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PRECISE LARGE DEVIATIONS OF AGGREGATE LOSS PROCESS IN A RISK MODEL BASED ON THE POLICY ENTRANCE PROCESS

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ABSTRACT. In this paper, we introduce a risk model based on the policy entrance process with n kinds of independent policies. Aiming at the model in which each kind of policy is issued according to a non-homogeneous Poisson process with heavy-tailed distributed claim sizes, we study the large deviations for aggregate loss process of the risk model.

1. Introduction. Let $\{X_k; k \geq 1\}$ be a sequence of random variables (rv's) with common distribution function F and mean μ independent of $\{N(t); t \geq 0\}$. Suppose that $\{N(t); t \geq 0\}$ is a nonnegative integer-valued process with mean function $EN(t) = \lambda(t)$. Mainstream on precise large deviations has been concentrated on the study of the asymptotics

$$\Pr\left(\sum_{k=1}^{n} X_k - n\mu\right) \sim n\overline{F}(x) \tag{1.1}$$

and

$$\Pr\left(\sum_{k=1}^{N(t)} X_k - \lambda(t)\mu\right) \sim \lambda(t)\overline{F}(x), \tag{1.2}$$

respectively, which hold uniformly for some x-region. Throughout, we let $\overline{F} = 1-F$. Heyde [**4, 5**] studied the asymptotics (1.1) with regularly varying tails. Cline and Hsing [**2**] obtained (1.1) for a larger class, the so-called ERV (extended regularly varying) class. Later, Klüppelberg and Mikosch [**6**] considered (1.2) for the ERV class. We restate their result as follows.

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Proposition 1.1 (Klüppelberg and Mikosch [**6**]). *If* $F \in \mathrm{ERV}\,(-\alpha, -\beta)$ *with* $1 < \alpha \le \beta < \infty$, *and* $N(t)$ *satisfies that*

Assumption A. $$\frac{N(t)}{\lambda(t)} \xrightarrow{\mathrm{P}} 1$$

and that, for some $\varepsilon > 0$ *and any* $\delta > 0$,

Assumption B. $$\sum_{k>(1+\delta)\lambda(t)} (1+\varepsilon)^k \mathrm{Pr}\,(N(t) > k) = o(1),$$

then, for any $\gamma > 0$, (1.2) *holds uniformly for* $x \ge \gamma\lambda(t)$.

Recent advances in precise large deviations can be found in [**8, 9, 12, 13, 14, 15, 16**], among many others. It is worth mentioning that Wang and Wang [**17**] considered the large deviations for sums of rv's with consistently varying tails (the so-called $\mathcal{C}$ class) in multi-risk models.

However, the model shown in [**17**] concentrates on the claim number process. But it is easy to conceive that the claim number process is virtually driven by the policy entrance process, since whenever the insurer issues a policy, he will have to burden the potential claims entitled by the policy. In view of above idea, Li and Kong [**7**] considered a new risk model with n kinds of policies and obtained some weak convergence properties of the model under the condition that the claim sizes distribution is regularly varying. Based on [**7**], we study the precise large deviations of the loss process with ERV distributed claim sizes of the improved risk model. We give a detailed description of the model as follows.

For the ith kind of policy, $1 \le i \le n$. Suppose that the arrival time of the jth customer is σ_j^i, and $\{N_i(t); t \ge 0\}$ is the counting process associated with $\{\sigma_j^i\}_{j=1}^{\infty}$, i.e., $N_i(t) = \max\{j; \sigma_j^i \le t\}$. The premium charged by the insured and the validity time are supposed to be two constants, denoted by d_i and a_i, respectively. Let Y_{jk}^i denote the kth claim size of the jth customer and T_{jk}^i the duration time from S_j^i to the kth claim time of the jth insured. Let $\{M_j^i(s); s \ge 0\}$ be the counting process associated with $\{T_{jk}^i\}_{k=1}^{\infty}$, i.e., $M_j^i(s) = \max\{k; T_{jk}^i \le s\}$. It is obvious that the jth insured can claim at most $M_j^i(a_i)$ times. Thus, the aggregate loss process of the ith kind of policy up to time t is

$$S_i(t) = \sum_{j=1}^{N_i(t)} \left(\sum_{k=1}^{M_j^i(a_i)} Y_{jk}^i I\{T_{jk}^i + \sigma_j^i \leq t\} - d_i \right), \tag{1.3}$$

and the total loss process due to these n kinds of policies up to time t is

$$S(n,t) = \sum_{i=1}^{n} S_i(t). \tag{1.4}$$

Remark 1.1. We make the convention that

$$\sum_{k=1}^{0} Y_{jk}^i I\{T_{jk}^i + \sigma_j^i \leq t\} = 0.$$

Remark 1.2. $S_i(t)$ and $S(n,t)$ can be thought of as some shot noise processes.

The remaining part of this paper is organized as follows. Section 2 presents some assumptions on the model and our main results. Section 3 proves the main results, after showing some necessary lemmas.

2. Assumptions and main results. First of all, we recall some famous classes of heavy-tailed distributions.

We say that a distribution function F, by definition, has dominated varying tails (denoted by $\mathcal{D}$), if and only if

$$\limsup_{x\to\infty} \frac{\overline{F}(xy)}{\overline{F}(x)} < \infty \text{ for any } y \in (0,1) \text{ (or, equivalently, for } y = \tfrac{1}{2}).$$

A closely related class is the long-tailed class (denoted by $\mathcal{L}$). A distribution function F is in $\mathcal{L}$ if and only if

$$\lim_{x\to\infty} \frac{\overline{F}(x+y)}{\overline{F}(x)} = 1, \quad \text{for any } y > 0.$$

Another important subclass of heavy tails is the consistently varying class (denoted by $\mathcal{C}$). A distribution function F is in $\mathcal{C}$ if and only if

$$\lim_{y\searrow 1}\liminf_{x\to\infty}\frac{\overline{F}(xy)}{\overline{F}(x)}=1,$$

or, equivalently,

$$\lim_{y\nearrow 1}\limsup_{x\to\infty}\frac{\overline{F}(xy)}{\overline{F}(x)}=1.$$

