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Note on complete local integrity domains

By

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Previously some interesting results concerning prime ideals in rings of formal power series were proved by C. Chevalley [1]. In the present paper, we want to offer a new treatment on the similar assertions. We see on the way a new result that when $\mathfrak{o}$ is a complete (Noetherian) local integrity domain with a basic field k, $\mathfrak{o}$ is separably generated[1] over k if and only if there exists a system of parameters $x_1, \cdots, x_n$ of $\mathfrak{o}$ such that $\mathfrak{o}$ is separable over the ring $k\{x_1, \cdots, x_n\}$ (formal power series).

Throughout the present paper, a local ring means a Noetherian local ring which contains a field.

§1. Kroneckerian products.

Let $\mathfrak{o}_1$ and $\mathfrak{o}_2$ be complete local rings with basic fields k_1 and k_2 respectively. If K is a field containing both k_1 and k_2, we can define the Kroneckerian product of (k_1-algebra) $\mathfrak{o}_1$ and (k_2-algebra) $\mathfrak{o}_2$ over K, as was defined by C. Chevalley [2]. We denote this Kroneckerian product by $\mathfrak{o}_1/k_1 \times_K \mathfrak{o}_2/k_2$[2]. (For the detail, see Chevalley [2]). When $k_1=k_2=K$, we denote this by $\mathfrak{o}_1 \times_K \mathfrak{o}_2$.

We define further Kroneckerian products of complete local rings with discrete rings:

Let $\mathfrak{o}_1$ be a complete local ring with basic field k_1 and let $\mathfrak{o}_2$ be a discrete ring[3] which contains a field k_2. Assume that K is a field which contain both k_1 and k_2. We define the Kroneckerian product of k_1-algebra $\mathfrak{o}_1$ and discrete k_2-algebra $\mathfrak{o}_2$ over K as follows:

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1) For the definition, see Chevalley [1] or §2 in the present paper.

2) Though Chevalley [2] denotes this ring by $\mathfrak{o}_1 \times_K \mathfrak{o}_2$, we dare use a more complicated notation because the product depends on the choice of basic fields.

3) $\mathfrak{o}_2$ may be a topological ring which is not discrete; we only regard it as an abstract ring (or a discrete topological ring).

Let B_1 be a strong base of $\mathfrak{o}_1$ over k_1 and let B' be a linearly independent base of $\mathfrak{o}_2$ over k_2. We set $\mathfrak{o}=\{\sum(\sum a_{\lambda n}v_\lambda)u_n;\ a_{\lambda n}\in K, v_\lambda\in B', u_n\in B_1, \sum a_{\lambda n}v_\lambda$ is a finite sum$\}$. We introduce in $\mathfrak{o}$ operations of sum and product by the followings;

$$(\sum(\sum a_{\lambda n}v_\lambda)u_n)+(\sum(\sum b_{\lambda n}v_\lambda)u_n)=\sum(\sum(a_{\lambda n}+b_{\lambda n})v_\lambda)u_n,$$

$$(\sum(\sum a_{\lambda n}v_\lambda)u_n)(\sum b_{\lambda n}v_\lambda)u_n)=\sum(\sum(\sum a_{\lambda n}b_{\lambda' n'}c_{\lambda\lambda'\varkappa}d_{nn'm})v_\varkappa)u_m,$$

where $v_\lambda v_{\lambda'}=\sum c_{\lambda\lambda'\varkappa}v_\varkappa (c_{\lambda\lambda'\varkappa}\in k_2)$ and $u_nu_{n'}=\sum d_{nn'm}u_m (d_{nn'm}\in k_1)$. (Observe that $\sum a_{\lambda n}b_{\lambda' n'}c_{\lambda\lambda'\varkappa}d_{nn'm}$ is a finite sum).

It is easy to see that though this depends on the choice of strong base B_1 and linearly independent base B' of $\mathfrak{o}_1$ and $\mathfrak{o}_2$ respectively, the structure of $\mathfrak{o}$ does not depend on the choice of them. This ring $\mathfrak{o}$ is called the Kroneckerian product of (complete local) k_1-algebra $\mathfrak{o}_1$ and (discrete) k_2-algebra $\mathfrak{o}_2$ over K and we denote this by $\mathfrak{o}_1/k_1\times_K(\mathfrak{o}_2/k_2)_d$. When $k_1=k_2=K$, we donote this by $\mathfrak{o}_1\times_K(\mathfrak{o}_2)_d$. If $\mathfrak{o}_2=k$ is a field, we denote this by $\mathfrak{o}_1\times_K k$.

