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Two-phase free boundary problem for compressible viscous fluid motion

By

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§1. Introduction

Many arguments have been presented under the title of a "Stefan" problem or a free boundary problem. However, as seen from a real physical point of view, so far as the author knows, all of them seem to be unsatisfactory in at least two points, for it is natural and plausible that

(1) the movement necessarily accompanies heat change and vice versa

and that

(2) the movement of one fluid acts upon those of others.

Taking into account these, we consider a free boundary problem arising from the movement of a finite number of nonmiscible compressible viscous isotropic Newtonian fluids. We may give, as somewhat idealized physical examples of this problem, the movement of water wave adjacent to the atmosphere, that of gas bubbles in liquid and so on. In the present paper, for simplicity, we discuss the above problem in the case of there being only two fluids of such as mentioned above.

Let Ω [resp. Ω^{I}] be a bounded or unbounded domain in $\boldsymbol{R}^3$ [resp. Ω] with a boundary Γ [resp. Γ^{I}]; Γ and Γ^{I} are assumed to have no points in common; the exterior boundary Γ is assumed to be fixed. We denote the domain of the fluid at the time t which initially lies in Ω^{I} [resp. $\Omega^{\mathrm{II}}\equiv\Omega-\overline{\Omega^{\mathrm{I}}}$] by Ω_t^{I} [resp. Ω_t^{II}].

Then our aim in this problem is to find the domain Ω_t^{I}, Ω_t^{II}, and the function $(\rho^{\mathrm{I}}, v^{\mathrm{I}}, \theta^{\mathrm{I}}, \rho^{\mathrm{II}}, v^{\mathrm{II}}, \theta^{\mathrm{II}})(x, t)$ satisfying the equations

$$(1.1)^{\mathrm{I}}\quad \begin{cases} \left[\dfrac{D}{Dt}\right]_{\mathrm{I}}\rho^{\mathrm{I}}=-\rho^{\mathrm{I}}\nabla\cdot v^{\mathrm{I}}, \\ \left[\dfrac{D}{Dt}\right]_{\mathrm{I}}v_k^{\mathrm{I}}=\dfrac{1}{\rho^{\mathrm{I}}}\nabla_k[\mu'^{\mathrm{I}}(\nabla\cdot v^{\mathrm{I}})]+\dfrac{1}{\rho^{\mathrm{I}}}\nabla_l[\mu^{\mathrm{I}}(\nabla_l v_k^{\mathrm{I}}+\nabla_k v_l^{\mathrm{I}})]- \\ \qquad -\dfrac{1}{\rho^{\mathrm{I}}}\nabla_k p^{\mathrm{I}}+f_k^{\mathrm{I}} \qquad (k=1, 2, 3), \\ \left[\dfrac{D}{Dt}\right]_{\mathrm{I}}S^{\mathrm{I}}=\dfrac{1}{\rho^{\mathrm{I}}\theta^{\mathrm{I}}}\nabla\cdot(\kappa^{\mathrm{I}}\nabla\theta^{\mathrm{I}})+\dfrac{\mu^{\mathrm{I}}}{2\rho^{\mathrm{I}}\theta^{\mathrm{I}}}(\nabla_k v_l^{\mathrm{I}}+\nabla_l v_k^{\mathrm{I}})^2+\dfrac{\mu'^{\mathrm{I}}}{\rho^{\mathrm{I}}\theta^{\mathrm{I}}}(\nabla\cdot v^{\mathrm{I}})^2, \\ \qquad \text{in} \quad \mathscr{D}_T^{\mathrm{I}}\equiv\{(x, t)\in\boldsymbol{R}^4 | x\in\Omega_t^{\mathrm{I}}, \quad t\in(0, T]\}, \end{cases}$$

$$(1.1)^{\rm II}\quad \begin{cases} \left[\dfrac{D}{Dt}\right]_{\rm II}\rho^{\rm II}=-\rho^{\rm II}\nabla\cdot v^{\rm II}, \\ \left[\dfrac{D}{Dt}\right]_{\rm II}v_k^{\rm II}=\dfrac{1}{\rho^{\rm II}}\nabla_k[\mu'^{\rm II}(\nabla\cdot v^{\rm II})]+\dfrac{1}{\rho^{\rm II}}\nabla_l[\mu^{\rm II}(\nabla_l v_k^{\rm II}+\nabla_k v_l^{\rm II})]- \\ \qquad -\dfrac{1}{\rho^{\rm II}}\nabla_k p^{\rm II}+f_k^{\rm II}\quad (k=1,2,3), \\ \left[\dfrac{D}{Dt}\right]_{\rm II}S^{\rm II}=\dfrac{1}{\rho^{\rm II}\theta^{\rm II}}\nabla\cdot(\kappa^{\rm II}\nabla\theta^{\rm II})+\dfrac{\mu^{\rm II}}{2\rho^{\rm II}\theta^{\rm II}}(\nabla_k v_l^{\rm II}+\nabla_l v_k^{\rm II})^2+\dfrac{\mu'^{\rm II}}{\rho^{\rm II}\theta^{\rm II}}(\nabla\cdot v^{\rm II})^2, \\ \qquad \text{in}\ \ \mathscr{D}_T^{\rm II}\equiv\{(x,t)\in\boldsymbol{R}^4|x\in\Omega_t^{\rm II},\ \ t\in(0,T]\}, \end{cases}$$

and the initial-boundary conditions

$$(1.2)\quad \begin{cases} (\rho^{\rm I}, v^{\rm I}, \theta^{\rm I})_{|t=0}=(\rho_0^{\rm I}, v_0^{\rm I}, \theta_0^{\rm I})(x) & (x\in\Omega^{\rm I}), \\ (\rho^{\rm II}, v^{\rm II}, \theta^{\rm II})_{|t=0}=(\rho_0^{\rm II}, v_0^{\rm II}, \theta_0^{\rm II})(x) & (x\in\Omega^{\rm II}), \end{cases}$$

$$(1.3)\quad \begin{cases} v^{\rm I}=v^{\rm II},\ P^{\rm I}n(x,t)=P^{\rm II}n(x,t), \\ \theta^{\rm I}=\theta^{\rm II},\ (\kappa^{\rm I}\nabla\theta^{\rm I})\cdot n(x,t)=(\kappa^{\rm II}\nabla\theta^{\rm II})\cdot n(x,t), \end{cases}\quad \text{on}\ \ \partial\Omega_t^{\rm I},$$

$$(1.4)\quad v^{\rm II}=0,\ \theta^{\rm II}=\theta_e^{\rm II}\quad \text{on}\quad \Gamma_T\equiv\Gamma\times[0,T]$$