A slight small class is the extended regularly varying class (denoted by ERV). A distribution function F is in ERV $(-\alpha,-\beta)$ for some α, β with $0<\alpha\le\beta<\infty$ if and only if

$$y^{-\beta}\le\liminf_{x\to\infty}\frac{\overline{F}(xy)}{\overline{F}(x)}\le\limsup_{x\to\infty}\frac{\overline{F}(xy)}{\overline{F}(x)}\le y^{-\alpha}\quad\text{for any } y>1.$$

Clearly, the ERV class covers the famous class $\mathcal{R}_{-\alpha}$ of distributions with regularly-varying tails in the sense that the relation

$$\lim_{x\to\infty}\frac{\overline{F}(xy)}{\overline{F}(x)}=y^{-\alpha}$$

holds for some $\alpha>0$ and all $y>0$. Some related discussions on heavy-tailed distributions can be found in [**1, 3, 11**]. It is well known that these classes satisfy the following inclusions:

$$\mathcal{R}_{-\alpha}\in ERV\subset\mathcal{C}\subset\mathcal{D}\cap\mathcal{L}.$$

Set

$$\overline{F}_*(y)=\liminf_{x\to\infty}\frac{\overline{F}(xy)}{\overline{F}(x)}$$

and

$$\mathbb{J}_F=\inf\left\{-\frac{\log\overline{F}_*(y)}{\log y}:y>1\right\},$$

where $\mathbb{J}_F$ is called the upper Matuszewska index of the distribution function F. Clearly, if $F\in$ ERV $(-\alpha,-\beta)$, then $\alpha\le\mathbb{J}_F\le\beta$. For more details on the Matuszewska index see Bingham et al. [**1**].

Some assumptions are required for models (1.3) and (1.4) in present paper.

Assumption 2.1. $\{M_j^i(t); t \geq 0\}$, $i \geq 1$, $j \geq 1$, are independent homogeneous Poisson processes with mean function $EM_j^i(t) = \nu_i t$.

Assumption 2.2. $\{N_i(t); t \geq 0\}_{i=1}^n$ is an independent non-homogeneous Poisson process with intensity function $\lambda_i(t)$ and the accumulated intensity function $\Lambda_i(t) = \int_0^t \lambda_i(s)\,\mathrm{d}s$ satisfying $\Lambda_i(t) \to \infty$ as $t \to \infty$.

Assumption 2.3. For a given $i \geq 1$, $\{Y_{jk}^i, j \geq 1, k \geq 1\}$ are i.i.d. rv's with a common distribution function $F_i(\cdot) \in ERV(-\alpha, -\beta)$ for some $\alpha,\ \beta$ satisfying $1 < \alpha \leq \beta < \infty$.

Assumption 2.4. The sequences $\{Y_{jk}^i; j \geq 1, k \geq 1\}$, $\{M_j^i(t); t \geq 0\}$ and $\{N_i(t); t \geq 0\}$ are mutually independent.

For one kind of policy, namely, $n = 1$, we simplify the notation $N_i(t)$, Y_{jk}^i, T_{jk}^i, σ_j^i, $M_j^i(t)$, $F_i(\cdot)$, d_i in (1.3), respectively, as $N(t)$, Y_{jk}, T_{jk}, σ_j, $M_j(t)$, $F(\cdot)$, d. Thus, the aggregate loss process due to one kind of policy is denoted by

$$S(t) = \sum_{j=1}^{N(t)} \left(\sum_{k=1}^{M_j(a)} Y_{jk} I\{T_{jk} + \sigma_j \leq t\} - d \right). \tag{2.1}$$

Remark 2.1. Assumptions 2.1–2.4 hold, respectively, for (2.1). All subscripts or superscripts i of corresponding notation are omitted.

Henceforth, all limit relations, unless otherwise stated, are for $t \to \infty$, namely, $\Lambda(t) \to \infty$. For positive functions $a(x)$ and $b(x)$, we write $a(x) = o(b(x))$ if $\lim_{x\to\infty} a(x)/b(x) = 0$; $a(x) \lesssim b(x)$ if $\limsup_{x\to\infty} a(x)/b(x) \leq 1$; $a(x) \gtrsim b(x)$ if $\liminf_{x\to\infty} a(x)/b(x) \geq 1$ and $a(x) \sim b(x)$ if both $a(x) \lesssim b(x)$ and $a(x) \gtrsim b(x)$. Very often, we limit relationships with certain uniformity for our specific purposes. For instance, for two positive bivariate functions $a(\cdot,\cdot)$ and $b(\cdot,\cdot)$, we say that $a(x,t) \lesssim b(x,t)$ holds uniformly for $t \in \triangle \neq \emptyset$ if

$$\limsup_{x\to\infty} \sup_{t\in\triangle\neq\emptyset} \frac{a(x,t)}{b(x,t)} \leq 1.$$

Now, we give our main results. We start with the result below which can be thought of as a contribution of Proposition 1.1.

Theorem 2.1. *Suppose that Assumptions* 2.1–2.4 *hold. Then, for each* $\gamma > 0$,

$$\Pr\left(\sum_{j=1}^{N(t)}\sum_{k=1}^{M_j(a)} Y_{jk} - E\sum_{j=1}^{N(t)}\sum_{k=1}^{M_j(a)} Y_{jk} > x\right) \sim a\nu\Lambda(t)\overline{F}(x)$$

holds uniformly for $x \geq \gamma\Lambda(t)$.

Let

$$h_j = \sum_{k=1}^{M_j(a)} Y_{jk}, \quad j \geq 1.$$

Note that $M_j(a) < \infty$ and $\{h_j; j \geq 1\}$ are i.i.d. For $x \to \infty$, we derive

$$\begin{aligned}\overline{G}(x) &= \Pr(h_j > x)\\ &= \sum_{m=1}^{\infty} \Pr(M_j(a) = m)\Pr\left(\sum_{k=1}^{m} Y_{jk} > x\right) \sim a\nu\overline{F}(x).\end{aligned}$$

Thus, one can easily check that $\{h_j; j \geq 1\}$ belongs to ERV. Moreover, we can verify that $\{N(t); t \geq 0\}$ in our model (1.3) still satisfies Assumptions A and B. Thus, the proof of Theorem 2.1 is similar to that of Theorem 3.1 of Klüppelberg and Mikosch **[6]**. Therefore, we omit the proof here.