Next we explain an easy, but, important lemma:

Lemma 1. Let $\mathfrak{o}_1$ and $\mathfrak{o}_2$ be complete local rings with the same basic field K. Then $\mathfrak{o}_1\times_K\mathfrak{o}_2=\mathfrak{o}_1\times_K(\mathfrak{o}_2)_d$.

Proof. Let B_1 and B_2 be strong bases of $\mathfrak{o}_1$ and $\mathfrak{o}_2$ over K respectively and let B' be a linearly independent base of $\mathfrak{o}_2$ over K. Set $\mathfrak{o}'=\mathfrak{o}_1\times_K(\mathfrak{o}_2)_d$ and $\mathfrak{o}=\mathfrak{o}_1\times_K\mathfrak{o}_2$. Then

$$\mathfrak{o}'=\{\sum(\sum a_{\lambda n}v_\lambda)u_n;\ a_{\lambda n}\in K,\ v_\lambda\in B',\ u_n\in B_1,\ \sum a_{\lambda n}v_\lambda \text{ is a finite sum}\},$$

$$\mathfrak{o}=\{\sum b_{\mu n}w_\mu u_n;\ b_{\mu n}\in K,\ w_\mu\in B_2,\ u_n\in B_1\}.$$

Let ϕ be a mapping from $\mathfrak{o}'$ into $\mathfrak{o}$ as follows:

$$\phi(\sum(\sum a_{\lambda n}v_\lambda)u_n)=\sum b_{\mu n}w_\mu u_n,$$

where $\sum a_{\lambda n}v_\lambda=\sum b_{\mu n}w_\mu$ ($\sum b_{\mu n}w_\mu$ may be infinite sum).

Similarly, let ϕ^* be a mapping from $\mathfrak{o}$ into $\mathfrak{o}'$ as follows:

$$\phi^*(\sum b_{\mu n}w_\mu u_n)=\sum(\sum a_{\lambda n}v_\lambda)u_n,$$

where $\sum b_{\mu n}w_\mu=\sum a_{\lambda n}v_\lambda$ ($\sum a_{\lambda n}v_\lambda$ must be a finite sum).

Then we see easily that ϕ and ϕ^* are homomorphisms and that $\phi\circ\phi^*$ is the identity mapping. Therefore ϕ is an isomorphism.

§2. Separably generated extensions.

We denote hereafter by p the characteristic of the field of reference when it is not zero or the number 1 for the other case.

Definition. We say that a complete local integrity domain $\mathfrak{o}$

is separably generated over its basic field k if $\mathfrak{o}\times_k k^{p^{-1}}$ is an integrity domain.

Theorem 1. *Let $\mathfrak{o}$ be a complete local integrity domain with a basic field k. Then the following three conditions are equivalent to each other:*

(1) $\mathfrak{o}$ is separably generated over k.

(2) For any strongly linearly independent subset B of $\mathfrak{o}$ over k, $B_p=\{u^p\,;\,u\in B\}$ is also strongly linearly independent over k.

(3) For any integer m, $\mathfrak{o}\times_k k^{p^{-m}}$ is an integrity domain.

Proof. Assume that B is a strongly linearly independent set of $\mathfrak{o}$ over k and assume that B^p is not strongly linearly independent over k: There exist $u_n\in B$ such that $\sum a_n u_n^p=0$ ($u_i\neq u_j$ if $i\neq j$, $a_n\in k$), and therefore in $\mathfrak{o}\times_k k^{p^{-1}}$, $(\sum a_n^{p^{-1}}u_n)^p=0$ and $\sum a_n^{p^{-1}}u_n\neq 0$. Therefore $\mathfrak{o}\times_k k^{p^{-1}}$ is not an integrity domain. This proves that (2) follows from (1). Next we prove the converse: Assume that $\mathfrak{o}\times_k k^{p^{-1}}$ contains a divisor c of zero ($c\neq 0$). Since $c^p\in\mathfrak{o}$, we see that $c^p=0$. We write $c=\sum a_n u_n$($\{u_n\}$ is strongly linearly independent over k ($u_n\in\mathfrak{o}$), $a_n\in k^{p^{-1}}$). Then $\{u_n^p\}$ is not strongly linearly independent over k. These being settled, the eqivalence with (3) is evident.

