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On the regularity of solutions of a mixed problem for hyperbolic equations of second order in a domain with corners

By

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§ 0. Introduction.

Mixed problems for hyperbolic equations have been studied by many authors. In the case when the domain is a quarter space and the coefficients are constant, S. Osher studies a mixed problem for hyperbolic systems ([6], [7]). On the other hand some authors treat with a mixed problem for hyperbolic equations with discontinuous boundary conditions in the case when the boundary of a domain is smooth ([1], [3]). K. Hayashida showed that a mixed problem of (1. 1)-(1. 4) has a unique solution which satisfies the boundary conditions weakly (See § 1. Theorem 1). A mixed problem with discontinuous boundary conditions and a mixed problem in a domain with corners seem to be similar.

In this paper we extend the result of K. Hayashida [1] in the case when the boundary of a domain has corners and we study the regularity of solutions.

§ 1. Statement of the results.

Let Ω be a domain in the n-dimensional Euclidean space $\boldsymbol{R}^n$. We assume that Ω and its boundary S satisfy the following three conditions;

i) S is compact,

ii) $S=\Gamma_1\cup\Gamma_2\cup L$, $\Gamma_i\cap L=\phi$ $(i=1,2)$, $\Gamma_1\cap\Gamma_2=\phi$, Γ_i is the $(n-1)$-dimensional C^∞-manifold $(i=1,2)$, and L is the $(n-2)$-dimensional compact C^∞-manifold, and

iii) for every point x_0 on L there exist a neighborhood $V(x_0)$ of x_0 in R_x^n, a neighborhood W of the origin in R_y^n and a regular C^∞-mapping $y=\varphi(x)$ such that $\varphi(L)\subset\{y_1=y_2=0\}$ and

$$\varphi: V(x_0)\cap\bar{\Omega}\rightrightarrows W\cap\overline{R_+^n}\quad \text{(case 1)}$$

or

$$\varphi: V(x_0)\cap\bar{\Omega}\rightrightarrows W\cap\overline{R_{1/4}^n}\quad \text{(case 2)}$$

or

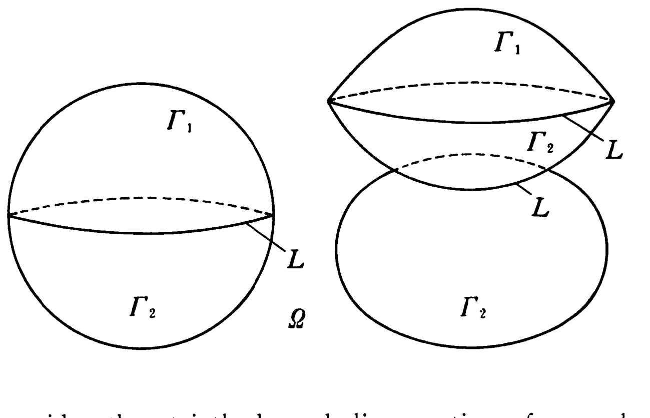
$$\varphi: V(x_0)\cap\bar{\Omega}\rightrightarrows W\cap\overline{R_{3/4}^n}\quad \text{(case 3)}$$

where $R_+{}^n=\{(y_1, y_2, y'')\,;\, y_1>0,\ (y_2, y'')\in \boldsymbol{R}^{n-1}\}$

$$R_{1/4}^n=\{(y_1, y_2, y'')\,;\, y_1>0, y_2>0 \text{ and } y''\in\boldsymbol{R}^{n-2}\}$$

$$R_{3/4}^n=\{(y_1, y_2, y'')\,;\, y_1>0 \text{ or } y_2<0, y''\in\boldsymbol{R}^{n-2}\}$$

and we mean the diffeomorphism by $\rightrightarrows$.



We consider the strictly hyperbolic equation of second order;

$$\text{(1.1)}\qquad \left\{\frac{\partial^2}{\partial t^2}+a_1\left(x;\frac{\partial}{\partial x}\right)\frac{\partial}{\partial t}+a_2\left(x;\frac{\partial}{\partial x}\right)\right\}u(t,x)=f(t,x)$$

in $[0,T]\times\Omega$,

$$a_1\left(x; \frac{\partial}{\partial x}\right) = 2\sum h_i(x)\frac{\partial}{\partial x_i} + c_1(x),$$

$$a_2\left(x; \frac{\partial}{\partial x}\right) = -\sum \frac{\partial}{\partial x_i} a_{ij}(x)\frac{\partial}{\partial x_j} + \sum b_i(x)\frac{\partial}{\partial x_i} + c_2(x),$$

where $h_i(x)$ and $a_{ij}(x)$ $(=a_{ji}(x))$ are real functions, all the coefficients of the equation (1. 1) belong to $\mathcal{B}(\bar{\Omega})$[1] and $\sum a_{ij}(x)\xi_i\xi_j \geqq \delta|\xi|^2$ for all $(x, \xi) \in \bar{\Omega} \times \boldsymbol{R}^n$ $(\delta > 0)$. Further let us impose the initial condition (1. 2) and the boundary conditions (1. 3) and (1. 4).

(1. 2) $$\left(\frac{\partial}{\partial t}\right)^j u(0, x) = u_j(x) \quad (j=0, 1)$$

(1. 3) $$u(t, x) = 0 \text{ in } [0, T] \times \Gamma_1$$

(1. 4) $$\left(\frac{\partial}{\partial n} - \langle h, \gamma \rangle \frac{\partial}{\partial t} + \sigma(x)\right) u(t, x) = 0 \text{ in } [0, T] \times \Gamma_2,$$

$$\frac{\partial}{\partial n} = \sum a_{ij} \cos(\nu, x_i) \frac{\partial}{\partial x_j}, \quad \nu = \text{the unit outer normal of } \Gamma_2,$$

$$\langle h, \gamma \rangle = \sum h_i \cos(\nu, x_i),$$

where $\sigma(x)$ is a real C^∞-function on $\bar{\Gamma}_2$.

We denote by $H^k(\Omega)$ the Sobolev space and by $K(\Omega)$ the completion of all $u(x)$ each of which belongs to $C_0^\infty(\bar{\Omega})$[2] and vanishes in a neighborhood of $\bar{\Gamma}_1$ with $H^1(\Omega)$-norm. Let us define two weak boundary conditions (B_1) and (B_2).

Definition 1. We assume that $a_2(x; \partial/\partial x)u(t, x)$ is in $L^2(\Omega)$ and $u(t, x)$ is in $\mathcal{E}_t^1(H^1(\Omega))$.[3] We say that $u(t, x)$ satisfies the weak boundary condition (B_1), if the following two conditions (1. 5) and (1. 6) are satisfied;

(1. 5) $u(t, x)$ belongs to $\mathcal{E}_t^1(K(\Omega))$,

and

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1) $\mathcal{B}(\bar{\Omega})$ is the set of all functions defined in the closure $\bar{\Omega}$ of Ω such that their derivatives of any order are continuous and bounded.

2) $C_0^\infty(E)$ is the set of all functions in $C^\infty(E)$ which have a compact support in E, where E is either open set or not.

3) "$u(t, x) \in \mathcal{E}_t^k(B)$" means that $u(t, x)$ is k-times continuously differentiable in t as B-valued function, where B is a Banach space.

(1. 6) $$\left(-\sum{}^{'}\frac{\partial}{\partial x_i}a_{ij}\frac{\partial}{\partial x_j}u, \varphi\right)=\sum{}^{'}\left(a_{ij}\frac{\partial u}{\partial x_j}, \frac{\partial \varphi}{\partial x_i}\right)$$

$$+\int_{\Gamma_2}\left(\sigma u-\langle h, \gamma\rangle\frac{\partial u}{\partial t}\right)\bar{\varphi}dS \text{ for every } \varphi(x)\in K(\Omega).^{4)}$$

Remark 1. For $u(t,x)\in C_0^\infty([0,T]\times\bar{\Omega})$, (1. 3) and (1. 5) are equivalent, and (1. 4) and (1. 6) are equivalent.

Definition 2. We assume that $\{u(x), v(x)\}$ is in $H^1(\Omega)\times H^1(\Omega)$ and $a_2(x;\partial/\partial x)u(x)$ belongs to $L^2(\Omega)$. $\{u(x), v(x)\}$ is said to satisfy the weak boundary condition (B_2), if the following two conditions (1. 7) and (1. 8) are satisfied;

(1. 7) $u(x)$ and $v(x)$ are in $K(\Omega)$,

and

(1. 8) $$\left(-\sum{}^{'}\frac{\partial}{\partial x_i}a_{ij}\frac{\partial}{\partial x_j}u, \varphi\right)=\sum{}^{'}\left(a_{ij}\frac{\partial u}{\partial x_j}, \frac{\partial \varphi}{\partial x_i}\right)$$

$$+\int_{\Gamma_2}(\sigma u-\langle h, \gamma\rangle v)\bar{\varphi}dS \text{ for every } \varphi(x)\in K(\Omega).$$

Theorem 1. (K. Hayashida [1]) *Let* $\{u_0, u_1\}$ *be in* $H^1(\Omega)\times H^1(\Omega)$ *and* $a_2(x;\partial/\partial x)u_0(x)$ *belong to* L^2. *If* $\{u_0, u_1\}$ *satisfies* (B_2) *and* $f(t,x)$ *belongs to* $\mathcal{E}_t^0(K(\Omega))$, *then there exists a unique solution* $u(t,x)$ *of* (1. 1) *in* $\mathcal{E}_t^1(K(\Omega))\cap\mathcal{E}_t^2(L^2(\Omega))$ *which satisfies* (1. 2) *and* (B_1), *and the following energy inequality holds;*

(1. 9) $$\|u(t)\|_1+\|u'(t)\|_0\leqq C_1e^{\beta t}(\|u_0\|_1+\|u_1\|_0+\int_0^t\|f(s)\|_0 ds).^{5)}$$

Remark 2. K. Hayashida proved Theorem 1 in the case when Ω is a bounded domain with a boundary S of class C^∞, but he did not assume that L is smooth. We can also prove Theorem 1 in the case when Ω and S satisfy our assumptions by the same way as his proof (See [1]). We omit the proof of Theorem 1, but in § 4 we

4) We denote an inner product in $L^2(\Omega)$ by (,).
5) $u'(t), u''(t)$ and $u^{(k)}(t)$ are $(\partial u/\partial t)$, $(\partial^2 u/\partial t^2)$ and $(\partial^k u/\partial t^k)$ respectively. And $\|\ \|_k$ is a norm in $H^k(\Omega)$.

make up for its proof.

Corollary of Theorem 1. *In Theorem 1, if $f(t,x)$ is not in $\mathcal{E}_t^0(K(\Omega))$ but in $\mathcal{E}_t^1(L^2(\Omega))$, then the same result as Theorem 1 holds, and further the energy inequality (1.10) holds;*

$$\|u'(t)\|_1+\|u''(t)\|_0\leqq C_2e^{\beta t}(\|u_1\|_1+\|a_2u_0+a_1u_1\|_0+\|f(0)\|_0+\int_0^t\|f'(s)\|_0 ds) \tag{1.10}$$

Remark 3. We do not prove Corollary of Theorem 1 in this paper, but we can prove it by the same way as [2] or [4] (See [4] pp. 28 Théorème 2. 1).

In order to consider the regularity of solutions of (1. 1), (1. 2), (1. 3) and (1. 4), we introduce some spaces of functions.

Definition 3. Let k be an integer, then we define for $k\geqq 1$

$$G^k(\Omega)=\left\{u(x)\,;\,u\in L^2(\Omega) \text{ and } \left(\frac{r}{1+r}\right)^{|\mu|-1}\left(\frac{\partial}{\partial x}\right)^{\mu}u\in L^2(\Omega) \text{ for } 1\leqq|\mu|\leqq k\right\}^{6)}$$

and we define for $k\geqq 0$

$$F^k(\Omega)=\left\{u(x)\,;\,\left(\frac{r}{1+r}\right)^{|\mu|}\left(\frac{\partial}{\partial x}\right)^{\mu}u\in L^2(\Omega) \text{ for } |\mu|\leqq k\right\}$$

where $r=\text{distance}(x, L)$.

Remark 4. We easily see that $G^1(\Omega)=H^1(\Omega)$ and $F^0(\Omega)=L^2(\Omega)$ and that $G^k(\Omega)$ and $F^k(\Omega)$ are Hilbert spaces with their appropriate inner products. If $u(x)$ is in $G^{k+1}(\Omega)$, then $\partial u/\partial x$ is in $F^k(\Omega)$, and if $\alpha(x)$ is in $\mathcal{B}(\bar{\Omega})$ and $u(x)$ is in $G^k(\Omega)$ (resp. $F^k(\Omega)$), then αu belongs to $G^k(\Omega)$ (resp. $F^k(\Omega)$).