Theorem 2.2. *Suppose that Assumptions* 2.1–2.4 *hold. Then*

$$\Pr(S(t) - ES(t) > x) \sim a\nu\Lambda(t)\overline{F}(x) \tag{2.2}$$

holds uniformly for $x \geq \gamma\Lambda(t)$ *and every* $\gamma > 0$.

For model (1.4), we obtain the following result.

Theorem 2.3. *Suppose that Assumptions* 2.1–2.4 *hold. Then, for each* $\gamma > 0$,

$$\Pr(S(n,t) - ES(n,t) > x) \sim \sum_{i=1}^{n} a_i\nu_i\Lambda_i(t) \tag{2.3}$$

holds uniformly for $x \geq \gamma\overline{\Lambda}(t)$, *where* $\overline{\Lambda}(t) = \max_{i\geq 1}\Lambda_i(t)$.

3. Proof of main results.

3.1. Several lemmas. Consider a Poisson shot noise process

$$W(t) = \sum_{i=i}^{N(t)} X_i(t - T_i),$$

where T_i are the points of a homogeneous Poisson process $N(t)$ and the processes $X_i, i \geq 1$ are i.i.d. with non-decreasing non-negative cadlag sample paths on R such that $X(t) = 0$ as for $t < 0$. The sequences (T_n) and (X_n) are also supposed to be independent. Miksoch and Nagaev **[10]** showed an elementary lemma which plays a key role in derivation of the large deviation results of the shot noise process $W(t)$. We restate their result as follows.

Lemma 3.1. *Let $f_n(x_1, \ldots, x_n)$, $n = 1, 2, \ldots$, be measurable R^d-valued functions which are symmetric in their arguments. Then, for every $t \geq 0$,*

$$f_{N(t)}(X_1(t - T_1), \ldots, X_{N(t)}(t - T_{N(t)})) \\ \stackrel{\mathrm{d}}{=} f_{N(t)}(X_1(t - U_1), \ldots, X_{N(t)}(t - U_{N(t)})),$$

where $U_1, \ldots, U_{N(t)}$ is a sequence of i.i.d. rv's with common distribution function $\Lambda(s)/\Lambda(t)$ for $0 \leq s \leq t$, independent of the Poisson process $N(t)$.

By Lemma 3.1, for every fixed $t > 0$, we conclude some important relations as follows:

$$S(t) \stackrel{\mathrm{d}}{=} \sum_{j=1}^{N(t)} \left(\sum_{k=1}^{M_j(a)} Y_{jk} I\{U_{jk} + U_j \leq t\} - d \right), \tag{3.1}$$

where $\{U_j; j \geq 1\}$ is a sequence of i.i.d. rv's with common distribution function $\Lambda(s)/\Lambda(t)$ $(0 \leq s \leq t)$, independent of the non-homogeneous Poisson process $N(t)$ and all other sources of randomness; $\{U_{jk}; j \geq 1, k \geq 1\}$ is a sequence of i.i.d. uniformly distributed on $(0, a)$, independent of the homogeneous Poisson process $M_j(s)$ and all other sources of randomness.

$$S(n,t) \overset{\mathrm{d}}{=} \sum_{i=1}^{n} \sum_{j=1}^{N_i(t)} \left(\sum_{k=1}^{M_j^i(a_i)} Y_{jk}^i I\{U_{jk}^i + U_j^i \le t\} - d_i \right), \tag{3.2}$$

where $\{U_j^i, j \ge 1\}$ is a sequence of i.i.d. rv's with common distribution function $\Lambda_i(s)/\Lambda_i(t)$ for $0 \le s \le t$, independent of the non-homogeneous Poisson process $N_i(t)$ and all other sources of randomness; $\{U_{jk}^i; j \ge 1, k \ge 1\}$ is a sequence of i.i.d. rv's uniformly distributed on $(0, a_i)$, independent of the homogeneous Poisson process $M_j^i(s)$ and all other sources of randomness.

Observe that, for $j \ge 1$, $k \ge 1$,

$$\Pr(Y_{jk} I\{U_{jk} \le t - U_j\} > x) = \overline{F}(x) \int_0^t \min\left\{\frac{t-s}{a}, 1\right\} \Pr(U_j \in \mathrm{d}s).$$

Denote $g(t) = \int_0^t \min\{\frac{t-s}{c}, 1\} \Pr(U_j \in \mathrm{d}s)$. Then,

$$\begin{aligned} \Pr(Y_{jk} I\{U_{jk} \le t - U_j\} > x) &= g(t)\overline{F}(x); \\ E(Y_{jk} I\{U_{jk} \le t - U_j\}) &= g(t) EY_{11}; \end{aligned}$$

and

$$\mu(t) \triangleq ES(t) = \Lambda(t)(a\nu g(t) EY_{11} - d).$$

It is easy to see that, for $j \ge 1$,

$$\lim_{t \to \infty} g(t) = 1.$$

For fixed $t > 0$, we write

$$H_j(t) = \sum_{k=1}^{M_j(a)} Y_{jk} I\{U_{jk} + U_j < t\}.$$

Li and Kong **[7]** showed an equivalent relation between $\sum_{j=1}^{N(t)} H_j(t)$ and $\sum_{j=1}^{N(t)} h_j$ as follows.

Lemma 3.2. *If* $E[Y_{jk}]^\beta < \infty$, *for some* $0 < \beta \le 1$, *then for any* $\delta > 0$,

$$\frac{1}{\Lambda_i^\delta(t)} \left[\sum_{j=1}^{N(t)} H_j(t) - \sum_{j=1}^{N(t)} h_j \right] \xrightarrow{P} 0.$$

With the help of Lemma 3.2, we obtain the following result of the weak law of large numbers.

