ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 13, Number 4, Fall 1983

THE PEDERSEN IDEAL AND THE REPRESENTATION OF C*-ALGEBRAS

C. W. BAKER

ABSTRACT. Let A be a C^*-algebra, Z the center of A, and K the Pedersen ideal of A. It is proved that if ZA is dense in A, then K is equal to $(K \cap Z)A$. It is known from the Dauns-Hofmann representation theory that given a C^*-algebra A, there exists a C^*-bundle such that A is isometrically *-isomorphic to the ring of sections which vanish at infinity. This, together with the above characterization of the Pedersen ideal, is used to prove that if ZA is dense in A, then K is isometrically *-isomorphic to the ring of sections with compact support. Under the same assumption it is observed that $M(A)$, the multiplier algebra of A, is isometrically *-isomorphic to the ring of bounded sections and that $M(K)$, the multiplier algebra of K, is *-isomorphic to the ring of all sections.

1. Introduction. Let A be a C^*-algebra. If A is commutative, then $A = C_\infty(X)$, the continuous, complex-valued functions which vanish at infinity on a locally compact, Hausdorff space X. The algebra A contains the ideal $C_K(X)$, the functions with compact support. The multiplier algebra of A, $M(A)$, is equal to $C_b(X)$, the bounded, continuous functions on X and the multiplier algebra of $C_K(X)$ is equal to $C(X)$, the space of all continuous functions on X. The purpose of this note is to develop a non-commutative analogue of these relationships in terms of sections in a C^*-bundle. This will be done by use of the Pedersen ideal.

In order to develop an integration theory for arbitrary C^*-algebras, G.K. Pedersen introduced in **[11]** an ideal which is generally accepted as the non-commutative analogue of $C_K(X)$. This ideal will be referred to as the Pedersen ideal. Extensive studies of the Pedersen ideal and its multiplier algebra have been made by Lazar and Taylor **[8]**, **[9]**, Pedersen and Petersen **[13]**, Akemann, Pedersen, and Tomiyama **[1]**.

This paper formed part of the author's dissertation at the University of Kentucky. I would like to take this opportunity to express my very deep appreciation to Professor John E. Mack for many helpful suggestions and encouragement concerning this work.

The notation in this note is approximately that of **[3]**. The letter A will

Received by the editors on March 12, 1981.

denote a C^*-algebra. The algebra A will not necessarily have an identity. The symbol $\tilde{A}$ will signify the algebra A with an identity adjoined. If B is any C^*-algebra contained in A, the Pedersen ideal of B will be denoted by $K(B)$. When the algebra is understood, we will write K in place of $K(B)$. The center of A will be given by Z. If B is any subset of A, the positive elements of B will be signified by B^+. If B and C are subsets of A, by BC we will mean the set of all products bc where b is an element of B and c is an element of C. If B is a subalgebra of A, the multiplier algebra (double centralizer) of B will be given by $M(B)$. For any unexplained notation concerning bundless see [4], [5], or [10].

2. The Pedersen ideal of a C*-algebra. It will be proved that if ZA is dense in A, then $K = (K \cap Z)A$.

The Pedersen ideal of a C^*-algebra A is the minimal dense, order related, ideal in A. This definition is implicit in the statement of Theorem 1.3 in [11]. It has since been shown [7] that the Pedersen ideal is the minimal dense ideal in A. In [12] Pedersen gives the following, often more useful, characterization of the ideal. Let the sets $K_1^+(A)$ and $K_2^+(A)$ be given by $K_1^+(A) = \{a \in A^+\colon$ there is an element b in A^+ such that $ab = a\}$ and $K_2^+(A) = \{a \in A^+\colon$ there are elements a_j in $K_1^+(A)$ $(j = 1, 2, \ldots, n)$ with $a \leqq \sum a_j\}$. Then $K(A)$, the Pedersen ideal of A, is the linear span of $K_2^+(A)$.

It is clear from this characterization that if A has an identity, then $K(A) = A$.