[$\partial\Omega_t^{\rm I}$, boundary of $\Omega_t^{\rm I}$; ρ^j, density; $v^j=(v_1^j, v_2^j, v_3^j)$, velocity; θ^j, absolute temperature; $\mu^j(\rho^j,\theta^j)$, coefficient of viscosity; $\mu'^j(\rho^j,\theta^j)$, second coefficient of viscosity; $\kappa^j(\rho^j,\theta^j)$, coefficient of heat conductivity; $p^j(\rho^j,\theta^j)$, pressure; $S^j(\rho^j,\theta^j)$, entropy; $\left[\dfrac{D}{Dt}\right]_j=\dfrac{\partial}{\partial t}+v^j\cdot\nabla$; $P^j=[-p^j+\mu'^j(\nabla\cdot v^j)]\mathrm{I}_3+\mu^j E^j$, $E^j=(\nabla_k v_l^j+\nabla_l v_k^j)$; I_n, $n\times n$ identity matrix; $n(x,t)$, unit normal vector at $x\in\partial\Omega_t^{\rm I}$ pointing into the interior of Ω_t^j $(j={\rm I, II})$].

We use the summation convention throughout this paper. And we assume that the compatibility conditions are always valid although we do not write them down.

Besides the boundary conditions (1.3) and (1.4), we suppose that the boundary surface $\partial\Omega_t^{\rm I}$ consists of one and the same fluid particles at any moment, which is formulated by

$$(1.5)\qquad \left[\frac{D}{Dt}\right]_{\rm I}F^{\rm I}=0\quad \text{on}\quad \partial\Omega_t^{\rm I},$$

if $\partial\Omega_t^{\rm I}$ is represented by the equation $F^{\rm I}(x,t)=0$. Of course, on the fixed exterior boundary Γ, an equation analogous to (1.5) holds.

As in [3, 4], we use the characteristic transformation $\Pi_{x_0,t_0}^{x,t}$: $(x,t)\mapsto(x_0(x,t), t_0=t)$ from $\overline{D_T^{\rm I}}$ [resp. $\overline{D_T^{\rm II}}$] to $\overline{Q_T^{\rm I}}\equiv\overline{\Omega^{\rm I}}\times[0,T]$ [resp. $\overline{Q_T^{\rm II}}\equiv\overline{\Omega^{\rm II}}\times[0,T]$], especially $\Pi_{x_0,t_0}^{x,t}(\partial\Omega_t^{\rm I})=\Gamma^{\rm I}$, which is one-to-one and onto so far as $\Gamma\cap\partial\Omega_t^{\rm I}=\emptyset$. Here x_0 is related to x by the equation

$$x=x_0+\int_0^{t_0}\hat{v}^{\rm I}(x_0,\tau)d\tau\quad \text{or}\quad x=x_0+\int_0^{t_0}\hat{v}^{\rm II}(x_0,\tau)d\tau$$

according as x belongs to $\Omega_t^{\rm I}$ or $\Omega_t^{\rm II}$($(\hat{v}^{\rm I},\hat{v}^{\rm II})=\Pi_{x_0,t_0}^{x,t}(v^{\rm I},v^{\rm II})$).

By this transformation, (1.1)$\sim$(1.4) are rewritten as follows:

$$(1.6)^j \quad \begin{cases} \dfrac{\partial}{\partial t_0}\hat{\rho}^j = -\hat{\rho}^j(\Gamma_{\hat{v}^j}\cdot\hat{v}^j), \\ \dfrac{\partial}{\partial t_0}\hat{v}^j_k = \dfrac{1}{\hat{\rho}^j}\Gamma_{\hat{v}^j,k}[\mu'^j(\Gamma_{\hat{v}^j}\cdot\hat{v}^j)] + \dfrac{1}{\hat{\rho}^j}\Gamma_{\hat{v}^j,l}[\mu^j(\Gamma_{\hat{v}^j,l}\hat{v}^j_k + \Gamma_{\hat{v}^j,k}\hat{v}^j_l)] - \\ \qquad -\dfrac{1}{\hat{\rho}^j}\Gamma_{\hat{v}^j,k}p^j + \hat{f}^j_k \quad (k=1, 2, 3), \\ \dfrac{\partial}{\partial t_0}\hat{\theta}^j = \dfrac{1}{\hat{\rho}^j\hat{\theta}^j S^j_{\hat{\theta}^j}}\Gamma_{\hat{v}^j}\cdot(\kappa^j\Gamma_{\hat{v}^j}\hat{\theta}^j) + \dfrac{\mu^j}{2\hat{\rho}^j\hat{\theta}^j S^j_{\hat{\theta}^j}}(\Gamma_{\hat{v}^j,k}\hat{v}^j_l + \Gamma_{\hat{v}^j,l}\hat{v}^j_k)^2 + \\ \qquad + \dfrac{\mu'^j}{\hat{\rho}^j\hat{\theta}^j S^{jj}_{\hat{\theta}}}(\Gamma_{\hat{v}^j}\cdot\hat{v}^j)^2 + \dfrac{\hat{\rho}^j S^j_{\hat{\rho}^j}}{S^j_{\hat{\theta}^j}}(\Gamma_{\hat{v}^j}\cdot\hat{v}^j), \\ \qquad\qquad \text{in } Q^j_T \quad (j=\mathrm{I}, \mathrm{II}), \end{cases}$$