Lemma 3.3. *The weak law of large numbers*

$$\frac{\sum_{j=1}^{[\Lambda(t)]} H_j(t) - [\Lambda(t)]EH_1(t)}{\Lambda(t)} \xrightarrow{\mathrm{P}} 0$$

holds.

Proof. For every $\varepsilon > 0$, we conclude that

$$\begin{aligned}
&\Pr\left(|\sum_{j=1}^{[\Lambda(t)]} H_j(t) - [\Lambda(t)]EH_1(t)| > \varepsilon\Lambda(t)\right)\\
&\quad = \Pr\Bigg(\Big|\sum_{j=1}^{[\Lambda(t)]} (H_j(t) - h_j) + [\Lambda(t)]\\
&\qquad\qquad (Eh_j - EH_j(t)) + \sum_{j=1}^{[\Lambda(t)]} h_j - [\Lambda(t)]Eh_j\Big| > \varepsilon\Lambda(t)\Bigg)\\
&\quad \le \Pr\left(\sum_{j=1}^{[\Lambda(t)]} \big|H_j(t) - h_j\big| > \frac{\varepsilon\Lambda(t)}{2} - [\Lambda(t)]\big|EH_j(t) - Eh_j\big|\right)\\
&\qquad + \Pr\left(\Big|\sum_{j=1}^{[\Lambda(t)]} h_j - [\Lambda(t)]Eh_j\Big| > \frac{\varepsilon\Lambda(t)}{2}\right)\\
&\quad = \Pr\left(\sum_{j=1}^{[\Lambda(t)]} \big|H_j(t) - h_j\big| > \Lambda(t)\left(\frac{\varepsilon}{2} - Eh_j\big|g(t) - 1\big|\right)\right)\\
&\qquad + \Pr\left(\Big|\sum_{j=1}^{[\Lambda(t)]} h_j - [\Lambda(t)]Eh_j\Big| > \frac{\varepsilon\Lambda(t)}{2}\right)\\
&\quad = I_1 + I_2,
\end{aligned}$$

where $[\Lambda(t)]$ stands for its integer part. Recall that $g(t) \to 1$ as $t \to \infty$, and we can find a small constant $\varepsilon' > 0$ satisfying $\varepsilon' = \frac{\varepsilon}{2} - Eh_j|g(t) - 1|$; hence, by Lemma 3.2, $I_1 \to 0$. Furthermore, by the Khinchine law of large numbers, we have $I_2 \to 0$. This ends the proof of Lemma 3.3. $\square$

The lemma below is a direct consequence of Su et al. [**14**].

Lemma 3.4. *If the distribution function* $F \in \mathrm{ERV}\,(-\alpha, -\beta)$ *for some* $1 < \alpha \le \beta < \infty$. *Then, for any* $1 < \alpha' < \alpha \le \beta < \beta' < \infty$, *we have* $EX^{\alpha'} < \infty$ *and*

$$c_2 x^{-\beta'} \le \overline{F}(x) \le c_1 x^{-\alpha'}$$

for all large $x > 0$, *where the constants* $c_1 = c_1(\alpha')$ *and* $c_2 = c_2(\beta')$ *are independent of* x.

It follows from Lemma 3.4, for each $\gamma > 0$, that

$$\Lambda(t)\mathrm{Pr}\,(Y_{11} I\{U_{11} \le t - U_1\} > \gamma t) \le c_1 \Lambda(t) t^{-\alpha'} \longrightarrow 0, \quad \text{as } t \to \infty. \tag{3.3}$$

Lemma 3.5. *If the distribution function* $F \in \mathrm{ERV}\,(-\alpha, -\beta)$ $(1 < \alpha \le \beta < \infty)$, *then* $\overline{F}(x + o(x)) \sim \overline{F}(x)$ *holds.*

Proof. For any $\epsilon > 0$ and large x,

$$\frac{\overline{F}((1+\epsilon)x)}{\overline{F}(x)} \le \frac{\overline{F}(x + o(x))}{\overline{F}(x)} \le \frac{\overline{F}((1-\epsilon)x)}{\overline{F}(x)}.$$

By the definition of ERV, we obtain that

$$\begin{aligned}(1+\epsilon)^{-\beta} &\le \liminf_{x\to\infty} \frac{\overline{F}((1+\epsilon)x)}{\overline{F}(x)} \le \liminf_{x\to\infty} \frac{\overline{F}(x + o(x))}{\overline{F}(x)} \\ &\le \limsup_{x\to\infty} \frac{\overline{F}(x + o(x))}{\overline{F}(x)} \le \limsup_{x\to\infty} \frac{\overline{F}((1-\epsilon)x)}{\overline{F}(x)} \\ &\le (1-\epsilon)^{-\alpha}.\end{aligned}$$

Let $\epsilon \to 0$, and the proof is obtained immediately. □

The following two lemmas are crucial for our main results.

Lemma 3.6. *Suppose that Assumption* 2.3 *holds. Then, for* $m \ge 1$, *the following relation*

$$\mathrm{Pr}\left(\sum_{k=1}^{m} Y_{jk} I\{U_{jk} + U_j \le t\} > x\right) \sim m\overline{F}(x) g(t)$$

holds for $x \to \infty$.