If A is commutative, $K(A)$ is isometrically $*$-isomorphic to $C_K(X)$ where X is the space of maximal, modular ideals of A (see [11]).

Proposition 2.1. *$K(Z)A$ is an ideal.*

Proof. It suffices to show that $K(Z)A$ is closed under addition. There is a locally compact, Hausdorff space X such that $K(Z)$ is isometrically $*$-isomorphic to $C_K(X)$. Denote the mapping from $K(Z)$ onto $C_K(X)$ by $k \to \tilde{k}$. Let k_1, k_2 be in $K(Z)$ and a_1, a_2 in A. Then there exists $\tilde{k}$ in $C_K(X)$ such that $\tilde{k} \equiv 1$ on the union of the supports of $\tilde{k}_1$ and $\tilde{k}_2$. So $k_1a_1 + k_2a_2 = k(k_1a_1 + k_2a_2)$ and $k_1a_1 + k_2a_2 \in K(Z)A$.

Proposition 2.2. *If ZA is dense in A, then $K(Z)A$ is dense in A.*

Proof. Let $z \in Z$ and $a \in A$. Since $K(Z)$ is dense in Z, there is a sequence $\{k_j\}$ in $K(Z)$ such that $\|k_j - z\| \to 0$. Then $\|k_ja - za\| \leqq \|k_j - z\| \cdot \|a\| \to 0$. So $K(Z)A$ is dense in ZA and thus is dense in A.

Corollary 2.3. *If ZA is dense in A, then $K(A)$ is contained in $K(Z)A$.*

Proof. We have shown that $K(Z)A$ is a dense ideal in A and $K(A)$ is the minimal such ideal.

PROPOSITION 2.4. *If A and B are C^*-algebras and $\phi: A \to B$ is a *-homomorphism from A into B, then $\phi(K(A))$ is contained in $K(B)$.*

PROOF. Since ϕ is a *-homomorphism, it preserves positive elements. Let $a \in K_1^+(A)$. Then there exists b in A^+ such that $ab = a$. Thus $\phi(a)\phi(b) = \phi(ab) = \phi(a)$ and hence $\phi(K_1^+(A))$ is contained in $K_1^+(B)$. It follows that $\phi(K(A))$ is contained in $K(B)$.

PROPOSITION 2.5. *The set $K(Z)A$ is contained in $K(A)$.*

PROOF. In Proposition 2.4 let ϕ be the inclusion map of Z into A and obtain that $K(Z)$ is contained in $K(A)$. Then observe that $K(A)$ is an ideal in A.

Combining Propositions 2.3 and 2.5 we have the following theorem.

THEOREM 2.6. *If ZA is dense in A, then $K(A) = K(Z)A$.*

Note that $K(Z) \subset K(A) \cap Z$. If ZA is dense in A, then equality holds here. To see this, let $a \in K(A) \cap Z$. By Theorem 2.6, $a = ka_1$ for some $a_1 \in A$, $k \in K(Z)$. Then since $K(Z)$ is isometrically *-isomorphic to $C_K(X)$, where X is a locally compact, Hausdorff space, there is an element k_1 in $K(Z)$ such that $k_1k = k$. Then $a = ka_1 = (k_1k)a_1 = k_1(ka_1) = k_1a$. Since $k_1 \in K(Z)$ and $a \in Z$, we have that a is in $K(Z)$. Theorem 2.6 may be restated as follows.

THEOREM 2.7. *If ZA is dense in A, then $K = (K \cap Z)A$.*

This section is concluded with several examples.

EXAMPLE 2A. Let A be the C^*-algebra of all compact operators on a Hilbert space H. Then the center of A is zero and $K(A)$ is the algebra of all operators with finite rank.

The next example is a non-commutative C^*-algebra which satisfies the condition required in Theorem 2.7.

EXAMPLE 2B. Let X be a locally compact, Hausdorff space and A the algebra of all 2×2 matrices over the ring $C_\infty(X)$. Then ZA is dense in A.

The following example shows that it is possible for the center of a C^*-algebra to be non-trivial and still be too small for our purpose.

EXAMPLE 2C. Let B be the compact operators on the Hilbert space ℓ_2 and let B_1 be the subset of B consisting of the operators which are diagonal with respect to the usual basis for ℓ_2. Then define A as follows:

$$A = \{f \in C([-1, 1], B): f(x) \in B_1 \text{ if } x \geqq 0\}.$$

In this case $Z = \{f \in A: f(x) = 0 \text{ if } x < 0\}$ and $ZA = Z$.

3. The Representation of C*-algebras. Let A be a C^*-algebra with ZA dense in A. In this section a C^*-bundle is constructed for which the base space will be the set of all maximal, modular ideals of the center of A with the hull-kernel topology and the stalks will be quotients of A. It will be proved that K, the Pedersen ideal of A, is isometrically *-isomorphic to the ring of all sections with compact support; that $M(A)$, the multiplier algebra of A, is isometrically *-isomorphic to the ring of all bounded sections; and that $M(K)$, the multiplier algebra of K, is *-isomorphic to the ring of all sections.

The center of A, Z, is isometrically *-isomorphic to $C_\infty(X)$ where X is a locally compact, Hausdorff space that is homeomorphic to the space of maximal, modular ideals of Z with the hull-kernel topology. Denote the mapping of Z onto $C_\infty(Z)$ by $z \to \tilde{z}$. For each x in X denote the corresponding maximal modular ideal in Z by M_x ($M_x = \{z \in Z: \tilde{z}(x) = 0\}$). Let $I_x = \overline{M_x A}$. To see that I_x is an ideal in A, let O_x be the set of all z in Z such that $\tilde{z}$ vanishes on a neighborhood of x. Then $\overline{O_x A} = \overline{M_x A}$ and the proof that $\overline{O_x A}$ is an ideal is similar to the proof that $K(Z)A$ is an ideal in §1 (see Proposition 2.1). We claim that each of the ideals I_x is modular. For x in X let e_x be an element of Z such that $\tilde{e}_x(x) = 1$. Then it is easily seen that $e_x + I_x$ is an identity for the algebra A/I_x.

Finally let $E = \bigcup_x \{x\} \times A/I_x$. Define the map $p: E \to X$ by $p(x, a + I_x) = x$ for $(x, a + I_x)$ in E. Let $X \times A$ have the product topology and let the map ϕ from $X \times A$ onto E be given by $\phi(x, a) = (x, a + I_x)$ for $(x, a) \in X \times A$. Give E the quotient topology with respect to ϕ. Observe that ϕ is an open map. For $(x, a + I_x)$ in E define the norm of $(x, a + I_x)$ by $\|(x, a + I_x)\| = \|a + I_x\|$, the norm of $a + I_x$ as an element of the C^*-algebra A/I_x. Then (E, p, X) is a C^*-bundle.

DEFINITION 3.1. Let (E, p, X) be a C^*-bundle. A section is a continuous function $\sigma: X \to E$ such that $p \circ \sigma$ is the identity mapping on X. A section σ is said to vanish at infinity provided the function from X into R given by $x \to \|\sigma(x)\|$ vanishes at infinity. The algebra of all sections which vanish at infinity will be denoted by Σ_∞. Similarly a section σ is said to have compact support if the function $x \to \|\sigma(x)\|$ has compact support. We will use Σ_K to signify the algebra of all sections which have compact support.

We will require that the map $E \to R$ given by the norm on each stalk of a bundle be only upper semi-continuous. Otherwise the definitions and terminology for bundles will be the same as in [5]. For the remainder of this paper (E, P, X) will denote the specific bundle constructed above.

The selection 1 defined by $1(x) = (x, e_x + I_x)$ is a section and is the identity for Σ. The subset $\{\alpha 1(x): x \in X, \alpha \in \mathbf{C}\}$ of E is homeomorphic to $X \times \mathbf{C}$. (The preceding set is also equal to $\phi(X \times Z)$, since $\phi(x, z) =$

$\tilde{z}(x)1(x)$ for (x, z) in $X \times Z$). If A is commutative $E = \{\alpha 1(x)\colon x \in X, \alpha \in \mathbf{C}\}$ and hence E is homeomorphic to $X \times \mathbf{C}$.