$$(1.7) \quad (\hat{\rho}^j, \hat{v}^j, \hat{\theta}^j)|_{t_0=0} = (\rho^j_0, v^j_0, \theta^j_0)(x_0)(x_0\in\Omega^j) \quad (j=\mathrm{I}, \mathrm{II}),$$

$$(1.8) \quad \begin{cases} \hat{v}^{\mathrm{I}} = \hat{v}^{\mathrm{II}}, \quad \dfrac{\hat{P}^{\mathrm{I}}\cdot\mathscr{G}^{\mathrm{I}}n(x_0)}{|\mathscr{G}^{\mathrm{I}}\mathring{\Gamma}F^{\mathrm{I}}_0|} = \dfrac{\hat{P}^{\mathrm{II}}\cdot\mathscr{G}^{\mathrm{II}}n(x_0)}{|\mathscr{G}^{\mathrm{II}}\mathring{\Gamma}F^{\mathrm{I}}_0|}, \quad \hat{\theta}^{\mathrm{I}} = \hat{\theta}^{\mathrm{II}}, \\ \dfrac{\kappa^{\mathrm{I}}\mathscr{G}^{\mathrm{I}}n(x_0)}{|\mathscr{G}^{\mathrm{I}}\mathring{\Gamma}F^{\mathrm{I}}_0|}\cdot(\Gamma_{\hat{v}^{\mathrm{I}}}\hat{\theta}^{\mathrm{I}}) = \dfrac{\kappa^{\mathrm{II}}\mathscr{G}^{\mathrm{II}}n(x_0)}{|\mathscr{G}^{\mathrm{II}}\mathring{\Gamma}F^{\mathrm{I}}_0|}\cdot(\Gamma_{\hat{v}^{\mathrm{II}}}\hat{\theta}^{\mathrm{II}}), \quad \text{on} \quad \Gamma^{\mathrm{I}}_T \equiv \Gamma^{\mathrm{I}}\times[0, T], \end{cases}$$

$$(1.9) \quad \hat{v}^{\mathrm{II}} = 0, \quad \hat{\theta}^{\mathrm{II}} = \theta_e \quad \text{on} \quad \Gamma_T$$

$[\hat{P}^j = [-p^j + \mu'^j(\Gamma_{\hat{v}^j}\cdot\hat{v}^j)]\mathrm{I}_3 + \mu^j\hat{E}^j, \quad \hat{E}^j = (\Gamma_{\hat{v}^j,k}\hat{v}^j_l + \Gamma_{\hat{v}^j,l}\hat{v}^j_k) \quad (j=\mathrm{I}, \mathrm{II});$

$F^{\mathrm{I}}_0(x_0)=0$ represents the boundary Γ^{I}; $n(x_0)$ in (1.8), unit normal vector at $x_0\in\Gamma^{\mathrm{I}}$ pointing into the interior of Ω^{I}; $\mathscr{G}^j = (g^j_{kl}) = (\partial x/\partial x_0)^{-1}|_{d^j_T}$, $\Gamma_{\hat{v}^j} = (\Gamma_{\hat{v}^j,1}, \Gamma_{\hat{v}^j,2}, \Gamma_{\hat{v}^j,3}) = \mathscr{G}^j\mathring{\Gamma}$, $\mathring{\Gamma} = (\mathring{\Gamma}_1, \mathring{\Gamma}_2, \mathring{\Gamma}_3)$, $\mathring{\Gamma}_k = \partial/\partial x_{0,k}$ $(j=\mathrm{I}, \mathrm{II})]$.

Remark. From (1.5), we derive the relation

$$\hat{F}^{\mathrm{I}}(x_0, t_0) = \Pi^{x,t}_{x_0,t_0}F^{\mathrm{I}}(x, t) = F^{\mathrm{I}}_0(x_0).$$

Since the first equation in $(1.6)^j$ is uniquely solved by

$$(1.10) \quad \hat{\rho}^j(x_0, t_0) = \rho^j_0(x_0)\cdot\exp\left[-\int_0^{t_0}\Gamma_{\hat{v}^j}\cdot\hat{v}^j(x_0, \tau)d\tau\right] \quad (j=\mathrm{I}, \mathrm{II})$$

if $\hat{v}^j\in C^{2+\alpha, 1+\alpha/2}_{x_0,t_0}(\overline{Q^j_T})$ be given, the problem (1.6)$\sim$(1.9) can be considered as the following initial-boundary value problem with respect to $w^j = {}^t(\hat{v}^j - v^j_0, \hat{\theta}^j - \theta^j_0)$:

$$(1.11) \quad \begin{cases} \dfrac{\partial}{\partial t_0}w^j = \mathscr{A}^j(x_0, t_0, w^j; \mathring{\Gamma})w^j + \mathscr{B}^j(x_0, t_0, w^j) \quad \text{in} \quad Q^j_T \quad (j=\mathrm{I}, \mathrm{II}), \\ \qquad (w^{\mathrm{I}}, w^{\mathrm{II}})|_{t_0=0} = (0, 0), \\ \begin{pmatrix} \mathrm{I}_3 \\ \dfrac{B^{\mathrm{I}}(x_0, t_0, w^{\mathrm{I}}; \mathring{\Gamma})}{|\mathscr{G}^{\mathrm{I}}\mathring{\Gamma}F^{\mathrm{I}}_0|} \end{pmatrix} w^{\mathrm{I}} - \begin{pmatrix} \mathrm{I}_3 \\ \dfrac{B^{\mathrm{II}}(x_0, t_0, w^{\mathrm{II}}; \mathring{\Gamma})}{|\mathscr{G}^{\mathrm{II}}\mathring{\Gamma}F^{\mathrm{I}}_0|} \end{pmatrix} w^{\mathrm{II}} = \end{cases}$$

$$
\left|\begin{array}{l}
=\begin{pmatrix} w_0^{\mathrm{II}} \\ \dfrac{\phi^{\mathrm{II}}(x_0, t_0, w^{\mathrm{II}})}{|\mathscr{G}^{\mathrm{II}}\mathring{\nabla} F_0^{\mathrm{I}}|} \end{pmatrix} - \begin{pmatrix} w_0^{\mathrm{I}} \\ \dfrac{\phi^{\mathrm{I}}(x_0, t_0, w^{\mathrm{I}})}{|\mathscr{G}^{\mathrm{I}}\mathring{\nabla} F_0^{\mathrm{I}}|} \end{pmatrix} \quad \text{on} \quad \Gamma_T^{\mathrm{I}}, \\
w^{\mathrm{II}} = (0, \hat{\theta}_e - \theta_0^{\mathrm{II}}) \quad \text{on} \quad \Gamma_T.
\end{array}\right.
$$

Here the principal term $\mathscr{A}^j$, the lower order term $\mathscr{B}^j$, and the boundary operator B^j ($j=\mathrm{I}$, II) are defined by A, B and B, respectively, in [3], §2.1 and [4], §1 with the corresponding replacements;

$$
\phi^j(x_0, t_0, w^j) = \begin{pmatrix} -p^j\mathscr{G}^j n(x_0) \\ 0 \end{pmatrix} + B^j(x_0, t_0, w^j; \mathring{\nabla}) w_0^j \quad \left(w_0^j = \begin{pmatrix} v_0^j \\ \theta_0^j \end{pmatrix} \right) \quad (j=\mathrm{I}, \mathrm{II}).
$$

Now we state our assumptions: for any $\alpha \in (0, 1)$,

$$
(1.12) \qquad \Gamma, \Gamma^{\mathrm{I}} \in C^{2+\alpha},
$$

$$
(1.13) \qquad (\rho_0^j, v_0^j, \theta_0^j) \in H^{1+\alpha}(\overline{\Omega^j}) \times H^{2+\alpha}(\overline{\Omega^j}) \times H^{2+\alpha}(\overline{\Omega^j}) \quad (j=\mathrm{I}, \mathrm{II})
$$

$$
(0 < \underline{\rho}_0^j \equiv \inf_{\overline{\Omega^j}} \rho_0^j(x) \leqq \rho_0^j(x) \leqq \bar{\rho}_0^j \equiv |\rho_0^j|_{\Omega^j}^{(0)} < +\infty,
$$

$$
0 < \underline{\theta}_0^j \equiv \inf_{\overline{\Omega^j}} \theta_0^j(x) \leqq \theta_0^j(x) \leqq \bar{\theta}_0^j \equiv |\theta_0^j|_{\Omega^j}^{(0)} < \infty),
$$

$$
(1.14) \qquad f^j \in B^1(\overline{R_T^3} \equiv \boldsymbol{R}^3 \times [0, T]), \quad \sum_{r+|s|=1} |D_t^r D_t^s f^j|_{x, \overline{R_T^3}}^{(L)} < +\infty (j=\mathrm{I}, \mathrm{II}),
$$

$$
(1.15) \qquad (\mu^j, \mu'^j, \kappa^j, p^j, S^j) \in O_{\mathrm{loc}}^{2+L}((0, \infty) \times (0, \infty)),
$$

$$
\mu'^j + \frac{2}{3}\mu^j > 0, \quad \mu^j, \kappa^j, p^j, S_{\theta^j}^j > 0 \quad (j=\mathrm{I}, \mathrm{II}).
$$

Then our result is as follows:

Theorem A. *Under the assumptions* (1.12)$\sim$(1.15), *there exists a unique solution* $(w^{\mathrm{I}}, w^{\mathrm{II}}) \in C_{x_0, t_0}^{2+\alpha, 1+\alpha/2}(Q_{T'}^{\mathrm{I}}) \times {}_{x_0, t_0}^{2+\alpha, 1+\alpha/2}(Q_{T'}^{\mathrm{II}})$ *of* (1.11) *for some* $T' \in (0, T]$.

In the same manner as theorems in [4], Theorem A implies:

Theorem B. *If we assume* (1.12)$\sim$(1.15) *and define* $(\rho^{\mathrm{I}}, v^{\mathrm{I}}, \theta^{\mathrm{I}}, \rho^{\mathrm{II}}, v^{\mathrm{II}}, \theta^{\mathrm{II}})$, Ω_t^{I}, Ω_t^{II} *by the formulae*

$$
\begin{cases}
(\rho^{\mathrm{I}}, v^{\mathrm{I}}, \theta^{\mathrm{I}})(x, t) = \Pi_{x,t}^{x_0, t_0}(\hat{\rho}^{\mathrm{I}}, w^{\mathrm{I}} + w_0^{\mathrm{I}})(x_0, t_0), \\
(\rho^{\mathrm{II}}, v^{\mathrm{II}}, \theta^{\mathrm{II}})(x, t) = \Pi_{x,t}^{x_0, t_0}(\hat{\rho}^{\mathrm{II}}, w^{\mathrm{II}} + w_0^{\mathrm{II}})(x_0, t_0), \\
\Omega_t^{\mathrm{I}} = \Pi_{x,t}^{x_0, t_0}\Omega^{\mathrm{I}}, \quad \Omega_t^{\mathrm{II}} = \Pi_{x,t}^{x_0, t_0}\Omega^{\mathrm{II}},
\end{cases}
$$

then $(\rho^{\mathrm{I}}, v^{\mathrm{I}}, \theta^{\mathrm{I}}, \rho^{\mathrm{II}}, v^{\mathrm{II}}, \theta^{\mathrm{II}})$ *is a unique solution of* (1.1)–(1.4), *which belongs to* $B^{1+\alpha}(\overline{\mathscr{D}_{T'}^{\mathrm{I}}}) \times C_{x,t}^{2+\alpha, 1+\alpha/2}(\overline{\mathscr{D}_{T'}^{\mathrm{I}}}) \times C_{x,t}^{2+\alpha, 1+\alpha/2}(\overline{\mathscr{D}_{T'}^{\mathrm{I}}}) \times B^{1+\alpha}(\overline{\mathscr{D}_{T'}^{\mathrm{II}}}) \times C_{x,t}^{2+\alpha, 1+\alpha/2}(\overline{\mathscr{D}_{T'}^{\mathrm{II}}}) \times C_{x,t}^{2+\alpha, 1+\alpha/2}(\overline{\mathscr{D}_{T'}^{\mathrm{II}}})$$(0 < \rho^{\mathrm{I}}, \rho^{\mathrm{II}} < \bar{\rho} = constant, 0 < \theta^{\mathrm{I}}, \theta^{\mathrm{II}} < \bar{\theta} = constant)$ *for some* $T' \in (0, T]$.

Remark. Referring to the result in [4], we can discuss the above problem even if the exterior boundary be free. Of course, the boundary condition concerning θ^{II} in (1.4) can be replaced by the Neumann or the third boundary one.

§ 2. Linearized problem

In this section we consider the following linearized problem of (1.11):

$$(2.1)_1^j \quad \frac{\partial}{\partial t_0}\tilde{w}^j = \mathscr{A}^j(x_0, t_0, w^j; \mathring{V})\tilde{w}^j + \mathscr{B}^j(x_0, t_0, w^j) \quad \text{in} \quad Q_T^j \quad (j = \mathrm{I}, \mathrm{II}),$$

$$(2.1)_2 \quad (\tilde{w}^{\mathrm{I}}, \tilde{w}^{\mathrm{II}})_{|t_0=0} = (0, 0)$$

$$(2.1)_3 \quad \begin{pmatrix} \mathrm{I}_3 \\ \dfrac{B^{\mathrm{I}}(x_0, t_0, w^{\mathrm{I}}; \mathring{V})}{|\mathscr{G}^{\mathrm{I}}\mathring{V}F_0^{\mathrm{I}}|} \end{pmatrix} \tilde{w}^{\mathrm{I}} - \begin{pmatrix} \mathrm{I}_3 \\ \dfrac{B^{\mathrm{II}}(x_0, t_0, w^{\mathrm{II}}; \mathring{V})}{|\mathscr{G}^{\mathrm{II}}\mathring{V}F_0^{\mathrm{I}}|} \end{pmatrix} \tilde{w}^{\mathrm{II}} \equiv$$

$$\equiv \tilde{B}^{\mathrm{I}}(x_0, t_0, w^{\mathrm{I}}; \mathring{V})\tilde{w}^{\mathrm{I}} - \tilde{B}^{\mathrm{II}}(x_0, t_0, w^{\mathrm{II}}; \mathring{V})\tilde{w}^{\mathrm{II}} =$$

$$= \begin{pmatrix} w_0^{\mathrm{II}} \\ \dfrac{\phi^{\mathrm{II}}(x_0, t_0, w^{\mathrm{II}})}{|\mathscr{G}^{\mathrm{II}}\mathring{V}F_0^{\mathrm{I}}|} \end{pmatrix} - \begin{pmatrix} w_0^{\mathrm{I}} \\ \dfrac{\phi^{\mathrm{I}}(x_0, t_0, w^{\mathrm{I}})}{|\mathscr{G}^{\mathrm{I}}\mathring{V}F_0^{\mathrm{I}}|} \end{pmatrix} \equiv \psi^{\mathrm{II}} - \psi^{\mathrm{I}} \quad \text{on} \quad \Gamma_T^{\mathrm{I}},$$

$$(2.1)_4 \quad \tilde{w}^{\mathrm{II}} = (0, \hat{\theta}_e - \theta_0^{\mathrm{II}}) \equiv \phi_e \quad \text{on} \quad \Gamma_T;$$

here we also assume that the compatibility conditions are valid and that $(w^{\mathrm{I}}, w^{\mathrm{II}})$ belonging to

$$(2.2) \quad \mathfrak{S}_T \equiv \{(w^{\mathrm{I}}, w^{\mathrm{II}}) \in C_{x_0, t_0}^{2+\alpha, 1+\alpha/2}(\overline{Q_T^{\mathrm{I}}}) \times C_{x_0, t_0}^{2+\alpha, 1+\alpha/2}(\overline{Q_T^{\mathrm{II}}}) |$$
$$(w^{\mathrm{I}}, w^{\mathrm{II}})_{|t_0=0} = (0, 0), \quad \|w^j\|_{Q_T^j}^{(2)} < M_1^j, \quad |\mathring{V}\mathring{V}w^j|_{x_0, \overline{Q_T^j}}^{(\alpha)} < M_2^j \quad (j = \mathrm{I}, \mathrm{II})\}$$

is given, where M_1^j is an arbitrary positive number and M_2^j is a positive number to be determined later $(j = \mathrm{I}, \mathrm{II})$.

2.1. Parabolicity and complementing condition

As was shown in [3], the following lemma holds.