Proof. For $x \to \infty$, we have

$$\begin{aligned}
&\Pr\left(\sum_{k=1}^{m} Y_{jk} I\{U_{jk} + U_j \le t\} > x\right) \\
&= \int_0^t \Pr\left(\sum_{k=1}^{m} Y_{jk} I\{U_{jk} \le t-s\} > x\right) \Pr\left(U_j \in \mathrm{d}s\right) \\
&= \int_0^{t-a} \Pr\left(\sum_{k=1}^{m} Y_{jk} > x\right) \Pr\left(U_{jk} \le a\right) \Pr(U_j \in \mathrm{d}s) \\
&\quad + \int_{t-a}^{t} \sum_{r=1}^{m} \binom{m}{r} \left(\frac{t-s}{a}\right)^r \left(1 - \frac{t-s}{a}\right)^{m-r} \\
&\qquad \Pr\left(\sum_{k=1}^{r} Y_{jk} > x\right) \Pr\left(U_j \in \mathrm{d}s\right) \\
&\quad \sim m\Pr\left(Y_{jk} > x\right) \int_0^{t-a} \Pr\left(U_j \in \mathrm{d}s\right) \\
&\quad + m\Pr\left(Y_{jk} > x\right) \int_{t-a}^{t} \frac{t-s}{a} \Pr\left(U_j \in \mathrm{d}s\right) \\
&= m\Pr\left(Y_{jk} > x\right) \int_0^t \min\left\{\frac{t-s}{a}, 1\right\} P(U_j \in \mathrm{d}s) \\
&= m\overline{F}(x) g(t).
\end{aligned}$$

This ends the proof of Lemma 3.6. □

Lemma 3.7. *Under the conditions in Lemma* 3.6*, for fixed* $t > 0$*, the following relation*

$$\Pr\left(\sum_{k=1}^{M_j(a)} Y_{jk} I\{U_{jk} + U_j \le t\} > x\right) \sim a\nu\overline{F}(x) g(t)$$

holds for $x \to \infty$.

Proof. For $0 < m_0 < \infty$, we have

$$\Pr\left(\sum_{k=1}^{M_j(a)} Y_{jk} I\{U_{jk} + U_j \le t\} > x\right) = \sum_{m_j=1}^{\infty} \Pr(M_j(a) = m_j)$$

$$\Pr\left(\sum_{k=1}^{m_j} Y_{jk} I\{U_{jk} + U_j \le t\} > x\right) = \left(\sum_{m_j=1}^{m_0} + \sum_{m_j=m_0+1}^{\infty}\right) \Pr(M_j(a) = m_j)$$

$$\Pr\left(\sum_{k=1}^{m_j} Y_{jk} I\{U_{jk} + U_j \le t\} > x\right) = I_3 + I_4.$$

It follows from Lemma 3.6 that

$$\begin{aligned} I_3 &\sim \sum_{m_j=1}^{m_0} m_j \Pr(M_j(a) = m_j) \Pr(Y_{jk} I\{U_{jk} + U_j \le t\} > x) \\ &= E[M_j(a) I\{M_j(a) \le m_0\}] \overline{F}(x) g(t). \end{aligned}$$

For I_4, by Kesten's inequality, it holds for each $\epsilon > 0$ and some $K > 0$, that

$$\begin{aligned} I_4 &\le \sum_{m_j=m_0+1}^{\infty} \Pr(M_j(a) = m_j) \Pr\left(\sum_{k=0}^{m_j} Y_{jk} > x\right) \\ &\le K\overline{F}(x) \sum_{m_j=m_0+1}^{\infty} \Pr(M_j(a) = m_j)(1+\epsilon)^{m_j}, \end{aligned}$$

since $EM_j(c) < \infty$; hence, $I_4 = o(I_3)$ as m_0 large enough. We conclude that

$$\Pr\left(\sum_{k=1}^{M_j(a)} Y_{jk} I\{T_{jk} + U_j \le t\} > x\right) \sim a\nu \overline{F}(x) g(t).$$

Furthermore, for any fixed $\gamma > 0$, it follows for $x \ge \gamma t$, that

$$\Pr\left(\sum_{k=1}^{M_j(a)} Y_{jk} I\{U_{jk} + U_j \le t\} > x\right) \sim a\nu \overline{F}(x), \quad t \to \infty. \tag{3.4}$$

This ends the proof of Lemma 3.7. □

3.2. Proof of Theorem 2.2.

Proof. Firstly, we estimate the lower bound of $\Pr(S(t)-ES(t) > x)$. Denote

$$L_n(t) = \sum_{j=1}^{n} H_j(t), \qquad \widetilde{L}_n(t) = L_n(t) - EL_n(t).$$

By the law of large numbers of the Poisson process, there exists a positive function $\varepsilon_t \to 0$ as $t \to \infty$ such that

$$\Pr(|N(t) - \Lambda(t)| \le \varepsilon_t \Lambda(t)) \longrightarrow 1.$$

For $x \ge \gamma\Lambda(t)$, we have

$$\begin{aligned}
&\Pr(S(t) - ES(t) > x) \\
&= \sum_{n=1}^{\infty} \Pr(N(t) = n) \Pr\left(\sum_{j=1}^{n} (H_j(t) - d) - \mu(t) > x\right) \\
&\ge \sum_{|n-\Lambda(t)| \le \varepsilon_t \Lambda(t)} \Pr\big(L_n(t) - \mu(t) > x + nd\big) \\
&\ge \sum_{|n-\Lambda(t)| \le \varepsilon_t \Lambda(t)} \Pr(N(t) = n) \\
&\quad \cdot \Pr\big(L_{[\Lambda(t)(1-\varepsilon_t)]}(t) > x + \mu(t) + \Lambda(t)(1+\varepsilon_t)d\big) \\
&= (1+o(1)) \Pr\big(L_{[\Lambda(t)(1-\varepsilon_t)]}(t) - [a\nu\Lambda(t)(1-\varepsilon_t)] g(t) EY_{11}) \\
&\qquad > x + \gamma_t\big) \\
&= (1+o(1)) \Pr\big(\widetilde{L}_{[\Lambda(t)(1-\varepsilon_t)]}(t) > x + \gamma_t\big),
\end{aligned} \tag{3.5}$$

where $\gamma_t = \varepsilon_t \Lambda(t)(a\nu g(t) EY_{11} + d)$.