For a in A let $\hat{a}$ be the selection given by $\hat{a}(x) = \phi(x, a)$. The mapping $a \to \hat{a}$ will be called the *Gelfand mapping*. Observe that for a in A, the selection $\hat{a}$ is easily seen to be a section. The following result is due to Dauns and Hofmann.

THEOREM 3.2. [6]. *The Gelfand mapping is an isometric *-isomorphism of A onto Σ_∞.*

To facilitate the following proofs we will characterize the Gelfand image of the center of A. As already remarked Z, the center of A, is isometrically *-isomorphic to $C_\infty(X)$. We have also noted that E contains a homeomorphic copy of $X \times \mathbf{C}$. If z is in Z, the Gelfand image of z will be the section $\hat{z}$ given by $\hat{z}(x) = \phi(x, z) = \tilde{z}(x)1(x)$ for x in X. Note that $\hat{z}$ will have compact support as a section precisely when $\tilde{z}$ has compact support as a complex valued function.

If B is any normed *-algebra with an approximate identity, we will consider the elements of $M(B)$, the multiplier algebra of B, to be two-sided functions m on B which satisfy the condition that $(am)b = a(mb)$ for all a, b in B. The elements of $M(B)$ are actually two sided linear operators on B and under the operation $m^*a = (a^*m)^*$, $am^* = (ma^*)^*$ $a \in B$, $m \in M(B)$, $M(B)$ is a *-algebra. For m in $M(B)$ define $\|m\|$ by $\|m\| = \sup\{\|ma\|\colon a \in B, \|a\| \leqq 1\} = \sup\{\|am\|\colon a \in B, \|a\| \leqq 1\}$. If $\|m\| < \infty$, the multiplier m is said to be *bounded*. The bounded elements of $M(B)$ with the norm given above form a Banach *-subalgebra of $M(B)$ [8]. If B is a C^*-algebra, then all elements of $M(B)$ are bounded and $M(B)$ is a C^*-algebra. For a more detailed description of multiplier algebras see [8] or [2].

THEOREM 3.3. *If ZA is dense in A, then K is isometrically *-isomorphic to Σ_K.*

PROOF. Let $a \in K$. By Theorem 2.7 we have $K = (K \cap Z)A$. Thus $a = ha$ for some h in $K \cap Z$ and $\hat{a} = (ha)\hat{} = \hat{h}\hat{a}$, where $\hat{a}$ denotes the image of a under the Gelfand mapping. Since $\hat{h}$ has compact support, it follows that $\hat{h}\hat{a}$ has compact support. Thus the Gelfand mapping takes K into Σ_K.

Let $\sigma \in \Sigma_K$. Note that $\Sigma_K \subset \Sigma_\infty$. By Theorem 3.2 there is an element a in A such that $\hat{a} = \sigma$. There is an h in Z such that $\hat{h}(x) = 1(x)$ for each x in the support of a and such that $\hat{h}$ has compact support. Note that h is in K. Then $(ha)\hat{}(x) = \hat{h}(x)\hat{a}(x) = \hat{a}(x)$ for x in X. Thus $(ha)\hat{} = \hat{a}$. Since the Gelfand mapping is an isomorphism, $ha = a$ and hence a is an element of K. Thus the Gelfand mapping when restricted to K is an isometric *-isomorphism of K onto Σ_K.

The following lemma will be required.

LEMMA 3.4. *If D is a dense subset of A, x, $y \in A$ and $dx = dy$ for all d in D, then $x = y$.*

PROOF. Let x, $y \in A$ and $\{d_n\}$ a sequence in D which converges to $(x - y)^*$. Observe that $\{d_n(x - y)\}$ converges to $(x - y)^*(x - y)$, but $d_n(x - y) = 0$ for each positive integer n. It follows that $x = y$.

THEOREM 3.5. *If $z \in Z \cap K$ and $m \in M(K)$, then $zm = mz$.*

PROOF. Let $k \in K$. Recall that K is dense in A. Then note that $k(zm) = (kz)m = (zk)m = z(km) = (km)z = k(mz)$. Thus by Lemma 3.4, $zm = mz$.

THEOREM 3.6. *If ZA is dense in A, then $M(K)$ is *-isomorphic to Σ.*

PROOF. Let $m \in M(K)$, $x \in X$. Then there is an element $h \in K \cap Z$ such that $\hat{h}(t) = 1(t)$ for each t in a neighborhood of x. Define the selection $\hat{m}$: $X \to E$ by $\hat{m}(x) = (mh)\hat{}(x)$ for each x in X where $(mh)\hat{}$ is the image of the element mh under the Gelfand mapping.

To see that the definition of $\hat{m}$ is independent of the choice of h, let h_1 and h_2 be in $K \cap Z$ such that $\hat{h}_1(x) = \hat{h}_2(x) = 1(x)$. Then since $h_1 - h_2 \in K$, there is an element k in K such that $k(h_1 - h_2) = h_1 - h_2$. Thus

$$\begin{aligned}(mh_1)\hat{}(x) - (mh_2)\hat{}(x) &= (mh_1 - mh_2)\hat{}(x)\\ = (m(h_1 - h_2))\hat{}(x) &= (m(k(h_1 - h_2)))\hat{}(x)\\ = ((mk)(h_1 - h_2))\hat{}(x) &= (mk)\hat{}(x)(h_1 - h_2)\hat{}(x)\\ = (mk)\hat{}(x)(\hat{h}_1(x) - \hat{h}_2(x)) &= 0.\end{aligned}$$

The selection $\hat{m}$ is continuous since if h is in $K \cap Z$ such that $\hat{h}(t) = 1(t)$ for all t in a neighborhood of x, then $\hat{m}(t) = (mh)\hat{}(t)$ for all t in the neighborhood.

Thus $:m \to \hat{m}$ is a mapping from $M(K)$ into Σ. We will show that this mapping is a *-isomorphism. To see that the map is surjective, let $\sigma \in \Sigma$ and define m_σ by $m_\sigma a = b$ where $\hat{b} = \sigma\hat{a}$, for $a \in K$, and $am_\sigma = c$ where $\hat{c} = \hat{a}\sigma$, for $a \in K$. Observe that $\sigma\hat{a}$, $\hat{a}\sigma \in \Sigma_K$ which is isometrically *-isomorphic to K. To see that m_σ is a multiplier of K, it suffices to show that $a(m_\sigma b) = (am_\sigma)b$ for a, $b \in K$. Observe that $(a(m_\sigma b))\hat{} = \hat{a}(m_\sigma b)\hat{} = \hat{a}(\sigma\hat{b}) = (\hat{a}\sigma)\hat{b} = (am_\sigma)\hat{}\hat{b} = ((am_\sigma)b)\hat{}$. Since the Gelfand mapping is an isomorphism, $a(m_\sigma b) = (am_\sigma)b$. Also observe that $\hat{m}_\sigma(x) = (m_\sigma h)\hat{}(x) = (\sigma\hat{h})(x) = \sigma(x)\hat{h}(x) = \sigma(x)1(x) = \sigma(x)$. Thus $\hat{m}_\sigma = \sigma$ and the map $:m \to \hat{m}$ is surjective.

To see that the map is injective, let $k \in K$, m_1, $m_2 \in M(K)$, and $x \in X$. Suppose $\hat{m}_1 = \hat{m}_2$. Then $(m_1k)\hat{}(x) = \hat{m}_1(x)\hat{k}(x) = \hat{m}_2(x)\hat{k}(x) = (m_2k)\hat{}(x)$. Since x is an arbitrary element of X, $(m_1k)\hat{} = (m_2k)\hat{}$, and since the Gelfand mapping is an isomorphism $m_1k = m_2k$. Similarly $km_1 = km_2$.

