily seen that $P_s L^p(B, d\tau) \subset l^{1,s-1}$. Let $m > n-1$. Given f in $L^p(B, d\tau)$, by Fubini's theorem and the reproducing property of the kernel functions, we easily obtain

$$
\begin{aligned}
R_s^m P_s f(x) &= c_s \int_B (1-|y|^2)^{s-1} K_{s+m}(x,y) P_s f(y)\, d\nu(y) \\
&= c_s^2 \int_B (1-|y|^2)^{s-1} K_{s+m}(x,y)\, d\nu(y) \\
&\quad \cdot \int_B (1-|\xi|^2)^{s-1} K_s(y,\xi) f(\xi)\, d\nu(\xi)
\end{aligned}
$$

$$= c_s^2 \int_B (1-|\xi|^2)^{s-1} f(\xi)\, d\nu(\xi)$$
$$\cdot \int_B (1-|y|^2)^{s-1} K_s(\xi,y) K_{s+m}(y,x)\, d\nu(y)$$
$$= c_s \int_B (1-|\xi|^2)^{s-1} K_{s+m}(x,\xi) f(\xi)\, d\nu(\xi)$$
$$= R_s^m f(x).$$

Thus,

$$(1-|x|^2)^m R_s^m P_s f(x) = E_{m,s} f(x).$$

By Theorem 3.1, the operator $E_{m,s}$ is bounded on $L^p(B, d\tau)$ for all $1 \le p \le \infty$. Thus, the function $(1-|x|^2)^m R_s^m P_s f(x)$ is in $L^p(B, d\tau)$ and hence $P_s f$ is in B^p, by Theorem 3.2.

That P_s maps $L^p(B, d\tau)$ onto B^p follows from the fact that if $f \in B^p$, then $f(x) = c_{s+m} c_s^{-1} P_s((1-|x|^2)^m R_s^m f(x))$.

A slight modification of the previous arguments gives the following:

Theorem 4.2. *The Bergman projection P_s, $s > 0$, maps $C_0(B)$ boundedly onto B_0.*

5. Duality. In this section we deal with duality. The main result is the following.

Theorem 5.1. *Let $1 \le p < \infty$, $m > n-1$ and $s = m-n+1$. The integral pairing*

$$\langle f, g\rangle_\tau = \int_B E_{m,s} f(y) E_{m,s} g(y)\, d\tau(y)$$

induces the following dualities

(a) $(B^p)^\star = B^q$, *where* $1/p + 1/q = 1$,

(b) $(B_0)^\star = B^1$.

Proof. (a) By Theorem 3.2, $f \in B^p$ if and only if $E_{m,s} f \in L^p(B, d\tau)$; thus, the above pairing is well defined and we clearly have $B^q \subset (B^p)^\star$ under the above pairing.

Conversely, assume that λ is a bounded linear functional on B^p; we show that λ arises from a function in B^q. Since $E_{m,s}$ maps B^p into $L^p(B, d\tau)$ isometrically, $\lambda \circ E_{m,s}^{-1}$ is a bounded linear functional on the image space of $E_{m,s}$ in $L^p(B, d\tau)$. By the Hahn-Banach theorem $\lambda \circ E_{m,s}^{-1}$ extends to a bounded linear functional on $L^p(B, d\tau)$. Thus there exists a function $\phi \in L^q(B, d\tau)$ such that

$$\lambda \circ E_{m,s}^{-1}(f) = \int_B f(y)\phi(y)\, d\tau(y), \quad f \in L^p(B, d\tau).$$

When f is in B^p, $E_{m,s}f$ is $L^p(B, d\tau)$. Therefore,

$$\lambda(f) = \int_B E_{m,s}f(y)\phi(y)\, d\tau(y), \quad f \in B^p.$$

Let $h = P_s\phi$. Then $h \in B^q$ by Theorem 4.1. Using Fubini's theorem, we obtain

$$\begin{aligned} E_{m,s}h(x) &= (1-|x|^2)^m R_s^m h(x) = (1-|x|^2)^m R_s^m (P_s\phi)(x) \\ &= (1-|x|^2)^m (R_s^m \phi)(x) = E_{m,s}\phi(x). \end{aligned}$$

To finish the proof of Theorem 5.1, it remains to show that

$$\langle E_{m,s}f, E_{m,s}\phi\rangle_\tau = \langle E_{m,s}^2 f, \phi\rangle_\tau$$

and that

$$c_{m+s}E_{m,s}^2 f = c_s E_{m,s} f.$$

This follows easily from Fubini's theorem and reproducing property of P_s. Note that $s = m - n + 1$. We leave the details to the interested reader. Thus,

$$\lambda(f) = \int_B E_{m,s}f(y) E_{m,s}g(y)\, d\tau(y)$$

for all $f \in B^p$ where $g = c_{m+s}c_s^{-1}h \in B^q$.