Corollary. If a complete local integrity domain $\mathfrak{o}$ is separably generated over its basic field k, then for any integer m, $\mathfrak{o}\times_k k^{p^{-m}}$ is separably generated over $k^{p^{-m}}$.

Remark 1. It is evident that if $\mathfrak{o}$ is a complete local ring with a basic field k and if K is a field which contains k, then $\mathfrak{o}\times_k K$ is a complete semi-local ring; we have the identity

$$\mathfrak{o}\times_k K=\mathfrak{o}/k\times_K k/k=\mathfrak{o}/k\times_K K/K.$$

Remark 2. By Lemma 1, we see that when $[k:k^p]<\infty$, a complete local integrity domain $\mathfrak{o}$ with a basic field k is separably generated over k if and only if the quotient field of $\mathfrak{o}$ is separably generated over k.[4] In general case, we see easily that the tensor product $\mathfrak{o}\otimes_k k^{p^{-1}}$ is a subring $\mathfrak{o}\times_k k^{p^{-1}}$ and therefore we see that if $\mathfrak{o}$ is separably generated over k, then the quotient field of $\mathfrak{o}$ is

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4) We say, according to C. Chevalley [1], that a field K is separably generated over its subfield k if the tensor product $K\otimes_k k^{p^{-1}}$ is an integrity domain. This is equivalent to that every finitely generated extension field of k contained in K has a separating transcendence base over k.

separably generated over k. The converse is not true as is easily seen.

§3. Derivations.

Definition. Let $\mathfrak{o}$ be a complete local integrity domain. A derivation D of $\mathfrak{o}$ is a linear operator to the quotient field L of $\mathfrak{o}$ which satisfies the following conditions:

i) $D(xy)=xDy+yDx$ (for any $x, y \in L$),

ii) There exists an element $d(\neq 0)$ of $\mathfrak{o}$ such that $dDx \in \mathfrak{o}$ for any $x \in \mathfrak{o}$ and if $\sum u_n$ is a convergent series in $\mathfrak{o}$ then $\sum dDu_n$ is also convergent (therefore $\sum Du_n$ has a meaning in L) and $D(\sum u_n)=\sum Du_n$.

A derivation D, for which it holds that $Da=0$ if a is in a subring $\mathfrak{o}'$ of $\mathfrak{o}$, is called a derivation of $\mathfrak{o}$ over $\mathfrak{o}'$.

It is evident that the totality of derivations of $\mathfrak{o}$ (over a subring) form an $\mathfrak{o}$-module. Linear dependency of derivations is defined in this sense.

Lemma 2. Let $\mathfrak{o}$ be the ring of formal power series in $x_1, \cdots, x_n$ over a field k. Then the partial dervations $D_i=\partial/\partial x_i$ $(i=1, \cdots, n)$ form a maximal linearly independent set of derivations of $\mathfrak{o}$ over k.

Proof is easy.

Lemma 3. Let $\mathfrak{o}$ be a complete local integrity domain with a basic field k. Let L be the quotient field of $\mathfrak{o}$. Assume that the characteristic p of $\mathfrak{o}$ is not zero. Let M be the subfield of L generated by L^p and k. Take an integer r such that $[L:M]=p^r$. Then any maximal set of linearly independent derivations of $\mathfrak{o}$ over k consists of r derivations.

Proof. If a is in L^p, $Da=0$ for any derivation D of $\mathfrak{o}$. Therefore, for any derivation D of $\mathfrak{o}$ over k and for any element a of M, we have $Da=0$. We take elements, $a_1, \cdots, a_r$ of L such that $L=M(a_1, \cdots, a_r)$. Then we can find derivations $D_1, \cdots, D_r$ of $\mathfrak{o}$ over k such that $D_i a_j=\delta_{ij}$ (Kroneckerian δ). That this is a maximal set of linearly independent derivations can be proved easily.