Hence $m_1 = m_2$ and the map $m\colon \to \hat{m}$ is injective.

In order to complete the proof that the map $:m \to \hat{m}$ is a *-isomorphism, the following equalities must be verified for $m_1, m_2 \in M(K)$, $\alpha \in \mathbf{C}$:

$$(m_1 + m_2)\hat{} = \hat{m}_1 + \hat{m}_2,$$
$$(m_1m_2)\hat{} = \hat{m}_1\hat{m}_2,$$

and

$$(\alpha m_1)\hat{} = \alpha\hat{m}_1,$$
$$(m_1{}^*)\hat{} = (\hat{m}_1)^*.$$

Let $x \in X$ and $h \in K \cap Z$ with the property that $\hat{h}(t) = 1(t)$ for all t in a neighborhood of x. Also assume that $h \geqq 0$. Then

$$\begin{aligned}(m_1 + m_2)\hat{}(x) &= (m_1 + m_2)h)\hat{}(x) = (m_1h + m_2h)\hat{}(x)\\ &= (m_1h)\hat{}(x) + (m_2h)\hat{}(x) = \hat{m}_1(x) + \hat{m}_2(x).\end{aligned}$$

Thus, since x is an arbitrary element of X, $(m_1 + m_2)\hat{} = \hat{m}_1 + \hat{m}_2$.

Next observe that

$$\begin{aligned}(m_1m_2)\hat{}(x) &= ((m_1m_2)h)\hat{}(x) = ((m_1m_2)h^2)\hat{}(x)\\ &= (m_1(m_2(h\cdot h)))\hat{}(x) = (m_1((m_2h)h))\hat{}(x)\\ &= (m_1(h(m_2h)))\hat{}(x) = ((m_1h)(m_2h))\hat{}(x)\\ &= (m_1h)\hat{}(x)\,(m_2h)\hat{}(x) = \hat{m}_1(x)\,\hat{m}_2(x)\\ &= (\hat{m}_1\hat{m}_2)(x).\end{aligned}$$

Thus $(m_1m_2)\hat{} = \hat{m}_1\hat{m}_2$.

Next note that

$$\begin{aligned}(\alpha m_1)\hat{}(x) &= ((\alpha m_1)h)\hat{}(x) = (\alpha(m_1h.)\hat{}(x)\\ &= \alpha((m_1h)\hat{}(x)) = \alpha(\hat{m}_1(x)).\end{aligned}$$

Hence $(\alpha m_1)\hat{} = \alpha\hat{m}_1$.

Finally observe that

$$(m_1^*)\hat{}(x) = (m_1^*h)\hat{}(x) = ((h^*m_1)^*)\hat{}(x).$$

Note that since $h > 0$, $h = h^*$, and by Theorem 3.5, $hm_1 = m_1h$, since h is in $K \cap Z$. Thus

$$\begin{aligned}((h^*m_1)^*)\hat{}(x) &= ((m_1h)^*)\hat{}(x) = ((m_1h)\hat{})^*(x)\\ &= ((m_1h)\hat{}(x))^* = (\hat{m}_1(x))^* = (\hat{m}_1)^*(x).\end{aligned}$$

Thus $(m_1^*)\hat{} = \bar{}(\hat{m}_1)^*$. This completes the proof that $M(K)$ is *-isomorphic to Σ.

We know from [9] that the map from $M(A)$ into $M(K)$ given by: $m \to$

$m|_K$ is an isometric *-isomorphism of $M(A)$ onto the C^*-algebra of bounded multipliers of K, denoted by $M^b(K)$. Thus in order to prove that $M(A)$ is isometrically *-isomorphic to Σ^b, it suffices to show that the mapping $:m \to \hat{m}$ given in Theorem 3.6 maps $M^b(K)$ onto Σ^b and that the mapping when restricted to $M^b(K)$ is an isometry.

