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E. STRAUS 1921–1983

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Last July after I returned to Hungary from a meeting on number theory in the Netherlands, I heard the sad news that my friend and collaborator, E. Straus, died of a heart attack on July 12, 1983. I had known for a long time that he had diabetes and in fact we were together in 1948 in Princeton when this was diagnosed. I also knew that he had several earlier heart attacks. Nevertheless, I did not expect that the end would come so soon. I cannot write at such short notice a complete description of his far-reaching mathematical activities, so I will restrict myself to the history of our friendship and collaboration.

This is a very strong restriction since his most important work was probably on the connection between arithmetic and algebraic properties of entire functions, a subject about which I could only write after considerable preparation and for which there is now no time. Since I have been asked to finish this report in two to three weeks I must rely a great deal on my poor old memory. This last restriction is really my own fault; but, enough of the excuses, and let me start my subject. I will begin at the end. Let me state two of our relatively recent results which are "lost"; i.e., the proofs were supposed to be in more or less complete form in Ernst's possession, but we could find no trace of the manuscript and there is little hope that they can be found. Most likely they never existed. First, a result due to Ernst, Selfridge and myself.

Let $n > n_0(\varepsilon)$. Then

$$n! = a_1 a_2 \cdots a_n, \frac{n}{e}(1 - \varepsilon) < a_1 \leqq \cdots \leqq a_n \tag{1}$$

is always solvable in integers $a_1, \ldots, a_n$. This result is certainly not of great importance, nevertheless, it pleased us since it is the best possible. Since $n!^{1/n} = (1 + o(1))\,(n/e)$, it is clear that in (1), $(n/e)\,(1 - \varepsilon) < a_1$ cannot be replaced by $(n/e)\,(1 + \varepsilon) < a_1$. Nevertheless, we managed to prove a slightly stronger form of (1). Let c be sufficiently large and $n > n_0(c)$. Then in (1), all the a's can be taken to be larger than $(n/e)\,(1 - c/\log n)$.

Ernst claimed that he had a nearly completed manuscript of the proof of (1). Perhaps this manuscript was lost or, perhaps, his memory deceived him. In any case, Selfridge and I will have to reconstruct our proof, which will be an onerous but not too difficult task. It might also be of some interest to determine the dependence of n_0 on ε; e.g., determine the smallest n_0 so that for every $n > n_0$, $n!$ is the product of n integers all greater than $n/3$. We have not yet carried out this task; but we hope to do so in time for our paper to be included in the collected papers of E. Straus, which I hope will be published in a few years.

Now to our second missing paper. This paper was supposed to have appeared in this Proceedings, and I should have prepared it after I heard of Ernst's death. Besides my laziness, I have an excuse; Ernst assured me that he had a nearly complete manuscript which was never found. Here is our result: Denote by $F(n; k)$ the least common multiple of $n + 1, \ldots,$ $n + k$ and by $f(n; k)$ the least common multiple of $n - 1, n - 2, \ldots$ $n - k$. We would expect that usually $F(n; k) > f(n; k)$ holds and, in fact if, say $k > \varepsilon n$, then for all, n, $F(n; k) > f(n; k)$ is easy to see. We prove that for almost all n there is a k so that

$$f(n; k) > F(n; k) \tag{2}$$

and conjectured with some trepidation that there are infinitely many integers n for which, for every $k < n$,

$$F(n; k) > f(n; k) \tag{3}$$

holds. We never came to a firm conclusion if (3) is true or not. It might be of some interest to try to determine the largest possible value of $k = k(n)$ for which $f(n; k) > F(n; h)$ can hold. It is easy to see that k must be $o(n)$ but must it be $o(n^{\varepsilon})$? Also, can one estimate the number of integers k for which (2) holds? We planned to investigate this in the future, but, unfortunately, fate prevented us. These problems are probably not of great importance; but it often happened before that seemingly special questions in number theory unexpectedly lead to interesting developments.

Now let me come back to the beginning. In 1944 Ernst Straus visited me with his fiancee, Louise, in Princeton. He was interested in various geometric problems about convex sets. These problems are not very popular now, but let me mention only one question which we then discussed and which is still open. Let J be a Jordan curve. Is it always possible to find four points on J which are the vertices of a square? I do not know who first formulated this pretty conjecture, and as far as I know it is still open. Let me add a little story which I remember. We had lunch together with the great algebraist Claude Chevalley, who was never much interested in elementary geometry but noticed "bosses", i.e., girls. He said about Louise: "What a pretty girl, I hope we will see more of her".