Notice that, for fixed $t > 0$, $\gamma_t = o(\Lambda(t))$. Hence, for arbitrary $\delta > 0$,

$$\begin{aligned}
&\Pr\big(\widetilde{L}_{[\Lambda(t)(1-\varepsilon_t)]}(t) > x + \gamma_t\big) \\
&\ge \Pr\Bigg(\bigcup_{k=1}^{[\Lambda(t)(1-\varepsilon_t)]} \big(\widetilde{L}_{[\Lambda(t)(1-\varepsilon_t)]}(t) > x + \gamma_t, H_k(t) > (1+\delta)x, \\
&\qquad \max_{\substack{j \ne k \\ j \le [\Lambda(t)(1-\varepsilon_t)]}} H_j(t) \le (1+\delta)x\big)\Bigg)
\end{aligned} \tag{3.6}$$

$$
\begin{aligned}
&\geq [\Lambda(t)(1-\varepsilon_t)]P(H_1(t) > (1+\delta)x) \\
&\Pr\big(\widetilde{L}_{[\Lambda(t)(1-\varepsilon_t)]-1}(t) > -\delta x + \gamma_t, \max_{j\leq[\Lambda(t)(1-\varepsilon_t)]-1} H_j(t) \\
&\qquad \leq (1+\delta)x\big),
\end{aligned}
$$

where the last step is obtained by the fact that, for fixed $t > 0$, $\{H_j(t)\}_j$ are independent.

With respect to Lemmas 3.5 and 3.7, for $x \geq \gamma\Lambda(t)$, we obtain that,

$$
\Pr(H_1(t) > (1+\delta)x) \sim a\nu\overline{F}\big(x(1+\delta)\big) \sim a\nu\overline{F}(x). \tag{3.7}
$$

By (3.3) and (3.7), for $x \to \infty$, we have

$$
\begin{aligned}
\Pr\left(\max_{j\leq[\Lambda(t)(1-\varepsilon_t)]} H_j(t) \leq (1+\delta)x\right) &= \Pr(H_1(t) \leq (1+\delta)x)^{[\Lambda(t)(1-\varepsilon_t)]} \\
&\geq \big[1 - \Pr(H_1(t) > (1+\delta)x)\big]^{\Lambda(t)} \\
&\sim \big[\big(1 - a\nu\overline{F}(x)\big)^{1/(a\nu\overline{F}(x))}\big]^{a\nu\Lambda(t)\overline{F}(x)} \\
&\longrightarrow 1.
\end{aligned} \tag{3.8}
$$

On the other hand, Lemma 3.3 shows that

$$
\Pr(\widetilde{L}_{[\Lambda(t)(1-\varepsilon_t)]-1}(t) > -\delta x + \gamma_t) \longrightarrow 1. \tag{3.9}
$$

In view of (3.5)–(3.9), we obtain the lower estimate

$$
\Pr(S(t) - \mu(t) > x) \gtrsim a\nu\Lambda(t)\overline{F}(x).
$$

Now we check the upper estimate using the truncation argument. For fixed $t > 0$, we write

$$
Y_{jk}^{\delta x} = \min\{Y_{jk}, \delta x\}, \qquad H_j^{\delta x}(t) = \sum_{k=1}^{M_j(a)} Y_{jk}^{\delta x} I\{U_{jk} + U_j < t\}
$$

and

$$
S^{\delta x}(t) = \sum_{j=1}^{N(t)} (H_j^{\delta x}(t) - d).
$$

For any $\delta \in (0,1)$, we have

$$
\Pr(S(t) - \mu(t) > x) \tag{3.10}
$$

$$
\begin{aligned}
&= \sum_{n=1}^{\infty} \Pr(N(t)=n) \Pr\left(\sum_{j=1}^{n}(H_j(t)-d)-\mu(t)>x\right) \\
&= \sum_{n=1}^{\infty} \Pr(N(t)=n) \\
&\qquad \left(\Pr\left(\sum_{j=1}^{n}(H_j(t)-d)-\mu(t)>x, \max_{j\le n} H_j(t)>\delta x\right)\right. \\
&\qquad \left. + \Pr\left(\sum_{j=1}^{n}(H_j(t)-d)-\mu(t)>x, \max_{j\le n} H_j(t)\le \delta x\right)\right) \\
&\le \Lambda(t)\Pr\big(H_1(t)>\delta x\big)+\Pr\big(S^{\delta x}(t)-\Lambda(t)(a\nu g(t)EY_{11}-d)>x\big) \\
&= \Lambda(t)\Pr\big(H_1(t)>\delta x\big)+\Pr\big(\widetilde{S}^{\delta x}(t)>x\big) \\
&= \Lambda(t)\Pr\big(H_1(t)>\delta x\big)+I_5.
\end{aligned}
$$

Recall that $F \in \mathrm{ERV}(-\alpha,-\beta)$. Thus, for $x \ge \gamma\Lambda(t)$, $t \to \infty$,

$$
\Pr(H_1(t)>\delta x) \sim a\nu \overline{F}(x). \tag{3.11}
$$

It thus remains to show that $I_5 = o(a\nu\Lambda(t)\overline{F}(x))$.

Set $b = -\ln(a\nu\Lambda(t)\overline{F}(x))$, $r = \frac{b-\tau\beta\ln b}{\delta x}$, $\tau > 1$. Lemma 3.4 implies that, for $x \ge \gamma\Lambda(t)$, $b \to \infty$, $r \to 0$.

Using Markov's inequality yields that

$$
\begin{aligned}
\frac{I_5}{a\nu\Lambda(t)\overline{F}(x)} &\le \exp\{-r\big(x+\Lambda(t)(a\nu g(t)EY_{11}-d)\big)+b\}E \\
&\qquad \cdot \exp\left\{r\left(\sum_{j=1}^{N(t)}(h_j^{\delta x}-d)\right)\right\} \\
&= \exp\Big\{-r\big(x+\Lambda(t)(a\nu g(t)EY_{11}-d)\big) \\
&\qquad + b-\Lambda(t)+\Lambda(t)Ee^{rh_j^{\delta x}}-r\Lambda(t)d\Big\} \\
&= \exp\Big\{-rx+b+\Lambda(t) \\
&\qquad \big[Ee^{rh_j^{\delta x}}-a\nu r EY_{11}g(t)-1\big]\Big\}.
\end{aligned} \tag{3.12}
$$

Recalling an inequality $e^u - 1 \le ue^u$ and by the fact that $F \in$

ERV $(-\alpha,-\beta)$, we divide $Ee^{rh_j^{\delta x}}-1$ into two parts as follows:

$$
\begin{aligned}
Ee^{rh_1^{\delta x}}-1 &\le \int_0^{\delta x/b^\tau}(e^{rs}-1)\Pr(h_1\in \mathrm{d}s)+\int_{\delta x/b^\tau}^{\delta x} e^{rs}\Pr(h_1\in \mathrm{d}s)\\
&\le re^{b^{1-\tau}}Eh_1+e^{r\delta x}\Pr\left(h_1>\frac{\delta x}{b^\tau}\right)\\
&= re^{b^{1-\tau}}a\nu g(t)EY_{11}+(1+o(1))e^{b-\beta\tau\ln b}a\nu\overline{F}\left(\frac{\delta x}{b^\tau}\right)\\
&\le re^{b^{1-\tau}}a\nu EY_{11}+\frac{a\nu}{a\nu\Lambda(t)\overline{F}(x)}\frac{1}{b^{\tau\beta}}\left(\frac{b^\tau}{\delta}\right)^\beta\overline{F}(x)\\
&= re^{b^{1-\tau}}a\nu EY_{11}+\frac{1}{\Lambda(t)}\frac{1}{\delta^\beta}. \qquad (3.13)
\end{aligned}
$$

Substituting (3.13) into (3.12) yields

$$
\frac{\Pr\left(\widetilde{S}^{\delta x}(t)>x\right)}{a\nu\Lambda(t)\overline{F}(x)}\le \exp\left\{-rx+b+a\nu r\Lambda(t)EY_{11}(e^{b^{1-\tau}}-g(t))+\delta^{-\beta}\right\}.
$$

Notice that $g(t)\to 1$, $e^{b^{1-\tau}}\to 1$ as $t\to\infty$. After some simple calculation, we see that $a\nu r\Lambda(t)EY_{11}(e^{b^{1-\tau}}-g(t))=o(b)$. Hence, it holds for $x\ge\gamma\Lambda(t)$ that

$$
\frac{\Pr\left(\widetilde{S}^{\delta x}(t)>x\right)}{a\nu\Lambda(t)\overline{F}(x)}\le C\exp(3.13)\left\{\left(1-\frac{1}{\delta}\right)b+o(b)(3.13)\right\}\longrightarrow 0
$$

with the coefficient C given by $C=e^{\delta^{-\beta}}$. This concludes the result (2.2). □

3.3. Proof of Theorem 2.3. Firstly, we establish some notation to be used later. For each $i=1,2,\ldots,n$, denote

$$
g_i(t)=\Pr(U_{jk}^i+U_j^i\le t)=\int_0^t\min\left\{\frac{t-s}{a_i},1\right\}P(U_1^i\in \mathrm{d}s),
$$

$$
R_i(t)=\sum_{j=1}^{N_i(t)}\sum_{k=1}^{M_j^i(a_i)}Y_{jk}\quad\text{and}\quad \widetilde{S}_i(t)=S_i(t)-ES_i(t).
$$

It follows from Theorem 2.2 that, for each $i=1,2,\ldots,n$,

$$
\Pr(S_i(t)-\mu_i(t)>x)\sim a_i\nu_i\Lambda_i(t)\overline{F}_i(x) \qquad (3.14)
$$

holds uniformly for $x \geq \gamma\Lambda_i(t)$, each $\gamma > 0$.

Proof of Theorem 2.3. Employing the arguments in Wang and Wang **[17]**, we use induction to prove (2.3). Since n stands for the amount of the policies, it is finite. Hence, we only need to prove (2.3) holds for the case in which $n = 2$.

The lower estimate. Recall an elementary inequality $\Pr(AB) \geq \Pr(A) + \Pr(B) - 1$ for all events A and B. It follows for any $0 < \varepsilon < 1$ that

$$\begin{aligned}
(3.15)\quad & \Pr(S(2;t) - ES(2;t) > x) \\
& \geq \Pr\Big(\big\{\widetilde{S}_1(t) > x + \varepsilon ES_2(t), \widetilde{S}_2(t) > -\varepsilon ES_2(t)\big\} \\
& \bigcup \big\{\widetilde{S}_2(t) > x + \varepsilon ES_1(t), \widetilde{S}_1(t) > -\varepsilon ES_1(t)\big\}\Big) \\
& \geq \Pr\Big(\widetilde{S}_1(t) > x + \varepsilon ES_2(t), \widetilde{S}_2(t) > -\varepsilon ES_2(t)\Big) \\
& \quad + \Pr\Big(\widetilde{S}_2(t) > x + \varepsilon ES_1(t), \widetilde{S}_1(t) > -\varepsilon ES_1(t)\Big) \\
& \quad - \Pr\Big(\widetilde{S}_1(t) > x + \varepsilon ES_2(t), \widetilde{S}_2(t) > x + \varepsilon ES_1(t)\Big) \\
& \geq \Pr\big(\widetilde{S}_1(t) > x + \varepsilon ES_2(t)\big) + \Pr\big(\widetilde{S}_2(t) > -\varepsilon ES_2(t)\big) - 1 \\
& \quad + \Pr\big(\widetilde{S}_2(t) > x + \varepsilon ES_1(t)\big) + \Pr\big(\widetilde{S}_1(t) > -\varepsilon ES_1(t)\big) - 1 \\
& \quad - \Pr\Big(R_1(t) - ES_1(t) > x + \varepsilon ES_2(t)\Big) \\
& \cdot \Pr\Big(R_2(t) - ES_2(t) > x + \varepsilon ES_1(t)\Big).
\end{aligned}$$

By virtue of (3.14) and Lemma 3.5, letting $\varepsilon \to 0$, we obtain that

$$\Pr\big(\widetilde{S}_1(t) > x + \varepsilon ES_2(t)\big) \sim a_1\nu_1\overline{F}_1(x). \tag{3.16}$$

By the weak law of large numbers of Lemma 3.3, we further can choose some positive constant ε and positive function $\epsilon_t \to 0$ such that $\epsilon_t/\varepsilon \to 0$,