THEOREM 3.7. *If ZA is dense in A, then $M(A)$ is isometrically *-isomorphic to Σ^b.*

PROOF. Recall that if m is a bounded multiplier of K, the norm of m is given by $\|m\| = \sup\{\|ma\| : a \in K, \|a\| \leqq 1\} = \sup\{\|am\| : a \in K, \|a\| \leqq 1\}$. Let $m \in M^b(K)$. For each x in X choose h_x in $K \cap Z$ such that $\hat{h}_x(t) = 1(t)$ for each t in a neighborhood of x, and $\|h_x\| = 1$. Then

$$\begin{aligned}\sup\{\|\hat{m}(x)\| : x \in X\} &= \sup\{\|(mh_x)\hat{\ }(x)\| : x \in X\} \\ &\leqq \sup\{\|(mh_x)\hat{\ }\| : x \in X\} = \sup\{\|mh_x\| : x \in X\} \\ &\leqq \sup\{\|ma\| : a \in K, \|a\| \leqq 1\} = \|m\|.\end{aligned}$$

Thus the mapping $:m \to \hat{m}$ takes $M^b(K)$ into Σ^b.

Let $\sigma \in \Sigma^b$. By Theorem 3.6 there is an m in $M(K)$ such that $\hat{m} = \sigma$. Then we have

$$\begin{aligned}&\sup\{\|ma\| : a \in K, \|a\| \leqq 1\} \\ &\quad = \sup\{\|(ma\hat{\ }\| : a \in K, \|a\| \leqq 1\} \\ &\quad = \sup\{\|\hat{m}\hat{a}\| : a \in K, \|a\| \leqq 1\} \\ &\quad \leqq \sup\{\|\hat{m}\| \cdot \|\hat{a}\| : a \in K, \|a\| \leqq 1\} \\ &\quad \leqq \|\hat{m}\|.\end{aligned}$$

Hence m is in $M^b(K)$ and the mapping $:m \to \hat{m}$ maps $M^b(K)$ onto Σ^b. It also follows from the preceding calculations that the mapping $:m \to \hat{m}$ when restricted to $M^b(K)$ is an isometry.

REFERENCES

1. C.A. Akemann, G.K. Pedersen, and J. Tomiyama, *Multipliers of C*-algebras*, Københavens Universitet, Matematisk Institut, 1972.

2. R.C. Busby, *Double centralizers and extension of C*-algebras*, Trans. A.M.S. **132** (1968), 79–99.

3. J. Dixmier, *Les C*-algebras et leurs representations*, Gauthier-Villars, Paris, 1964.

4. M.J. Dupré, *Classifying Hilbert bundles*, J. Functional Analysis **15** (1974), 244–278.

5. ———, *Hilbert bundles with infinite dimensional fibres*, Mem. A.M.S. **148** (1974), 165–176.

6. K.H. Hofmann, *Representation of algebras by continuous sections*, Bull. A.M.S. **78** (1972), 291–373.

7. K.B. Laursen and A.M. Sinclair, *Lifting matrix units in C*-algebras* II, Math. Scand. **37** (1975), 167–172.

8. A.J. Lazar and D.C. Taylor, *Multipliers of Pedersen's ideal*, Mem. A.M.S. **169** (1976).

9. ———, *Double centralizers of Pedersen's ideal of a C*-algebra*, Bull. A.M.S. **78** (1972), 992–997.

10. J. Mack, *Fields of topological spaces*, Pacific J. Math. **46** (1973), 457–466.

11. G.K. Pedersen, *Measure theory for C*-algebras*, Math. Scand. **19** (1966), 131–145.

12. ———, *An approach to non-commutative measure theory*, Copenhagen, 1971.

13. ———, and N.H. Petersen, *Ideals in a C*-algebra*, Københavens Universitet, Matematisk Institut, 1969.

DEPARTMENT OF MATHEMATICS, INDIANA UNIVERSITY SOUTHEAST, NEW ALBANY, IN 47150