Lemma 2.1. *Both the systems of differential equations* $(2.1)_1^{\mathrm{I}}$ *and* $(2.1)_1^{\mathrm{II}}$ *are uniformly parabolic in the sense of Petrowsky (modules of parabolicity* δ^{I} *and* δ^{II} *respectively) if T be chosen in such a way that*

$$(M_1^j + \|v_0^j\|_{\Omega^j}^{(2)})T < \min\left\{M_0, \frac{1}{2}dis(\Gamma, \Gamma^{\mathrm{I}}), \bar{\theta}_0^j\right\},$$

$$0 < C_1^j(T, M_1^j) < 1, \quad 1 - 6C_1^j(T, M_1^j) - 5C_1^j(T, M_1^j)^2 > 0 \quad (j = \mathrm{I}, \mathrm{II}),$$

where M_0 *is a positive root of* $1 - 3x - 6x^2 - 6x^3 = 0$, *since the estimates* $|g_{kl}^j - \delta_{kl}| < C_1^j(T, M_1^j)$ $(k, l = 1, 2, 3)$ *are true for some constant* $C_1^j(T, M_1^j)$ *which is increasing in each arguement and tends to zero as* $T \downarrow 0$ $(j = \mathrm{I}, \mathrm{II})$ $(\delta_{kl}$, *Kronecker's delta).*

It should be noted that $(\hat{\rho}^j, w_4^j)$ takes a value on

$$\mathscr{D}^j_{\rho,\theta}=(\bar{\rho}_0^j \exp[-b^j T],\ \bar{\bar{\rho}}_0^j \exp[+b^j T])\times(\theta_0^j-M_1^j,\ \bar{\theta}_0^j+M_1^j)$$

where $b^j=3(M_1^j+\|v_0^j\|^{(2)}_{\Omega^j})(1+3C_1^j)$ $(j=\mathrm{I},\mathrm{II})$.

Since the validity of the complementing condition for the boundary value problem $(2.1)_1^{\mathrm{II}}$, $(2.1)_4$ was proved in [3], we proceed to do so for $(2.1)_1^{\mathrm{I},\mathrm{II}}$, (2.3) in the case that $\Omega^{\mathrm{I}}=\boldsymbol{R}^3_+$ and $\Gamma^{\mathrm{I}}=\{x_0\in F_0^{\mathrm{I}}(x_0)\equiv x_{0,3}=0\}$. If we put

$$\tilde{w}(x_0,t_0)={}^t(\tilde{w}^{\mathrm{I}}(x_0,t_0),\ \tilde{w}^{\mathrm{II}}(x_0',-x_{0,3},t_0))(x_0'=(x_{0,1},x_{0,2})),$$

then $(2.1)_{1,2,3}$ are transformed into

$$(2.3)\quad \begin{cases} \dfrac{\partial}{\partial t_0}\tilde{w}=\mathscr{A}(x_0,t_0,w;\mathring{V})\tilde{w}+\mathscr{B}(x_0,t_0,w) \quad \text{in} \quad R_3^+\times(0,T],\\ \tilde{w}_{|t_0=0}=0,\\ \begin{pmatrix} \mathrm{I}_3 & \mathrm{I}_3\\ \dfrac{B^{\mathrm{I}}(x_0,t_0,w^{\mathrm{I}};\mathring{V})}{|g^{\mathrm{I}}_{h3}|} & -\dfrac{B^{\mathrm{II}}(x_0',-x_{0,3},t_0,w^{\mathrm{II}};\mathring{V}_1,\mathring{V}_2,-\mathring{V}_3)}{|g^{\mathrm{II}}_{h3}|}\end{pmatrix}\tilde{w}|_{x_{0,3}=0}\\ \quad\equiv B(x_0,t_0,w;\mathring{V})\tilde{w}_{|x_{0,3}=0}={}^t\psi^{\mathrm{II}}(x_0',t_0)-{}^t\psi^{\mathrm{I}}(x_0',t_0)\end{cases}$$

where

$$\mathscr{A}(x_0,t_0,w;\mathring{V})=\begin{pmatrix}\mathscr{A}^{\mathrm{I}}(x_0,t_0,w^{\mathrm{I}};\mathring{V}) & 0\\ 0 & \mathscr{A}^{\mathrm{II}}(x_0',-x_{0,3},t_0,w^{\mathrm{II}};\mathring{V}_1,\mathring{V}_2,-\mathring{V}_3)\end{pmatrix}$$

$$B(x_0,t_0,w)={}^t(B^{\mathrm{I}}(x_0,t_0,w^{\mathrm{I}}),\quad B^{\mathrm{II}}(x_0',-x_{0,3},t_0,w^{\mathrm{II}})).$$

Lemma 2.1 implies that the system (2.3) is uniformly parabolic in the sense of Petrowsky with a module of parabolicity $\delta^{\mathrm{I}}\delta^{\mathrm{II}}$. $\det[\mathscr{A}(x_0,t_0,w;i\xi)-\nu\mathrm{I}_8]=0$ is equivalent to $(g^{\mathrm{I}}_{k1}\xi_1+g^{\mathrm{I}}_{k2}\xi_2+g^{\mathrm{I}}_{k3}\xi_3)^2=-a_r^{\mathrm{I}}\nu$, $(g^{\mathrm{II}}_{k1}\xi_1+g^{\mathrm{II}}_{k2}\xi_2-g^{\mathrm{II}}_{k3}\xi_3)^2=-a_r^{\mathrm{II}}\nu$ $(r=1,2,3,4)$ where

$$a_1^j=a_2^j=\hat{\rho}^j/\mu^j,\quad a_3^j=\hat{\rho}^j/(2\mu^j+\mu'^j),\quad a_4^j=\hat{\rho}^j(w_4^j+\theta_0^j)S^j_{\theta j}/\kappa^j \quad (j=\mathrm{I},\mathrm{II}).$$

Therefore the values ξ_3's satisfying it are given by

$$(2.4)\quad \begin{cases}\xi_3^{\pm(r)\mathrm{I}}(\xi',\nu)=(g^{\mathrm{I}}_{h3})^{-2}[-g^{\mathrm{I}}_{k3}(g^{\mathrm{I}}_{k1}\xi_1+g^{\mathrm{I}}_{k2}\xi_2)\pm(A_r^{\mathrm{I}}+iB_r^{\mathrm{I}})],\\ \xi_3^{\pm(r)\mathrm{II}}(\xi',\nu)=-(g^{\mathrm{II}}_{h3})^{-2}[-g^{\mathrm{II}}_{k3}(g^{\mathrm{II}}_{k1}\xi_1+g^{\mathrm{II}}_{k2}\xi_2)\pm(A_r^{\mathrm{II}}+iB_r^{\mathrm{II}})],\end{cases}$$

where

$$(2.5)\quad \begin{cases} B_r^j(\xi',\nu)=\left[\dfrac{1}{2}(|D_r^j|-\mathrm{Re}\,D_r^j)\right]^{1/2},\quad A_r^j=\mathrm{Im}\,D_r^j/(2B_r^j),\\ D_r^j(\xi',\nu)=-\dfrac{1}{2}[(g^j_{k3}g^j_{m1}-g^j_{m3}g^j_{k1})\xi_1+(g^j_{k3}g^j_{m2}-g^j_{m3}g^j_{k2})\xi_2]^2-a_r^j\nu(g^j_{h3})^2\\ \qquad\equiv-\dfrac{1}{2}(D^j_{km})^2-a_r^j\nu(g^j_{h3})^2\equiv-D^j-a_r^j\nu(g^j_{h3})^2 \quad (j=\mathrm{I},\mathrm{II}).\end{cases}$$

We give without proof:

Lemma 2.2. *If for any $c\in(0, 1)$*

$$\mathrm{Re}\, v \geqq -c \min\left\{\frac{D^{\mathrm{I}}}{a_1^{\mathrm{I}}|g_{h3}^{\mathrm{I}}|^2}, \frac{D^{\mathrm{II}}}{a_1^{\mathrm{II}}|g_{h3}^{\mathrm{II}}|^2}\right\}, \quad \xi'^4+|v|^2>0,$$

$(\xi'^2=\xi_1^2+\xi_2^2)$, *then the following inequalities hold:*

$$\begin{cases} (2B_r^jB_s^k)^2-a_r^ja_s^k|g_{h3}^j|^2|g_{h3}^k|^2(\mathrm{Im}\, v)^2-4(1-c)\sqrt{D^jD^k}B_r^jB_s^k\geqq 0, \\ \sqrt{a^j}B_1^j\leqq B_3^j\leqq(1-c)^{-1/2}B_1^j \quad (j, k=\mathrm{I}, \mathrm{II}; r, s=1, 3; a^j\equiv a_3^ja_1^j). \end{cases}$$

In the proof of the above lemma, remark the relation $0<a^j\leqq 3/4$.
The following lemma is essential in our investigation.

Lemma 2.3. *There exists a positive constant δ smaller than $\delta^{\mathrm{I}}\delta^{\mathrm{II}}$ such that for any $\xi'=(\xi_1, \xi_2)\in \boldsymbol{R}^2$ and any $v\in C^1$ satisfying*

$$\mathrm{Re}\, v\geqq -\delta\xi'^2, \quad \xi'^4+|v|^2>0,$$

the row vectors of matrix $B(x_0, t_0, w; i\xi)\hat{\mathscr{A}}(x_0, t_0, w; i\xi, v)$ $((x_0, t_0)\in\Gamma_T^{\mathrm{I}}$, fixed) are linearly independent modulo $M\equiv\prod_{r=1}^{4}(\xi_3-\xi_3^{+(r)\mathrm{I}})(\xi_3-\xi_3^{-(r)\mathrm{II}})$ where $\hat{\mathscr{A}}(x_0, t_0, w; i\xi, v)$ is an adjugate matrix of $\mathscr{A}(x_0, t_0, w; i\xi)-vI_8$.

Proof. The proof is devided into two steps.
1°. Let $\sum_{s=1}^{8}\alpha^{(s)}\xi_3^{s-1}$ be the remainder term when we devide $B(x_0, t_0, w; i\xi)\hat{\mathscr{A}}(x_0, t_0, w; i\xi, v)$ by M. In this step, we calculate $\det\alpha^{(8)}$. Referring to the results in [3, 4], we can easily obtain

$$\alpha^{(8)}=\begin{pmatrix} \alpha^{(8)}(\mathrm{I}, \mathrm{I}) & \alpha^{(8)}(\mathrm{I}, \mathrm{II}) \\ \alpha^{(8)}(\mathrm{II}, \mathrm{I}) & \alpha^{(8)}(\mathrm{II}, \mathrm{II}) \end{pmatrix} \quad (\alpha^{(8)}(j, k)(j, k=\mathrm{I}, \mathrm{II}), 4\times 4\text{matrix})$$

as follows:

$$(\alpha^{(8)}(\mathrm{I}, \mathrm{I}))_{jk}=[(a_1^{\mathrm{II}})^2a_3^{\mathrm{II}}a_4^{\mathrm{II}}]^{-1}|g_{h3}^{\mathrm{I}}|^4|g_{h3}^{\mathrm{II}}|^8\times$$

$$\times\begin{cases} \dfrac{1}{a_4^{\mathrm{I}}a_5^{\mathrm{I}}}(\xi_3^{+(1)\mathrm{I}}-\xi_3^{+(3)\mathrm{I}})^{-1}('\alpha^{(8)}(\mathrm{I}, \mathrm{I}))_{jk} \quad (j, k=1, 2, 3;\ v\neq 0), \\ \text{the limit value as } v\to 0 \text{ of the preceding expression} \quad (j, k=1, 2, 3;\ v=0), \\ -\dfrac{|g_{h3}^{\mathrm{I}}|^2}{(a_1^{\mathrm{I}})^2a_3^{\mathrm{I}}}(\xi_3^{+(4)\mathrm{I}}-\xi_3^{-(1)\mathrm{I}})^2(\xi_3^{+(4)\mathrm{I}}-\xi_3^{-(3)\mathrm{I}})(\xi_3^{+(4)\mathrm{I}}-\xi_3^{+(1)\mathrm{II}})^2\times \\ \quad\times(\xi_3^{+(4)\mathrm{I}}-\xi_3^{+(3)\mathrm{II}})(\xi_3^{+(4)\mathrm{I}}-\xi_3^{+(4)\mathrm{II}}) \quad (j=k=4), \\ 0 \quad (\text{otherwise}), \end{cases}$$