Theorem 2. *Let $\mathfrak{o}$ be a complete local integrity domain of dimension n and with a basic field k. Then the number of menbers of a maximal set of linearly independent derivations of $\mathfrak{o}$ over k is at least n. It is n if and only if $\mathfrak{o}$ is separably generated over k.*

Proof. Let $x_1, \cdots, x_n$ be a system of parameters of $\mathfrak{o}$ and set $\mathfrak{r}=k\{x_1, \cdots, x_n\}$. Let $\mathfrak{o}'$ be the totality of separably algebraic (integral) elements of $\mathfrak{o}$ over $\mathfrak{r}$.

1) We first show that a maximal set of linearly independent derivations of $\mathfrak{o}'$ over k consists of just n numbers: It is true for $\mathfrak{r}$ by Lemma 2; since $\mathfrak{o}'$ is separable over $\mathfrak{r}$, we see that this is also true for $\mathfrak{o}'$ (similarly to the case of the theory of fields).

2) Next we show that $\mathfrak{o}'$ is separably generated over k: Let c be an element of $\mathfrak{o}'$ such that $\mathfrak{o}'$ is contained in the quotient field of $\mathfrak{o}[c]$ and let $f(x)$ be the irreducible monic polynomial over $\mathfrak{o}$ satisfied by c. If $\mathfrak{o}' \times_k k^{p^{-1}}$ is not an integrity domain, we see that $f(x)$ is reducible over $k^{p^{-1}}\{x_1, \cdots, x_n\}$, which is impossible because $f(x)$ is separable and $k^{p^{-1}}\{x_1, \cdots, x_n\}$ is purely inseparable over $k\{x_1, \cdots, x_n\}$.

3) We prove the general case by induction on $[\mathfrak{o} : \mathfrak{o}']$[5] ($p \geq 2$): Let $c_1, \cdots, c_r$ be elements of $\mathfrak{o}$ such that $c_i^p \in \mathfrak{o}'[c_1, \cdots, c_{i-1}]$, and $[\mathfrak{o}'[c_1, \cdots, c_s] : \mathfrak{o}'] = p^s$ ($[\mathfrak{o} : \mathfrak{o}'] = p^r$). We set $\mathfrak{s} = k\{x_1^p, \cdots, x_n^p\}[\mathfrak{o}'^p][c_1^p, \cdots, c_r^p]$, $\mathfrak{o}'' = \mathfrak{o}'[c_1, \cdots, c_{r-1}]$, $\mathfrak{s}'' = k\{x_1^p, \cdots, x_n^p\}[\mathfrak{o}'^p][c_1^p, \cdots, c_{r-1}^p]$. Then $[\mathfrak{s} : \mathfrak{s}''] \leq p$, $[\mathfrak{o} : \mathfrak{o}''] = p$ and $[\mathfrak{o}'' : \mathfrak{s}''] \geq p^n$ (by induction assumption), which shows $[\mathfrak{o} : \mathfrak{s}] = [\mathfrak{o} : \mathfrak{s}'']/[\mathfrak{s} : \mathfrak{s}''] = [\mathfrak{o} : \mathfrak{o}''][\mathfrak{o}'' : \mathfrak{s}'']/[\mathfrak{s} : \mathfrak{s}''] \geq p^n$. Thus we see that there exists a system of n linearly independent derivations of $\mathfrak{o}$ over k.

4) We assume that $\mathfrak{o}$ is separably generated oven k. We use the same notations as in 3). Since our assertion is true for $\mathfrak{o}'$, we prove our assertion by induction on $[\mathfrak{o} : \mathfrak{o}']$. Since $\mathfrak{o}''$ is a subspace of $\mathfrak{o}$, we see that $\mathfrak{o}''$ is also separably generated. Therefore by our induction assumption we see that $[\mathfrak{o}'' : \mathfrak{s}''] = p^n$. Therefore we have only to show that $[\mathfrak{s} : \mathfrak{s}''] = p$. Let B be a strong base of $\mathfrak{o}''$ over k. Then $\{B, Bc_r, \cdots, Bc_r^{p-1}\}$ form a strong base of $\mathfrak{o}''[c]$ over k. If $[\mathfrak{s} : \mathfrak{s}''] = 1$, we must have a relation $\sum_{i=0}^{p-1} b_i c_r^{pi} = 0$ ($b_i \in \mathfrak{s}''$, $b_0 \neq 0$). Then we must have a relation $\sum a_{\lambda i} u_\lambda^p c_r^{pi} = 0$ ($a_{\lambda i} \in k$, $u_\lambda \in B$, $0 \leq i \leq p-1$, $\sum a_{\lambda i} u_\lambda^p = b_i$), which is a contradiction to our assumption that $\mathfrak{o}$ is separably generated over k (because $\mathfrak{o}''[c]$ is a subspace of $\mathfrak{o}$).