(b) Since $f \to E_{m,s}f$ is one-to-one for harmonic f and

$$\|f\|_{B^p} \cong \|E_{m,s}f\|_{L^p(B, d\tau)},$$

we clearly have $B^1 \subset (B_0)^\star$.

If λ is a bounded linear functional on B_0, then $\lambda \circ E_{m,s}^{-1} : E_{m,s}(B_0) \to C$ is a bounded linear functional on the closed subspace $E_{m,s}(B_0)$ of $C_0(B)$. By Hahn-Banach, $\lambda \circ E_{m,s}^{-1}$ extends to a bounded linear functional on $C_0(B)$. By Riesz representation, there exists a finite complex Borel measure $d\mu$ on B such that

$$\lambda \circ E_{m,s}^{-1}(f) = \int_B f(y)\, d\mu(y), \quad f \in C_0(B).$$

In particular,

$$\lambda(f) = \int_B E_{m,s} f(y)\, d\mu(y), \quad f \in B_0.$$

Let

$$g(x) = \int_B (1-|y|^2)^m K_s(x,y)\, d\mu(y).$$

Then g is harmonic in B and

$$\begin{aligned} R_s^m g(x) &= c_s \int_B (1-|y|^2)^{s-1} K_{s+m}(x,y) g(y)\, d\nu(y) \\ &= c_2 \int_B (1-|y|^2)^{s-1} K_{s+m}(x,y)\, d\nu(y) \\ &\quad \cdot \int_B (1-|\xi|^2)^m K_s(y,\xi)\, d\mu(\xi) \\ &= \int_B (1-|\xi|^2)^m\, d\mu(\xi) c_s \\ &\quad \cdot \int_B (1-|y|^2)^{s-1} K_s(y,\xi) K_{s+m}(x,y)\, d\nu(y) \\ &= \int_B (1-|\xi|^2)^m K_{s+m}(x,\xi)\, d\mu(\xi). \end{aligned}$$

Hence,

$$\begin{aligned} &\|(1-|x|^2)^m R_s^m g\|_{L^1(\tau)} \\ &\quad \le C \int_B (1-|x|^2)^{m-n} \left(\int_B (1-|\xi|^2)^m |K_{s+m}(x,\xi)| d|\mu|(\xi) \right) d\nu(x) \\ &\quad = C \int_B (1-|\xi|^2)^m\, d|\mu|(\xi) \int_B (1-|x|^2)^{m-n} |K_{s+m}(x,\xi)|\, d\nu(x) \\ &\quad \le C|\mu|(B). \end{aligned}$$

Thus, $g \in B^1$. Here we have used Lemma 2.4 again and the fact that $s+n-1=m$.

Since $\langle E_{m,s}f, E_{m,s}g\rangle_\tau = \langle f, E^2_{m,s}g\rangle_\tau$ and $c_{s+m}E^2_{m,s}g = c_s E_{m,sg}$, we have

$$\begin{aligned}
\langle f, g\rangle_\tau &= \int_B f(y)E^2_{m,s}g(y)\,, d\tau(y)\\
&= c_s c^{-1}_{s+m}\int_B f(y)E_{m,s}g(y)\,d\tau(y)\\
&= c_s c^{-1}_{s+m}\int_B f(y)(1-|y|^2)^{m-n}\,d\nu(y)\\
&\quad\cdot \int_B (1-|\xi|^2)^m K_{s+m}(y,\xi)\,d\mu(\xi)\\
&= c_s c^{-1}_{s+m}\int_B (1-|\xi|^2)^m\,d\mu(\xi)\\
&\quad\cdot \int_B (1-|y|^2)^{s-1}K_{s+m}(\xi,y)f(y)\,d\nu(y)\\
&= c^{-1}_{s+m}\int_B E_{m,s}f(\xi)\,d\mu(\xi).
\end{aligned}$$

Thus,

$$\lambda(f) = \int_B E_{m,s}f(y)E_{m,s}h(y)\,d\tau(y),$$

where $h = c_{s+m}g$.

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Matematički Fakultet, Studentski trg 16, 11 000 Beograd, Serbia
E-mail address: jevtic@matf.bg.ac.yu

Matematički Fakultet, Studentski trg 16, 11 000 Beograd, Serbia
E-mail address: