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SELECTED PROCEEDINGS FROM TWO SPECIAL SESSIONS ON COMMUTATIVE ALGEBRA SPONSORED BY AMERICAN MATHEMATICAL SOCIETY

Special Session of Free Resolutions
Fall 2007 AMS Central Section Meeting
De Paul University, Chicago, IL, October 5–6, 2007

Most of the talks in the Special Session on Free Resolutions were on graded finite free resolutions, infinite free resolutions over local rings, and Hilbert functions. The idea to associate a free resolution to a finitely generated module was introduced by Hilbert. Since then a lot of progress has been made on studying the structure of free resolutions and on using them in applications in Commutative Algebra, Algebraic Geometry, Combinatorics, Computational Algebra, Non-Commutative Algebra, and other mathematical fields. This has been a very active area of recent research. The Special Session on Free Resolutions provided a forum for the commutative algebraists working on resolutions (and related topics) to meet, present their latest results, and learn new techniques through formal talks and informal discussions. It also provided them with an opportunity to exchange open problems, share ideas, and explore in new directions. We had a lively exchange of ideas and methods that will foster further research.

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Special Session on Progress in Commutative Algebra
Joint Mathematics Meetings
San Diego, CA, January 6–9, 2008

The talks in the Special Session on Progress in Commutative Algebra reported on recent research on several aspects of commutative algebra. The seven papers included in this volume are representative of the talks given at the session; they focus on various aspects of the theory of Noetherian rings and their modules. What follows is a short introduction to the papers.

Applications of homological methods in commutative algebra revolve around vanishing of the derived functors Ext^i and Tor_i. Over regular rings, these functors vanish for sufficiently large values of i, but it is known from the theory of complete intersection rings that the homology modules $\mathrm{Tor}_i(M,N)$ may vanish for non-trivial reasons, that is, even if M and N are not of finite homological dimension. In "Minimal intersections and vanishing of (co)homology", Jorgensen and Moore identify a new class of rings that have modules exhibiting this behavior.

Complete intersection rings, and more generally Gorenstein rings, come equipped with a useful duality theory. Since every artinian ring is a quotient of a Gorenstein ring, one can approach problems over an artinian ring by lifting them to a Gorenstein ring. The utility of this approach depends on how closely the Gorenstein ring approximates the artinian ring. One measure hereof is studied by Ananthnarayan in "Computing Gorenstein colength".

A conceptually similar strategy is to shift problems to a more suitable ring via a faithful extension $\phi\colon R \to S$; the heselization and the $\mathfrak{m}$-adic completion of a local ring R with maximal ideal $\mathfrak{m}$ are particularly important examples. The work of Wiegand and Hassler in "Extended modules" is motivated by the necessity of understanding which modules over the target ring S are extended from the source ring R in order to understand which properties can be translated between R and S via ϕ.

Sather-Wagstaff and Spiroff approach the study of a ring homomorphism between normal integral domains A and B, by looking at the divisor class group in "Maps on divisor class groups induced by ring homomorphisms of finite flat dimension". They show that if the ring homomorphism is of finite flat dimension then it induces a group homomorphism between the class groups $\mathrm{Cl}(A)$ and $\mathrm{Cl}(B)$, and they investigate the kernel of this induced map.

It is a fact that we still do not understand the complete implications of the finiteness condition in the definition of a noetherian ring, and investigations in this direction lead to very strong results. For example one can in some cases capture the solution to the infinite set of equations which give the tight closure of an ideal I by computing the ideal $(IN : N)$, where N is an R-module that does not depend on I. It is even possible that N can be an ideal; Vraciu and Vassilev address this in the paper "When is tight closure determined by the test ideal?".

In another direction, the work of Matson presented in "Rings of finite rank and finitely generated ideals" aims to completely understand rings where a condition stronger than noetherianicity holds: every ideal is generated by at most a fixed number of generators n.

One strength of commutative algebra is its many connections with other fields. For instance, one can study objects like graphs and simplicial complexes via certain commutative rings constructed from the object's combinatorial data. The paper "Properties of cut ideals associated to ring graphs" by Nagel and Petrovic investigates ring-theoretic properties of one such construction, the cut ideal of a graph, which encodes information about all possible partitions of the graph's vertex set.

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