Our next contact with the Strauses was in the summer of 1948. Louise and Ernst were married by then and all three of us were in Princeton. We then made the following pretty conjecture. Is it true that, for every integer n, $4/n = 1/x + 1/y + 1/z$ is solvable in positive integers x, y, z? This interesting conjecture is still open. It is settled for many arithmetic progressions and also, if it fails, then the smallest such n must be quite large. Schinzel and Sierpinski have the following extension. For every a there is an $n_0(a)$ so that, for every $n > n_0(a)$,

$$\frac{a}{n} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

is solvable in positive integers x, y, z. In memory of Straus, I offer five hundred dollars for a proof or disproof of this conjecture.

The paper *On the representation of fractions as sum and difference of three simple fractions* (Jointly with M.V. Subbarao), deals with the Schinzel conjecture that, for every given positive integer a and all sufficiently large n, the equation $a/n = 1/x \pm 1/y \pm 1/z$ has integral solutions x, y, z. It has been proved there that the conjecture holds for all a less than 40, and that at least in the cases $a \leqq 35$, the fraction $1/x$ can be chosen among the three nearest neighbours of a/n. Further, the paper gives some conjectures each of which implies the Schinzel conjecture. One such conjecture states that

$$\lim_{n\to\infty} \sup_{s\geqq 0} \frac{d(n+s)}{s+1} = \infty.$$

Our first joint paper was in fact written when we both were at UCLA. Ernst was at the University of California and I at the Institute for Numerical Analysis. I have to add here a personal note (perhaps when my obiturary will be written (soon?) this should be mentioned). John Curtiss was then head of the Institute for Numerical Analysis and he created for me the so called Curtiss condition, i.e., I was paid only when I was there and I could leave without asking for a leave of absence but it was up to the Institute to decide if I got paid while I was away (since then I have been fortunate enough to have posts only under these conditions).

In our paper we answer the following question of Dvoretsky. Let x_1, $x_2, \ldots$ be an infinite sequence of unit vectors in a Banach space which are linearly independent in the algebraic sense. Is it then true that there is a subsequence which is linearly independent in a stronger sense? We prove, among other things, that there always is a subsequence for which $\sum_{i=1}^{\infty} \ell_{n_i} x_{n_i} = 0$ implies $\ell_{n_i} = 0$.

We were together again in Colorado in 1959 and 1963. We proved the following theorems: Let n_h be an increasing sequence of positive integers and assume $\limsup n_h^2/n_{h+1} \leqq 1$, $N_h < cn_{h+1}$ where N_h is the least common multiple of $n_1, \ldots, n_h$. Then $\sum 1/n_h$ is rational only if $n_{h+1} = n_h^2 - n_h + 1$ for all $h > h_0$. It is not impossible that the conditions $N_h < Cn_{h+1}$ are superfluous. We could weaken it but could not eliminate it entirely. We wrote several more papers on the irrationality of infinite series. Here I mention only one problem of ours which has never been published.

It is easy to see that if $n_1 \leqq \cdots$ is a sequence of integers for which $\sum_{k=1}^{\infty} 1/n_k$ is rational, then we must have

$$\lim n_k^{1/2^k} < \infty$$

and this is best possible. Let

$$\sum_{k=1}^{\infty} \frac{1}{n_k} \text{ and } \sum_{k=1}^{\infty} \frac{1}{n_k - 1}$$

both be rational. How fast can n_k increase? David Cantor observed that this holds for $n_k = \binom{k}{2}$ and we could never decide if n_k can increase exponentially or even faster. We observed that the set of points (x, y) in the plane ($n_1 < n_2 < \cdots$, n_k integers), given by

$$x = \sum_k \frac{1}{n_k}, y = \sum_k \frac{1}{n_k - 1},$$

contains open sets and this no doubt generalises for higher dimensions. Perhaps we missed the nicest conjecture (due to Stolarsky) which states as follows. Let $n_1 < n_2 < \cdots$ be an infinite sequence for which $\sum 1/n_k < \infty$.

Is it then true that there is an integer t for which $\sum_{n_k \neq t} 1/(n_k - t)$ is irrational?