(3.17)

$$\Pr\big(\widetilde{S}_i(t) > -\varepsilon ES_i(t)\big) \geq (1 + o(1))\Pr\Bigg(\sum_{j=1}^{[(1-\epsilon_t)\Lambda_i(t)]} H_j^i(t)$$

$$- [(1-\epsilon_t)\Lambda_i(t)]a_i\nu_i g_i(t)EY_{11}^i > [(-\varepsilon+\epsilon_t)\Lambda_i(t)]a_i\nu_i g_i(t)EY_{11}^i\Bigg)$$

$$= (1+o(1))\Pr\Bigg(\sum_{j=1}^{[(1-\epsilon_t)\Lambda_i(t)]} H_j^i(t) - [(1-\epsilon_t)\Lambda_i(t)]a_i\nu_i g_i(t)EY_{11}^i$$

$$> -\varepsilon\left(1-\frac{\epsilon_t}{\varepsilon}\right)\Lambda_i(t)a_i\nu_i g_i(t)EY_{11}^i\Bigg) \longrightarrow 1.$$

For $i \geq 1$, since $F_i \in \mathrm{ERV}$ and $ES_i(t) - ER_i(t) = o(\Lambda_i(t))$ as $t \to \infty$, then, Theorem 2.2 shows that

$$\begin{aligned} \Pr\left(R_1(t) - ES_1(t) > x + \varepsilon ES_2(t)\right) &\\ = \Pr\left(R_1(t) - ER_1(t) > x + \varepsilon ES_2(t) + o(\Lambda_1(t))\right) &\\ \sim a_1\nu_1\Lambda_1(t)\overline{F}_1(x). & \end{aligned} \tag{3.18}$$

By the fact that $a_i\nu_i\Lambda_i\overline{F}_i(x) \to 0$ as $x \geq \gamma\Lambda_i(t)$, $t \to \infty$, it is easy to check that

$$\begin{aligned} &\lim_{t\to\infty}\liminf_{x\geq\gamma\Lambda(t)} \frac{a_1\nu_1\Lambda_1(t)\overline{F}_1(x)a_2\nu_2\Lambda_2(t)\overline{F}_2(x)}{a_1\nu_1\Lambda_1(t)\overline{F}_1(x) + a_2\nu_2\Lambda_2(t)\overline{F}_2(x)} \\ &= \lim_{t\to\infty}\liminf_{x\geq\gamma\Lambda(t)} \frac{1}{1/(a_2\nu_2\Lambda_2(t)\overline{F}_2(x)) + 1/(a_1\nu_1\Lambda_1(t)\overline{F}_1(x))} = 0. \end{aligned} \tag{3.19}$$

Combining (3.19) with (3.18) yields that

$$\begin{aligned} &\Pr\left(R_1(t) - ES_1(t) > x + \varepsilon ES_2(t)\right) \\ &\qquad \Pr\Big(R_2(t) - ES_2(t) > x + \varepsilon ES_1(t)\Big) \\ &\qquad\qquad = o(a_1\nu_1\Lambda_1(t)\overline{F}_1(x) + a_2\nu_2\Lambda_2(t)\overline{F}_2(x)). \end{aligned} \tag{3.20}$$

Substituting (3.16)–(3.20) into (3.15) yields

$$\Pr(S(2;t) - ES(2;t) > x) \geq \sum_{i=1}^{2} a_i\nu_i\Lambda_i(t)\overline{F}_i(x) + o\left(\sum_{i=1}^{2} a_i\nu_i\Lambda_i(t)\overline{F}_i(x)\right).$$

Now, account for the upper estimate of (2.3).

$$\begin{aligned} &\Pr\left(S(2;t) - ES(2;t) > x\right) \\ &\leq \Pr\big(\{\widetilde{S}_1(t) > (1-\varepsilon)x\} \cup \{\widetilde{S}_2(t) > (1-\varepsilon)x\} \end{aligned} \tag{3.21}$$

$$\begin{aligned}
&\cup \{\widetilde{S}_2(t) > \varepsilon x, \tilde{S}_1(t) > \varepsilon x\}\big) \\
&\le \Pr\big(\widetilde{S}_1(t) > (1-\varepsilon)x\big) \\
&\quad + \Pr\big(\widetilde{S}_2(t) > (1-\varepsilon)x\big) + \Pr\big(\widetilde{S}_2(t) > \varepsilon x, \widetilde{S}_1(t) > \varepsilon x\big) \\
&\le \Pr\big(\widetilde{S}_1(t) > (1-\varepsilon)x\big) + \Pr\big(\widetilde{S}_2(t) > (1-\varepsilon)x\big) \\
&+ \Pr\big(R_1(t) - ES_1(t) > \varepsilon x\big)\Pr\big(R_2(t) - ES_2(t) > \varepsilon x\big).
\end{aligned}$$

With the arbitrariness of ε, it holds uniformly for $x \ge \gamma\overline{\Lambda}(t)$ that

$$\Pr\big(\widetilde{S}_1(t) > (1-\varepsilon)x\big) \sim a_1\nu_1\Lambda_1(t)\overline{F}_1(x). \tag{3.22}$$

Similarly as in (3.18), it holds uniformly for $x \ge \gamma\overline{\Lambda}(t)$ that

$$\Pr\big(R_1(t) - ES_1(t) > \varepsilon x\big) \sim a_1\nu_1\Lambda_1(t)\overline{F}_1(\varepsilon x).$$

Recalling that $F_i \in ERV \subset \mathcal{D}$, it follows that

$$\limsup_{x\ge\gamma\overline{\Lambda}(t)} \frac{\overline{F}_1(\varepsilon x)}{\overline{F}_1(x)} < \infty. \tag{3.23}$$

In view of (3.21)–(3.23), we conclude that

$$\Pr\left(S(2;t) - ES(2;t) > x\right) \le \sum_{i=1}^{2} a_i\nu_i\Lambda_i(t)\overline{F}_i(x) + o\bigg(\sum_{i=1}^{2} a_i\nu_i\Lambda_i(t)\overline{F}_i(x)\bigg).$$

The proof is accomplished. □

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