We define the compatibility condition (C_k) of order k for data

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6) $\mu=(\mu_1, \mu_2, \cdots, \mu_n)$, $|\mu|=\mu_1+\mu_2+\cdots+\mu_n$, and $(\partial/\partial x)^{\mu}=(\partial/\partial x_1)^{\mu_1}(\partial/\partial x_2)^{\mu_2}\cdots(\partial/\partial x_n)^{\mu_n}$.

$\{f, u_0, u_1\}$ of (1.1) and (1.2).

Definition 4. Let $f(t, x)$ be in $\mathcal{E}_t^1(F^k)\cap\mathcal{E}_t^2(F^{k-1})\cap\cdots\cap\mathcal{E}_t^{k+1}(L^2)$, $\{u_0, u_1\}$ be in $H^1(\Omega)\times H^1(\Omega)$, and $a_2(x;\partial/\partial x)u_0$ be in $L^2(\Omega)$. Then $\{f, u_0, u_1\}$ is said to satisfy the compatibility condition (C_k), if $\{u_j, u_{j+1}\}$ belongs to $H^1(\Omega)\times H^1(\Omega)$, $a_2(x;\partial/\partial x)u_j(x)$ belongs to $L^2(\Omega)$, and $\{u_j, u_{j+1}\}$ satisfies (B_2) for $j=0, 1, \cdots, k$.
where $u_j (j\geqq 2)$ is inductively defined as

$$u_{j+2}=f^{(j)}(0)-a_1\Big(x;\frac{\partial}{\partial x}\Big)u_{j+1}-a_2\Big(x;\frac{\partial}{\partial x}\Big)u_j \quad \text{for } j=0, 1, 2, \cdots.$$

Now we state our main theorem.

Theorem 2. *We assume that $f(t, x)$, $\{u_0, u_1\}$ and a_2u_0 belong to $\mathcal{E}_t^1(F^k)\cap\mathcal{E}_t^2(F^{k-1})\cap\cdots\cap\mathcal{E}_t^{k+1}(L^2)$, $H^1(\Omega)\times H^1(\Omega)$ and $L^2(\Omega)$ respectively, and that $\{f, u_0, u_1\}$ satisfies (C_k), then the solution $u(t, x)$ of (1.1) and (1.2) which satisfies (B_1) belongs to $\mathcal{E}_t^0(G^{k+2}\cap K)\cap\mathcal{E}_t^1(G^{k+1}\cap K)\cap\cdots\cap\mathcal{E}_t^{k+1}(K)\cap\mathcal{E}_t^{k+2}(L^2)$.*

Remark 5. If u_0 and u_1 are in $C_0^\infty(\Omega)$ and $f(t, x)$ is in $C_0^\infty((0, T)\times\Omega)$, then $\{f, u_0, u_1\}$ satisfies the compatibility condition of order ∞. And then we see from Theorem 2 that the singularity of the solution of (1.1)-(1.4) is located in a neighborhood of L.

§ 2. Proof of Theorem 2.

In this section we prove Theorem 2 in the same way as [2] or [4] using Lemma 1 proved in § 3.

We introduce the space of C^2-valued functions as follows; $E^k=\{{}^t(u, v); u\in G^{k+2}, v\in G^{k+1}, a_2u\in F^k \text{ and } \{u, v\} \text{ satisfies } (B_2)\}$[7] $(k\geqq 0)$ with the norm;

$$\|{}^t(u, v)\|_{E^k}=(\|u\|_{G^{k+2}}^2+\|v\|_{G^{k+1}}^2+\|a_2u\|_{F^k}^2)^{1/2}.$$

Then E^k is a Hilbert space. And we consider the following bounded operator P from E^k to $(G^{k+1}\cap K)\times F^k$;

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[7] ${}^t(u, v)=\begin{bmatrix} u \\ v \end{bmatrix}$.

$$P=\begin{bmatrix} 0 & , & -1 \\ a_2-e_1+\lambda, & & a_1 \end{bmatrix}$$

where $e_1(x;\partial/\partial x)=\sum_i^l b_i(x)\partial/\partial x_i+c_2(x)$.

Proposition 1. *P is a one-to-one and onto mapping, if we take a sufficiently large number as λ.*

Lemma 1. *If for $g\in G^{s+1}\cap K$ and $f\in F^s$ the following equality (2. 1) holds, $u(x)$ in $K(\Omega)$ belongs to $G^{s+2}(\Omega)$.*

$$B[u,\varphi]=(f,\varphi)+\int_{\Gamma_2} g\bar{\varphi}dS \quad \text{for all} \quad \varphi\in K(\Omega), \tag{2. 1}$$

where $B[u,\varphi]=\sum(a_{ij}(\partial u/\partial x_j),(\partial\varphi/\partial x_i))+(u,\varphi)$.

The proof of Lemma 1 is given in § 3.

Proof of Proposition 1.

For any given $g\in G^{k+1}$ and $f\in F^k$, we consider the equation;

$$PU=F \quad \text{i.e.} \quad \begin{cases} -v=g \\ (a_2-e_1+\lambda)u+a_1v=f, \end{cases} \tag{2. 2}$$

where $U=\begin{bmatrix} u \\ v \end{bmatrix}$ and $F=\begin{bmatrix} g \\ f \end{bmatrix}$. By (2. 2) and (1. 8) we have

$$\sum\left(a_{ij}\frac{\partial u}{\partial x_j},\frac{\partial\varphi}{\partial x_i}\right)+\lambda(u,\varphi)+\int_{\Gamma_2}\sigma u\bar{\varphi}dS=(a_1g+f,\varphi) - \int_{\Gamma_2}\langle h,\gamma\rangle g\bar{\varphi}dS \quad \text{for any} \quad \varphi\in K(\Omega). \tag{2. 3}$$

If λ is sufficiently large, then using Lax-Milgram's theorem, we see that there exists uniquely a function $u(x)$ in $K(\Omega)$ which satisfies (2. 3). Thus P is a one-to-one mapping. By (2. 3) we have

$$B[u,\varphi]=(a_1g+f+u-\lambda u,\varphi)+\int_{\Gamma_2}(-\sigma u-\langle h,\gamma\rangle g)\bar{\varphi}dS. \tag{2. 4}$$

Since $a_1g+f+u-\lambda u$ is in F^0 and $-\sigma u-\langle h,\gamma\rangle g$ is in G^1, using Lemma 1 ($s=0$), we see that u belongs to G^2. Therefore $a_1g+f+u-\lambda u$ is in F^1, $-\sigma u-\langle h,\gamma\rangle g$ is in G^2, and by Lemma 1 ($s=1$) u belongs to G^3. Repeatedly we see that $u\in G^{k+2}$. (2. 3) holds for every $\psi\in C_0^\infty(\Omega)$. Therefore setting $v=-g$, we see that ${}^t(u, v)$ satisfies (2. 2). By (2. 2) and (2. 3) $\{u, v\}$ satisfies (B_2). Thus P is an onto mapping. (Q.E.D.)

Proof of Theorem 2. By Proposition 1 and the closed graph theorem of Banach, there exists an inverse operator of P which is continuous. Therefore we have

$$\|u\|_{G^{k+2}}\leqq C_3(\|v\|_{G^{k+1}}+\|a_2u-e_1u+\lambda u+a_1v\|_{F^k}) \tag{2. 5}$$

$$\text{for every}\quad U={}^t(u, v)\in E^k.$$

It follows from (2. 5) that

$$\|u\|_{G^{k+2}}\leqq C_4(\|u\|_{G^{k+1}}+\|v\|_{G^{k+1}}+\|a_2u+a_1v\|_{F^k}) \tag{2. 6}$$

$$\text{for any}\quad U={}^t(u, v)\in E^k\quad (k\geqq 0).$$

Let $u(t,x)$ be a solution of (1. 1) and (1. 2) in $\mathcal{E}_t^1(K)\cap\mathcal{E}_t^2(L^2)$ which satisfies (B_1). Its existence is guaranteed by Corollary of Theorem 1. From (1. 8) and (1. 1) it follows that

$$\begin{aligned} B[u,\varphi] &= (a_2u-e_1u+u,\varphi)+\int_{\Gamma_2}\left(\langle h,\gamma\rangle\frac{\partial u}{\partial t}-\sigma u\right)\bar{\varphi}dS \\ &= \left(f-a_1\frac{\partial u}{\partial t}-\frac{\partial^2 u}{\partial t^2}-e_1u+u,\varphi\right) \\ &\quad+\int_{\Gamma_2}\left(\langle h,\gamma\rangle\frac{\partial u}{\partial t}-\sigma u\right)\bar{\varphi}dS\quad\text{for any}\quad\varphi\in K(\Omega). \end{aligned} \tag{2. 7}$$

Since $(f-a_1(\partial u/\partial t)-\partial^2u/\partial t^2-e_1u+u)$ is in $F^0=L^2$ and $(\langle h,\gamma\rangle\partial u/\partial t-\sigma u)$ is in $K(\Omega)$, we see that $u(t,x)$ belongs to G^2 from (2. 7) and Lemma 1 $(s=0)$. Therefore ${}^t(u,\partial u/\partial t)$ is in E^0. From (2. 6) $(k=0)$ it follows that $u(t,x)$ is in $\mathcal{E}_t^0(G^2\cap K)$ since $u(t,x)$ is in $\mathcal{E}_t^1(K)$ and $a_2u+a_1(\partial u/\partial t)=f(t,x)-\partial^2u/\partial t^2$ is in $\mathcal{E}_t^0(L^2)$.

Now we consider an equation;

$$\begin{cases} \dfrac{\partial^2 v_1}{\partial t^2}+a_1\dfrac{\partial v_1}{\partial t}+a_2v_1=f'(t,x) \\ v_1(0,x)=u_1(x) \\ \dfrac{\partial v_1}{\partial t}(0,x)=\dfrac{\partial^2 u}{\partial t^2}(0,x)=f(0,x)-a_1u_1(x)-a_2u_0(x)=u_2(x). \end{cases} \tag{2. 8}$$

By applying Corollary of Theorem 1 to (2.8), we see that there exists a solution $v_1(t,x)$ in $\mathcal{E}_t^1(K)\cap\mathcal{E}_t^2(L^2)$. From the above argument it follows that $v_1(t,x)$ belongs to $\mathcal{E}_t^0(G^2\cap K)$. Let us set

$$v(t,x)=u_0(x)+\int_0^t v_1(s,x)\,ds, \tag{2.9}$$

then $v(t,x)$ is nothing but $u(t,x)$. In fact we get from (2.8) and (2.9)

$$\begin{cases} \dfrac{\partial}{\partial t}\left\{\dfrac{\partial^2 v}{\partial t^2}+a_1\dfrac{\partial v}{\partial t}+a_2 v-f\right\}=0 \\ \left(\dfrac{\partial^2 v}{\partial t^2}+a_1\dfrac{\partial v}{\partial t}+a_2 v-f\right)\Big|_{t=0}=0 \\ v(0,x)=u_0(x) \\ \dfrac{\partial v}{\partial t}(0,x)=v_1(0,x)=u_1(x)\ . \end{cases} \tag{2.10}$$

From (2.10) and the uniqueness of solutions of (1.1), (1.2) and (B_1) in $\mathcal{E}_t^1(K)\cap\mathcal{E}_t^2(L^2)$, we see that $v(t,x)=u(t,x)$. Therefore $u(t,x)$ belongs to $\mathcal{E}_t^3(L^2)\cap\mathcal{E}_t^2(K)\cap\mathcal{E}_t^1(G^2\cap K)$. In (2.7) $(f-a_1(\partial u/\partial t)-\partial^2u/\partial t^2-e_1u+u)$ is in F^1 and $(\langle h,\gamma\rangle\partial u/\partial t-\sigma u)$ is in $G^2\cap K$. By using Lemma 1 $(s=1)$, we see that $u(t,x)$ belongs to G^3. Therefore ${}^t(u,\partial u/\partial t)$ is in E^1. Since $u(t,x)$ is in $\mathcal{E}_t^1(G^2\cap K)$ and $a_2u+a_1(\partial u/\partial t)=f(t,x)-\partial^2u/\partial t^2$ is in $\mathcal{E}_t^0(F^1)$, it follows from (2.6) $(k=1)$ that $u(t,x)$ belongs to $\mathcal{E}_t^0(G^3\cap K)$. Repeating this argument, finally we see that $u(t,x)$ belongs to $\mathcal{E}_t^0(G^{k+2}\cap K)\cap\mathcal{E}_t^1(G^{k+1}\cap K)\cap\cdots\cap\mathcal{E}_t^{k+1}(K)\cap\mathcal{E}_t^{k+2}(L^2)$. Theorem 2 has been proved. (Q.E.D.)

§3. Proof of Lemma 1.

By the assumption on Ω, S, Γ_1, Γ_2 and L, there exists an open covering $\{V_k\}_{k=1,\dots,N}$ of $\bar{\Omega}$ such that

(1) for $k=1,2,\cdots,N_1$, $V_k\cap L\neq\phi$ and there exists a regular C^∞-mapping $y=\varphi(x)$ from V_k into R_y^n satisfying (case 1) or (case 2) or (case 3) in § 1,

(2) for $k=N_1+1$, $N_1+2,\cdots,N-1$, $V_k\cap S\neq\phi$ and $\bar{V}_k\cap L=\phi$, and

(3) $\bar{V}_N \cap S = \phi$.

Let $\alpha_k(x)$ ($\geqq 0$) be in $C_0^\infty(V_k)$ such that $\sum_{k=1}^N \alpha_k \equiv 1$ in $\bar{\Omega}$. We get

$$(3.1)\qquad \left(a_{ij}\frac{\partial(\alpha_k u)}{\partial x_j}, \frac{\partial\varphi}{\partial x_i}\right) = \left(a_{ij}\frac{\partial u}{\partial x_j}, \frac{\partial(\alpha_k\varphi)}{\partial x_i}\right)$$

$$+\left(a_{ij}\frac{\partial\alpha_k}{\partial x_j}u, \frac{\partial\varphi}{\partial x_i}\right) - \left(a_{ij}\frac{\partial\alpha_k}{\partial x_i}\frac{\partial u}{\partial x_j}, \varphi\right)$$

and therefore from (2.1) it follows that

$$(3.2)\qquad B[\alpha_k u, \varphi] = (\alpha_k f, \varphi) + C_k[u, \varphi] + \int_{\Gamma_2} \alpha_k g\bar{\varphi}\, dS$$

$$\text{for any } \varphi \in K(\Omega)$$

where

$$(3.3)\qquad C_k[u, \varphi] = \sum\left\{\left(a_{ij}\frac{\partial\alpha_k}{\partial x_j}u, \frac{\partial\varphi}{\partial x_i}\right) - \left(a_{ij}\frac{\partial\alpha_k}{\partial x_i}\frac{\partial u}{\partial x_j}, \varphi\right)\right\}.$$

For $k \geqq N_1+1$, since $\alpha_k f$ is in H^s, $\alpha_k g$ is in H^{s+1} and supp $[\alpha_k u] \cap L = \phi$, we see that $\alpha_k u$ belongs to $H^{s+2}(\subset G^{s+2})$ by the well-known method (see [5] Chap. III). So we have only to verify that $\alpha_k u$ belongs to G^{s+2} for $k \leqq N_1$.

Now let us suppose that there exists a neighborhood W_k of the origin in R_y^n such that $V_k \cap \Omega$, $V_k \cap \Gamma_1$ and $V_k \cap \Gamma_2$ are mapped diffeomorphically onto $W_k \cap R_+^n$, $W_k \cap \{y_1 = 0, y_2 > 0\}$, and $W_k \cap \{y_1 = 0, y_2 < 0\}$ respectively. (In the other cases we can prove in a similar way.) From now on we omit the suffix k. Then we have

$$(3.4)\qquad C_5 \cdot r \leqq |y'| = \sqrt{y_1^2 + y_2^2} \leqq C_6 \cdot r.$$

Once more we change independent variables from $y = (y_1, y_2, y'')$ to (θ, τ, ω);

$$(3.5)\qquad \begin{cases} y_1 = e^{-\tau}\sin\theta \\ y_2 = e^{-\tau}\cos\theta \\ y_j = \omega_j \quad (3 \leqq j \leqq n) \end{cases}$$

We obtain the following rules of calculus;

$$(3.6)\qquad \begin{cases} \dfrac{\partial}{\partial\theta}=e^{-\tau}\left(\cos\theta\dfrac{\partial}{\partial y_1}-\sin\theta\dfrac{\partial}{\partial y_2}\right) \\ \dfrac{\partial}{\partial\tau}=-e^{-\tau}\left(\sin\theta\dfrac{\partial}{\partial y_1}+\cos\theta\dfrac{\partial}{\partial y_2}\right) \\ \dfrac{\partial}{\partial\omega_j}=\dfrac{\partial}{\partial y_j}\quad (3\leqq j\leqq n), \end{cases}$$

i.e.

$$\begin{cases} \dfrac{\partial}{\partial y_1}=e^{\tau}\left(\cos\theta\dfrac{\partial}{\partial\theta}-\sin\theta\dfrac{\partial}{\partial\tau}\right) \\ \dfrac{\partial}{\partial y_2}=-e^{\tau}\left(\sin\theta\dfrac{\partial}{\partial\theta}+\cos\theta\dfrac{\partial}{\partial\tau}\right) \\ \dfrac{\partial}{\partial y_j}=\dfrac{\partial}{\partial\omega_j}\quad (3\leqq j\leqq n), \end{cases}$$

$$(3.7)\qquad \left|\frac{\partial(x_1,x_2,\cdots,x_n)}{\partial(\theta,\tau,\omega)}\right|=e^{-2\tau}\times J\quad\text{where}\quad J=\left|\frac{\partial(x_1,x_2,\cdots,x_n)}{\partial(y_1,y_2,\cdots,y_n)}\right|.$$

By the above change of variables, we have

$$(3.8)\qquad B[u,\varphi]=\mathscr{B}[u,\varphi]\equiv\sum\langle e^{-2\tau}\tilde{a}_{ij}E_ju,E_i\varphi\rangle+\langle e^{-2\tau}Ju,\varphi\rangle,^{8)}$$

where $\tilde{a}_{ij}=J\times a_{ij}$ and $E_i=\dfrac{\partial}{\partial x_i}$.

Then it follows from (3.2) and (3.3) that

$$(3.9)\qquad \mathscr{B}[\alpha u,\varphi]=\langle e^{-2\tau}J\alpha f,\varphi\rangle+\mathscr{C}[u,\varphi]+\iint_{\theta=\pi}e^{-\tau}\rho\alpha g\bar{\varphi}d\tau d\omega$$

where

$$(3.10)\qquad \mathscr{C}[u,\varphi]=\sum\{\langle e^{-2\tau}\tilde{a}_{ij}(E_j\alpha)u,E_i\varphi\rangle-\langle e^{-2\tau}\tilde{a}_{ij}(E_j\alpha)E_iu,\varphi\rangle\}$$

and $dS=\rho(\tau,\omega)d\tau d\omega$ on S.

Set

8) "$u(\theta,\tau,\omega)\in L^2_{\theta,\tau,\omega}$" means that $u(\theta,\tau,\omega)$ is a square integrable with respect to the usual Lebesgue measure $d\theta d\tau d\omega$, and its inner product is denoted by $\langle\ ,\ \rangle$.

$$(3.11)\quad \begin{cases} \Delta_\tau g(\theta,\tau,\omega)=\dfrac{1}{h}\{g(\theta,\tau+h,\omega)-g(\theta,\tau,\omega)\}, \\ \Delta_{-\tau} g(\theta,\tau,\omega)=\dfrac{1}{-h}\{g(\theta,\tau-h,\omega)-g(\theta,\tau,\omega)\}, \\ \Delta_j g(\theta,\tau,\omega)=\dfrac{1}{h}\{g(\theta,\tau,\omega+h_j)-g(\theta,\tau,\omega)\}, \\ \Delta_{-j}(\theta,\tau,\omega)=\dfrac{1}{-h}\{g(\theta,\tau,\omega-h_j)-g(\theta,\tau,\omega)\}, \end{cases}$$

where $h_j=(\overset{3}{\check{0}},\cdots,\overset{j}{\check{0}},h,0,\cdots 0)$ $(3\leqq j\leqq n)$.

We can suppose that $\alpha\equiv 0$ in $\tau<-M$. (M is a large positive constant.) Since $\Delta_\tau\varphi$ belongs to $K(\Omega)$ for $\varphi\in K(\Omega)$, it follows from (3.9) that

$$(3.12)\quad \mathscr{B}[\alpha u,\Delta_\tau\varphi]=\langle e^{-2\tau}J\alpha f,\Delta_\tau\varphi\rangle+\mathscr{C}[u,\Delta_\tau\varphi] + \iint_{\theta=\pi} e^{-\tau}\rho\alpha g\Delta_\tau\bar{\varphi}d\tau d\omega.$$

On the other hand we have

$$(3.13)\quad \mathscr{B}[\alpha u,\Delta_\tau\varphi]=-\mathscr{B}[\Delta_{-\tau}(\alpha u),\varphi]+\mathscr{G}_h[u,\varphi]$$

where

$$(3.14)\quad \mathscr{G}_h[u,\varphi]=-\sum\{\langle\{\Delta_{-\tau}(e^{-2\tau}\tilde{a}_{ij}E_j)\}(\alpha u)(\tau-h),E_i\varphi\rangle + \langle e^{-2\tau}\tilde{a}_{ij}(\alpha u),(\Delta_\tau E_i)\varphi(\tau+h)\rangle\}.$$

By (3.13) and (3.14), we get

$$(3.15)\quad \mathscr{B}[\Delta_{-\tau}(\alpha u),\varphi]=-\langle e^{-2\tau}J\alpha f,\Delta_\tau\varphi\rangle - \iint_{\theta=\pi} e^{-\tau}\rho\alpha g\Delta_\tau\bar{\varphi}d\tau d\omega-\mathscr{C}[u,\Delta_\tau\varphi]+\mathscr{G}_h[u,\varphi].$$

Now $f\in F^0=L^2(\Omega)$, $g\in G^1$ and $u,\varphi\in K(\Omega)$, so by (3.6) and (3.7) each term of the right side of (3.15) convergers when h tends to $+0$.

There $\Delta_{-\tau}(\alpha u)$ weakly converges in $K(\Omega)$, because $B[u,\varphi]$ is a positive definite Hermitian form equivalent to the inner product of $K(\Omega)$. In the other view

$$\Delta_{-\tau}(\alpha u) \to \frac{\partial}{\partial \tau}(\alpha u) \quad \text{in} \quad \mathcal{D}'(\Omega),$$

so we see that $\partial(\alpha u)/\partial\tau$ belongs to $K(\Omega)$. Further if $f \in F^1(\Omega)$ and $g \in G^2(\Omega)$, it follows from (3.15), (3.10) and (3.14) that

$$(3.16) \qquad \mathcal{B}\left[\frac{\partial}{\partial\tau}(\alpha u), \varphi\right] = \left\langle \frac{\partial}{\partial\tau}(e^{-2\tau}J\alpha f), \varphi \right\rangle + \iint_{\theta=\pi} \left\{\frac{\partial}{\partial\tau}(e^{-\tau}\rho\alpha g)\right\} \overline{\varphi} d\tau d\omega - \mathscr{C}_1[u, \varphi] + \mathcal{G}_1[u, \varphi],$$

where

$$\mathscr{C}_1[u, \varphi] = \sum \left\{ -\left\langle \frac{\partial}{\partial\tau}(e^{-2\tau}\tilde{a}_{ij}(E_j\alpha)u), E_i\varphi \right\rangle - \left\langle e^{-2\tau}\tilde{a}_{ij}(E_j\alpha)u, \frac{\partial E_i}{\partial\tau}\varphi \right\rangle - \left\langle e^{-2\tau}\tilde{a}_{ij}(E_j\alpha)E_iu, \frac{\partial}{\partial\tau}\varphi \right\rangle \right\}$$

and

$$\mathcal{G}_1[u, \varphi] = -\sum \left\{ \left\langle \left\{ \frac{\partial}{\partial\tau}(e^{-2\tau}\tilde{a}_{ij}E_j) \right\}(\alpha u), E_i\varphi \right\rangle + \left\langle e^{-2\tau}\tilde{a}_{ij}E_j(\alpha u), \frac{\partial E_i}{\partial\tau}\varphi \right\rangle \right\}.$$

In a similar way we get for $3 \leqq l \leqq n$

$$(3.17) \qquad \mathcal{B}[e^{-\tau}\Delta_{-l}(\alpha u), \varphi] = -\langle e^{-3\tau}J\alpha f, \Delta_l\varphi \rangle - \iint_{\theta=\pi} e^{-2\tau}\rho\alpha g \Delta_l\overline{\varphi} d\tau d\omega - \mathscr{C}[u, e^{-\tau}\Delta_l\varphi] + \mathcal{G}_{h,l}[u, \varphi],$$

where

$$\mathcal{G}_{h,l}[u, \varphi] = -\sum \{ \langle \{\Delta_{-l}(e^{-2\tau}\tilde{a}_{ij}E_j)\}(\alpha u)(\omega - h_l), E_i(e^{-\tau}\varphi) \rangle + \langle e^{-2\tau}\tilde{a}_{ij}E_j(\alpha u), (\Delta_l E_i)(e^{-\tau}\varphi(\omega - h_l)) \rangle \}.$$

From (3.17), we see that $e^{-\tau}(\partial(\alpha u)/\partial\omega_l)$ belongs to $K(\Omega)$. If $f \in F^1$ and $g \in G^2$, we have from (3.17)

$$(3.18) \qquad \mathcal{B}\left[e^{-\tau}\frac{\partial}{\partial\omega_l}(\alpha u), \varphi\right] = \left\langle e^{-3\tau}\frac{\partial}{\partial\omega_l}(J\alpha f), \varphi \right\rangle + \iint_{\theta=\pi} e^{-2\tau}\left\{\frac{\partial}{\partial\omega_l}(\rho\alpha g)\right\}\overline{\varphi} d\tau d\omega - \mathscr{C}_2[u, \varphi] + \mathcal{G}_2[u, \varphi],$$

where

$$\mathscr{C}_2[u,\varphi]=\sum\Big\{-\Big\langle e^{-2\tau}\frac{\partial}{\partial\omega_l}(\tilde{a}_{ij}(E_j\alpha)u),\,E_i(e^{-\tau}\varphi)\Big\rangle$$

$$-\Big\langle e^{-2\tau}\tilde{a}_{ij}(E_j\alpha)u,\,\frac{\partial E_i}{\partial\omega_l}(e^{-\tau}\varphi)\Big\rangle$$

$$-\Big\langle e^{-2\tau}\tilde{a}_{ij}(E_j\alpha)E_iu,\,\frac{\partial}{\partial\omega_l}(e^{-\tau}\varphi)\Big\rangle\Big\}$$

and

$$\mathscr{Q}_2[u,\varphi]=-\sum\Big\{\Big\langle\Big(\frac{\partial}{\partial\omega_l}(e^{-2\tau}\tilde{a}_{ij}E_j)\Big)(\alpha u),\,E_i(e^{-\tau}\varphi)\Big\rangle$$

$$+\Big\langle e^{-2\tau}\tilde{a}_{ij}E_j(\alpha u),\,\frac{\partial E_i}{\partial\omega_l}(e^{-\tau}\varphi)\Big\rangle\Big\}.$$

Until now we have taken $\alpha(x)=\alpha_k(x)$, but the above argument holds for every $\alpha(x)$ in $C_0^\infty(V_k)$.

Repeating this argument for (3.16) and (3.18), finally we have

$$\left(e^{-\tau}\frac{\partial}{\partial\omega}\right)^{\gamma}\left(\frac{\partial}{\partial\tau}\right)^{\beta_2}(\alpha u)\in K(\Omega)\quad\text{for}\quad\beta_2+|\gamma|\leqq s+1. \tag{3.19}$$

From (3.19) it follows that

$$\left(e^{-\tau}\frac{\partial}{\partial\omega}\right)^{\gamma}\left(\frac{\partial}{\partial\tau}\right)^{\beta_2+1}(\alpha u) \tag{3.20}$$

$$\text{and}\quad\left(e^{-\tau}\frac{\partial}{\partial\omega}\right)^{\gamma}\left(\frac{\partial}{\partial\theta}\right)\left(\frac{\partial}{\partial\tau}\right)^{\beta_2}(\alpha u)\in L^2_{\theta,\tau,\omega}$$

$$\text{for}\quad\beta_2+|\gamma|\leqq s+1.$$

Let φ be in $C_0^\infty(V_k\cap\Omega)$ in (3.9), then we have from (3.6)

$$\frac{\partial^2}{\partial\theta^2}(\alpha u)=d_1\frac{\partial^2}{\partial\theta\partial\tau}(\alpha u)+d_2\frac{\partial^2}{\partial\tau^2}(\alpha u) \tag{3.21}$$

$$+e^{-\tau}\sum_{|\gamma|\leqq1}\Big\{d_{1,\gamma}\left(\frac{\partial}{\partial\omega}\right)^{\gamma}\frac{\partial}{\partial\theta}u+d_{2,\gamma}\left(\frac{\partial}{\partial\omega}\right)^{\gamma}\frac{\partial}{\partial\tau}u\Big\}$$

$$+e^{-2\tau}\sum_{|\gamma|\leqq2}d_\gamma\left(\frac{\partial}{\partial\omega}\right)^{\gamma}u+e^{-2\tau}d_3\alpha f,$$

where $d_1, d_2, d_{1,\gamma}, d_{2,\gamma}, d_\gamma$ and d_3 are C^∞-functions and their all partial derivatives of any order are bounded in (θ, τ, ω)-space, and $d_{1,\gamma}, d_{2,\gamma}$ and d_γ have $\alpha(x)$ or its derivative as a factor. Applying the operator $(e^{-\tau}(\partial/\partial\omega))^\gamma(\partial/\partial\tau)^{\beta_2}$ to both members of (3. 21), we see from (3. 20) that

$$\left(e^{-\tau}\frac{\partial}{\partial\omega}\right)^\gamma\left(\frac{\partial}{\partial\theta}\right)^2\left(\frac{\partial}{\partial\tau}\right)^{\beta_2}(\alpha u) \in L^\theta_{2,\tau,\omega} \quad \text{for} \quad \beta_2+|\gamma|\leqq s. \tag{3. 22}$$

Again applying the operator $(e^{-\tau}(\partial/\partial\omega))^\gamma(\partial/\partial\theta)(\partial/\partial\tau)^{\beta_2}$ $(\beta_2+|\gamma|\leqq s-1)$ to both members of (3. 21), we have

$$\left(e^{-\tau}\frac{\partial}{\partial\omega}\right)^\gamma\left(\frac{\partial}{\partial\theta}\right)^3\left(\frac{\partial}{\partial\tau}\right)^{\beta_2}(\alpha u) \in L^2_{\theta,\tau,\omega} \quad \text{for} \quad \beta_2+|\gamma|\leqq s-1. \tag{3. 23}$$

Repeating this, finally we get

$$\left(e^{-\tau}\frac{\partial}{\partial\omega}\right)^\gamma\left(\frac{\partial}{\partial\theta}\right)^{\beta_1}\left(\frac{\partial}{\partial\tau}\right)^{\beta_2}(\alpha u) \in L^2_{\theta,\tau,\omega} \quad \text{for} \quad |\beta|+|\gamma|\leqq s+2. \tag{3. 24}$$

Since $\alpha(x)=0$ in $\tau<-M$, it follows from (3. 24) that

$$\alpha u \in G^{s+2}(\Omega). \tag{3. 25}$$

Thus the proof of Lemma 1 has completed.

§ 4. Comments.

In this section we prove Lemma 4 (see K. Hayashida [1] Lemma 9) which is necessary for the proof of Theorem 1. At first we state the following two lemmas without proof.

Lemma 2. *For any $u(x)$ in $K(\Omega)$, there exists a sequence $\{\varphi_j(x)\}$ such that*

(1) *$\varphi_j(x)$ is in $C_0^\infty(\bar{\Omega})$ and vanishes in a neighborhood of $\overline{\Gamma_1 \cup L}$,*

(2) *$\varphi_j(x)\to u(x)$ in $H^1(\Omega)$ as $j\to\infty$.*

Considering that L is 1-polar set in Schwartz' sense, we can easily prove Lemma 2. (see L. Schwartz [8])

Lemma 3. (K. Hayashida [1] Lemma 7)

For $u(y)$ in $C_0^\infty(\overline{R_+^n})$, there is a sequence $\varphi_j(y)$ in $C_0^\infty(\overline{R_+^n})$ such that

(1) $\varphi_j(y) \to u(y)$ *in* $H^1(R_+^n)$ *as* $j \to \infty$,

(2) $(\partial/\partial y_1 + \sigma(y_2, y''))\varphi_j = 0$ *on* $y_1 = 0$,

(3) *if* $u(y_1, y_2, y'') = 0$ *for the fixed* (y_2, y'') *and any* y_1, *then each* $\varphi_j(y)$ *also vanishes for the* (y_2, y'') *and* y_1.

We set

$$D(A) = \Big\{U = {}^t(u, v)\,;\, u, v \in H^1(\Omega),\ a_2\Big(x, \frac{\partial}{\partial x}\Big)u \in L^2(\Omega), \tag{4.1}$$
$$\text{and } U \text{ satisfies } (B_2)\Big\}.$$

Lemma 4. *$D(A)$ is dense in $K(\Omega) \times L^2(\Omega)$.*

Proof. Let ${}^t(u, v)$ be in $K(\Omega) \times L^2(\Omega)$, then there exists a sequence $\{{}^t(u_j, v_j)\}$ converging to ${}^t(u, v)$ in $K(\Omega) \times L^2(\Omega)$ such that each v_j is in $C_0^\infty(\Omega)$ and u_j belongs to $C_0^\infty(\bar{\Omega})$ which vanishes in a neighborhood of $\overline{\Gamma_1 \cup L}$. So we can suppose that u belongs to $C_0^\infty(\bar{\Omega})$ and vanishes in a neighborhood of $\overline{\Gamma_1 \cup L}$ and v is in $C_0^\infty(\Omega)$. For u there exists an open covering $\{V_k\}_{k=1}^N$ of $\bar{\Omega}$ different from that in § 3 such that

(1) for $1 \leqq k \leqq N_1$, $\bar{V}_k \cap L = \phi$, $\bar{V}_k \cap \Gamma_1 = \phi$, $\overline{V_k \cap \Omega}$ can be mapped in a one-to-one C^∞ way into $\bar{R}_+^n$, and $\partial/\partial n$ is transformed into $\partial/\partial y_1$,

(2) for $N_1 + 1 \leqq k \leqq N_2$ $u = 0$ in V_k,

and

(3) for $N_2 + 1 \leqq k \leqq N$ $\bar{V}_k \subset \Omega$.

Let $\{\alpha_k\}$ be the partition of unity of class C^∞ corresponding to $\{V_k\}$. Applying Lemma 3 for $\alpha_k u$ on $\overline{R_+^n}$, we can find a sequence $\{\varphi_j^{(k)}\}$ $(1 \leqq k \leqq N_1)$ such that

$$\varphi_j^{(k)} = 0 \text{ in a neighborhood of } \overline{\Gamma_1 \cup L}, \tag{4.2}$$

$$\Big(\frac{\partial}{\partial n} + \sigma(x)\Big)\varphi_j^{(k)} = 0 \quad \text{on} \quad \Gamma_2 \tag{4.3}$$

and

$$\varphi_j^{(k)} \to \alpha_k u \quad \text{in} \quad H^1(\Omega) \quad \text{as} \quad j \to \infty. \tag{4.4}$$

Let us set

$$\varphi_j = \sum_{k=1}^{N_1} \varphi_j^{(k)} + \sum_{k=N_1+1}^{N} \alpha_k u, \tag{4.5}$$

then $\varphi_j \to u$ in $H^1(\Omega)$. Setting $v_j \equiv v$, ${}^t(\varphi_j, v_j)$ is in $D(A)$ because for ${}^t(u, v) \in C_0^\infty(\bar{\Omega}) \times C_0^\infty(\bar{\Omega})$ (1.7) and (1.8) are equivalent to the condition;

$$u(x) = v(x) = 0 \quad \text{on} \quad \Gamma_1, \tag{4.6}$$

and

$$\frac{\partial}{\partial n} u - \langle h, \gamma \rangle u + \sigma(x) u = 0 \text{ on } \Gamma_2, \text{ respectively.} \tag{4.7}$$

And ${}^t(\varphi_j, v_j) \to {}^t(u, v)$ in $K(\Omega) \times L^2(\Omega)$. Therefore $D(A)$ is dense in $K(\Omega) \times L^2(\Omega)$. (Q.E.D.)

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