Straus and I also considered the following question. Let a sequence $A: a_1 < a_2 < \ldots$ be called non-averaging if the arithmetic mean of any two or more members of A is not in A. What can be said about the growth properties of such sequences? We proved that if $a_1 < a_2 < \cdots < a_k \leqq n$ then $k = o(n^{2/3})$ and conjectured that $k < n^{\varepsilon}$ for every $\varepsilon > 0$. This was shown to be false by H. L. Abbott who showed that it is possible to have $k \geqq n^{1/10}$. Recently, Abbott improved this to $n^{1/5}$. Many interesting open questions remain here. (H. L. Abbott, *On a conjecture of Erdös and Straus on non-averaging sets of integers*, Proc. Fifth British Combinatorial Conference, Congressus Numerantium XV, (1975), 1–4).

In 1959 at the meeting at Boulder, Colorado, Bose, Parker and Shrikhande presented their disproof of Euler's conjecture, i.e., they proved that, for every $n > 6$, there are two pairwise orthogonal Latin squares. Inspired by their ideas, Chowla, Straus & I showed that the number of pairwise orthogonal Latin squares of order n is greater than cn^{ε} where our ε was $> 1/91$. R. Wilson considerably improved our result but perhaps further improvement will be possible in the future.

Now I discuss what I think is our most important and most original joint work, namely our papers on Euclidean Ramsey Theorems. I hope that these results, and even more the problems, will outlive the authors, hopefully, by centuries. A set k of Euclidean m space is called Ramsey if, for every r, there is an $n = n_0(k, r)$ so that, for every r coloring of Euclidean n space, there is a monochromatic configuration k' which is congruent to k. If congruent is replaced by similar, then Gallai proved that every finite set k is Ramsey. In our first paper on this subject we prove that every brick, i.e., every rectangular parallelepiped is Ramsey and we also prove that every set which is Ramsey must lie on an m dimensional sphere. We never could decide whether any of these conditions are necessary or sufficient. It is quite possible that the truth is somewhere in between. Perhaps the most interesting open problems are: Is the regular pentagon Ramsey? or is every triangle Ramsey or, in particular, is the triangle of angles 30, 30, 120 Ramsey (every acute angled triangle is the subset of a brick and is therefore Ramsey)? Also is it true that if we divide the plane into two subsets and T is any triangle, then at least one of the subsets contains a monochromatic congruent copy of T (i.e., the vertices of T) with a possible single exception of an equilateral T. We and L. Shader proved several special cases of this conjecture. As another nice problem, let S be a set in the plane, no two points of which are at distance 1. Is it then true that the complement $\bar{S}$ of S contains the vertices of a square? R. Juhasz proved our conjecture in a more general form. She proved that $\bar{S}$ contains a congruent copy of any configuration of four points. She further showed that four cannot be replaced by 12, but the exact value of this number is not known, e.g., is it true for 5? (R. Juhasz, *Ramsey type theorems in the plane*, J. Combinatorial Theory (1979), 152–170.)

To end this short obituary I just want to remark that Ernst was not only a first rate mathematician, but also a superior human being, both intellectually and morally. I remember one occasion when, with great tact, insight and intelligence, he smoothed over a potentially unpleasant disagreement between two excellent mathematicians. I was concerned since they were both friends of mine and I wrote congratulating him for a success which had eluded me. "Blessed are the peace makers". UCLA, Los Angeles, California and the world will never be the same for me without him. May his theorems live forever.

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3. ——— P. Erdos and B. Rothschild, *Polychromatic Euclidean Ramsey theorems*, to appear in J. Geometry.

4. ——— and Moshe Goldberg, *Combinatorial inequalities, matrix norms, and generalized numerical radii* II, to appear in *General Inequalities*, edited by E. F. Beckenbach.

5. ——— and David Lee Hilliker, *On Puiseux series whose curves pass through an infinity of algebraic lattice points*, to appear in Bull. Amer. Math. Soc.

6. ——— and P. Lockhart, *Entire functions which are infinitely integer-valued at a finite number of points*, submitted to Trans. Amer. Math. Soc.

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9. ——— and ———, *Multiplicativity factors for C-numerical radii*, to appear in LAA.

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ADDED IN PROOF. P. Frankl and V. Rodl proved that every triangle is Ramsey.