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Corrections and supplements to my paper "Differential modules and derivations of complete discrete valuation rings[*)]

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Some errors are found in the above paper. We shall correct them and make some improvements at the same time.

1. Read $\sum_{i=0}^{\infty}$ in p. 429, l.1 as $\sum_{i=1}^{\infty}$.

2. In the statement of Theorem 2 in p. 429 we should make an assumtion that R is of unequal characteristic.

3. Read $\min_{0 \leq i \leq e-1}((\Delta(f_i)+1)e+1)-v(f'(u))$ in the foot note in p. 429 as $\min_{0 \leq i \leq e-1}((\Delta(f_i)+1)e+i)-v(f'(u))$.

4. As for the definition of $\Delta_{K/K^*}(u)$ in p. 429, we should state that this number does not depend on the choice of the set of elements $\{a_\iota\}_{\iota \in I}$ contained in P. This can be proved in various ways. For instance, the proof is reached easily if we use Theorem 2 and Proposition 2 and prove that in case $\Delta_{K/K^*}(u) \geq 0$ we have $\Delta_{K/K^*}(u) = \min_{\partial} v(\partial u)$, where ∂ runs over $\mathrm{Der}(R, R)$. An alternative and more direct proof is obtained if we restate Neggers' original definition of $\Delta_{K/K^*}(u)$ without assuming $f(u)$ to be an Eisenstein polynomial, that is, if $f(U)=U^e+b_{e-1}U^{e-1}+\cdots+b_0$, $\Delta_{K/K^*}(u)$ is defined to be $\min_{0 \leq i \leq e-1}(\Delta(b_i)e+i)-v(f'(u))$. This definition depends only on P and u and it is easy to see that this is equivalent to the previous definition.

5. As for Proposition 9 in p. 431, we should have stated the fol-

lowing lemma.

Lemma. *The following four conditions are equivalent.*

(α) *R is residually perfect.*

(β) $\Omega_{R/m}=0$.

(γ) $\mathrm{Der}(R, R)=0$

(δ) $\Delta_{K/K^*}(u)=\infty$ *for every prime element u in R.*

Proof of Lemma. $(\alpha)\Leftrightarrow(\beta)$ is well-known. We shall prove $(\beta)\Leftrightarrow(\gamma)$. Assume that $\Omega_{R/m}=0$. Then I is an empty set and the relation (8) is $f'(u)\partial u=0$. Therefore $\mathrm{Der}(R, R)=0$, because $f'(u)\neq 0$. Assume that $\Omega_{R/m}\neq 0$. Then $\{a_\iota\}_{\iota\in I}$ is not an empty set. Hence the equation (7) is solvable, putting a set of nontrivial values in $\{c_\iota\}_{\iota\in I}$. Hence $\mathrm{Der}(R, R)\neq 0$. Next, we shall prove $(\beta)\Leftrightarrow(\delta)$. Assume that $\Omega_{R/m}\neq 0$. Then, $\Omega_P^*=0$ and $\Delta_{K/K^*}(u)=\infty$. Conversely, assume that $\Delta_{K/K^*}(u)=\infty$ and assume that $\Omega_{R/m}\neq 0$. Then $\{a_\iota\}_{\iota\in I}$ is not an empty set. Since a_ι is a unit in R, $u'=ua_\iota$ is a prime element in R. Since the relation (8) is $f'(u)\partial t=0$ in our case, there exists a derivation ∂ in $\mathrm{Der}(R, R)$ such that $\partial a_\iota=1$. Then, $v(\partial u')=v(a_\iota\partial u+u\partial a_\iota)=v(u)=1$. Hence $\Delta_{K/K^*}(u')=\min_{\partial} v(\partial u)\leqq 1<\infty$, which proves our assertion.

6. In Proposition 1 in *p*. 431, we should make an additional assumption that R is not residually perfect.

7. The following statement in *p*. 433, *l*. 19–20 is not generally correct. "Exactness of the sequence:

$$(R\otimes_P\Omega_P)^*\to\Omega_R^*\to\Omega_{R/P}\to 0$$

is always true." Only thing we can assert is, "In the sequence $(R\otimes_P\Omega_P)^*\xrightarrow{\rho^*}\Omega_R^*\longrightarrow\Omega_{R/P}$ the second homomorphism is surjective and its kernel is the closure of $\rho^*((R\otimes_P\Omega_P)^*)$". This follows from the fact that $\Omega_{R/P}$ is finitely generated. This change does not affect the proof of Proposition 4.

8. Read "Example 3" in *p*. 434, *l*. 13, as "Example 1".
9. Omit, "Let M and N are R-modules." in *p*. 434, *l*. 19.

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