5) Conversely, we assume that a maximal set of linearly independent derivations over k consists of just n members. We can take a set of linearly independent derivations $D_1, \cdots, D_n$ over k and a system of elements $c_1, \cdots, c_n$ of $\mathfrak{o}$ such that $D_i c_j = \delta_{ij}$ (Kroneckerian δ). We may assume that these c_i are unit in $\mathfrak{o}$, because if c_i is not a unit, we may take $1 + c_i$ instead of c_i. It is evident that $x_1^p, \cdots, x_n^p$ is a system of parameters of $\mathfrak{o}$. We set $y_i = c_i x_i^p$.

5) $[\mathfrak{o} : \mathfrak{o}']$ means the index of the quotient field of $\mathfrak{o}$ over that of $\mathfrak{o}'$.

Then $y_1, \cdots, y_n$ form a system of parameters of $\mathfrak{o}$. Then by our construction, we see easily that every derivations of $k\{y_1, \cdots, y_n\}$ over k can be uniquely extended to a derivation of $\mathfrak{o}$ over k. This shows that $\mathfrak{o}$ is separable over $k\{y_1, \cdots, y_n\}$. Now we see that $\mathfrak{o}$ is separably generated over k by virtue of 2) above.

We have proved in the same time (in 2) and 5)) the following

Theorem 3. *Let $\mathfrak{o}$ be a complete local integrity domain with a basic field k. Then $\mathfrak{o}$ is separably generated over k if and only if there exists a system of parameters $x_1, \cdots, x_n$ of $\mathfrak{o}$ such that $\mathfrak{o}$ is separable over $k\{x_1, \cdots, x_n\}$.*

Now we prove

Theorem 4. *Assume that a complete local integrity domain $\mathfrak{o}$ is separably generably generated over its basic field k. If K is an extension field of k such that k is separably algebraically closed in K, then $\mathfrak{o} \times_k K$ is an integrity domain.*

Proof. By virtue of Theorem 3, we can choose a system of parameters $x_1, \cdots, x_n$ of $\mathfrak{o}$ so that $\mathfrak{o}$ is separable over $k\{x_1, \cdots, x_n\}$. We choose an element c of $\mathfrak{o}$ so that $[\mathfrak{o} : k\{x_1, \cdots, x_n\}[c]]=1$ and let $f(x)$ be the irreducible monic polynomial over $k\{x_1, \cdots, x_n\}$ satisfied by c. If $\mathfrak{o} \times_k K$ is not an integrity domain, we have that $f(x)$ is reducible over $K\{x_1, \cdots, x_n\}$: $f(x)=g(x)h(x)$, where g and h are monic polynomials over $K\{x_1, \cdots, x_n\}$. Then every coefficients of g and h are integral over $k\{x_1, \cdots, x_n\}$, therefore they are in $k^{p^{-m}}\{x_1, \cdots, x_n\}$ for some integer m, which shows that $\mathfrak{o} \times_k k^{p^{-m}}$ is not an integrity domain and this is a contradiction to that $\mathfrak{o}$ is separably generated over k.

§4. Regular extensions.

Definition. A complete local integrity domain $\mathfrak{o}$ with a basic field k is said to be a regular extension of k if 1) k is algebraically closed in the quotient field of $\mathfrak{o}$ and 2) $\mathfrak{o}$ is separably generated over k.

Theorem 5. *Let $\mathfrak{o}$ be a complete local integrity domain with a basic field k. Then the following three conditions are equivalent to each other:*

(1) $\mathfrak{o}$ is a regular extension of k.

(2) $\mathfrak{o}$ is separably generated over k and for any finite separable extension k' of k, $\mathfrak{o} \times_k k'$ is an integrity domain.

(3) For any finite separable extension k'' of $k^{p^{-1}}$, $\mathfrak{o} \times_k k''$ is an integrity domain.

Proof is easy.

Remark. We see easily that if a complete local integrity domain $\mathfrak{o}$ with a basic field k is a regular extension of k, then the quotient field of $\mathfrak{o}$ is regular extension of k.[6] The converse is true if $[k:k^p]<\infty$. (See the remark at the end of §2.)

Theorem 6. *Let $\mathfrak{o}$ be a complete local integrity domain with a basic field k. Assume that $\mathfrak{o}$ is a regular extension of k.[6] Then for any field K containing k, $\mathfrak{o}\times_k K$ is an integrity domain. Further $\mathfrak{o}\times_k K$ is a regular extension of K.*

Proof. That $\mathfrak{o}\times_k K$ is an integrity domain can be proved by a similar way as in the proof of Theorem 4. Let K' be an arbitrary field containing K. Then $(\mathfrak{o}\times_k K)\times_K K'=\mathfrak{o}\times_k K$ is an integrity domain, which shows that $\mathfrak{o}\times_k K$ is a regular extension of K.

§5. An application.

Theorem 7. *Let $\mathfrak{o}_1$ and $\mathfrak{o}_2$ be complete local integrity domains with the same basic field k. Assume that $\mathfrak{o}_1$ is a regular extension of k. Then $\mathfrak{o}_1\times_k\mathfrak{o}_2$ is an integrity domain. In this case, if $\mathfrak{o}_2$ is also a regular extension of k, then $\mathfrak{o}_1\times_k\mathfrak{o}_2$ is a regular extension of k.*

Proof. Let L be the quotient field of $\mathfrak{o}_2$. Then $\mathfrak{o}_1\times_k L$ is an integrity domain. By Lemma 1, $\mathfrak{o}_1\times_k\mathfrak{o}_2$ is a subring of $\mathfrak{o}\times_k L$, which shows that $\mathfrak{o}_1\times_k\mathfrak{o}_2$ is an integrity domain. Now we assume that $\mathfrak{o}_2$ is also a regular extension of k. Let K be an arbitrary field containing k. Then $(\mathfrak{o}_1\times_k\mathfrak{o}_2)\times_k K=(\mathfrak{o}_1\times_k K)\times_K(\mathfrak{o}_2\times_k K)$. Since $\mathfrak{o}_1\times_k K$ is a regular extension of K and since $\mathfrak{o}_2\times_k K$ is an integrity domain, we see that $(\mathfrak{o}_1\times_k\mathfrak{o}_2)\times_k K$ is an integrity domain. This shows that $\mathfrak{o}_1\times_k\mathfrak{o}_2$ is a regular extension of k.

Corollary 1. Let $\mathfrak{o}_1$ and $\mathfrak{o}_2$ be complete local integrity domains with basic fields k_1 and k_2 respectively. Assume that K is a field containing both k_1 and k_2. Then $\mathfrak{o}_1/k_1\times_K\mathfrak{o}_2/k_2$ is a regular extension of K if $\mathfrak{o}_1$ and $\mathfrak{o}_2$ are regular extensions of k_1 and k_2 respectively.

Corollary 2. Let $x_1, \cdots, x_n, y_1, \cdots, y_m$ be analytically independent elements over a field k and let $\mathfrak{p}_1$ and $\mathfrak{p}_2$ be prime ideals of $\mathfrak{o}_1=k\{x_1, \cdots, x_n\}$ and $\mathfrak{o}_2=k\{y_1, \cdots, y_m\}$ respectively. Then $(\mathfrak{p}_1, \mathfrak{p}_2)k\{x_1, \cdots x_n, y_1, \cdots, y_m\}$ is prime if $\mathfrak{o}_1/\mathfrak{p}_1$ is a regular extension

6) We say that a field K is a regular extension of its subfield k if the tensor produst $K\otimes_k\bar{k}$ of K and the algebraic closure $\bar{k}$ of k over k is an integrity domain, or equivalently, if K is separably generated over k and if k is algebraically closed in K.

of $\boldsymbol{k}$. If furthermore, $\mathfrak{o}_2/\mathfrak{p}_2$ is also a regular extension of $\boldsymbol{k}$, then $\boldsymbol{k}\{x_1, \cdots, x_n, y_1, \cdots, y_m\}/(\mathfrak{p}_1, \mathfrak{p}_2)\boldsymbol{k}\{x_1, \cdots, x_n, y_1, \cdots y_m\}$ is a regular extension of $\boldsymbol{k}$.

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