$$('\alpha^{(8)}(\mathrm{I}, \mathrm{I}))_{jk}=(\xi_3^{+(1)\mathrm{I}}-\xi_3^{-(1)\mathrm{I}})(\xi_3^{+(1)\mathrm{I}}-\xi_3^{-(4)\mathrm{I}})(\xi_3^{+(1)\mathrm{I}}-\xi_3^{+(1)\mathrm{II}})^2\times$$

$$\times(\xi_3^{+(1)\mathrm{I}}-\xi_3^{+(3)\mathrm{II}})(\xi_3^{+(1)\mathrm{I}}-\xi_3^{+(4)\mathrm{II}})v\delta_{jk}+\frac{1}{a_1^{\mathrm{I}}}[P_j^{\mathrm{I}}P_k^{\mathrm{I}}(\xi_3^{+(1)\mathrm{I}}-\xi_3^{-(1)\mathrm{I}})\times$$

$$\times(\xi_3^{+(1)\mathrm{I}}-\xi_3^{-(4)\mathrm{I}})(\xi_3^{+(1)\mathrm{I}}-\xi_3^{+(1)\mathrm{II}})^2(\xi_3^{+(1)\mathrm{I}}-\xi_3^{+(3)\mathrm{II}})\times(\xi_3^{+(1)\mathrm{I}}-\xi_3^{+(4)\mathrm{II}})$$

$$-Q_j^{\mathrm{I}}Q_k^{\mathrm{I}}(\xi_3^{+(3)\mathrm{I}}-\xi_3^{-(1)\mathrm{I}})(\xi_3^{+(3)\mathrm{I}}-\xi_3^{-(4)\mathrm{I}})\times$$

$$\times(\xi_3^{+(3)\mathrm{I}}-\xi_3^{+(1)\mathrm{II}})^2(\xi_3^{+(3)\mathrm{I}}-\xi_3^{+(3)\mathrm{II}})(\xi_3^{+(3)\mathrm{I}}-{}_3^{+(4)\mathrm{II}})],$$

$$P_j^{\mathrm{I}}=g_{j1}^{\mathrm{I}}\xi_1+g_{j2}^{\mathrm{I}}\xi_2+g_{j3}^{\mathrm{I}}\xi_3^{+(1)\mathrm{I}},\quad Q_j^{\mathrm{I}}=g_{j1}^{\mathrm{I}}\xi_1+g_{j2}^{\mathrm{I}}\xi_2+g_{j3}^{\mathrm{I}}\xi_3^{+(3)\mathrm{I}},\quad a_5^{\mathrm{I}}=\frac{\hat{\rho}^{\mathrm{I}}}{\mu^{\mathrm{I}}+\mu'^{\mathrm{I}}};$$

$\alpha^{(8)}(\mathrm{I},\mathrm{II})$ = [the right-hand side of $\alpha^{(8)}(\mathrm{I},\mathrm{I})$ as obtained by replacing a_r^{I}, g_{h3}^{I} and $\xi_3^{\pm(r)\mathrm{I}}$ by a_r^{II}, g_{h3}^{II} and $-\xi_3^{\mp(r)\mathrm{II}}$, respectively and vice versal] $(a_5^{\mathrm{II}}=\hat{\rho}^{\mathrm{II}}/(\mu^{\mathrm{II}}+\mu'^{\mathrm{II}}))$;

$$(\alpha^{(8)}(\mathrm{II},\mathrm{I}))_{jk}=i[(a_1^{\mathrm{II}})^2a_3^{\mathrm{II}}a_4^{\mathrm{II}}]^{-1}|g_{h3}^{\mathrm{I}}|^3|g_{h3}^{\mathrm{II}}|^8\times$$

$$\times\begin{cases}\dfrac{1}{a_4^{\mathrm{I}}a_5^{\mathrm{I}}}(\xi_3^{+(1)\mathrm{I}}-\xi_3^{+(3)\mathrm{I}})^{-1}('\alpha^{(8)}(\mathrm{II},\mathrm{I}))_{jk}\quad(j,k=1,2,3;\ v\neq0),\\ \text{the limit value as } v\to0 \text{ of the preceding expression}\quad(j,k=1,2,3;\ v=0),\\ -\dfrac{\kappa^{\mathrm{I}}|g_{h3}^{\mathrm{I}}|^2}{(a_1^{\mathrm{I}})^2a_3^{\mathrm{I}}}g_{m3}^{\mathrm{I}}(g_{m1}^{\mathrm{I}}\xi_1+g_{m2}^{\mathrm{I}}\xi_2+g_{m3}^{\mathrm{I}}\xi_3^{+(4)\mathrm{I}})(\xi_3^{+(4)\mathrm{I}}-\xi_3^{-(1)\mathrm{I}})^2\times\\ \quad\times(\xi_3^{+(4)\mathrm{I}}-\xi_3^{-(3)\mathrm{I}})(\xi_3^{+(4)\mathrm{I}}-\xi_3^{+(1)\mathrm{II}})^2(\xi_3^{+(4)\mathrm{I}}-\xi_3^{+(3)\mathrm{II}})(\xi_3^{+(4)\mathrm{I}}-\xi_3^{+(4)\mathrm{II}})\\ \qquad\qquad(j=k=4),\\ 0\quad(\text{otherwise}),\end{cases}$$

$$('\alpha^{(8)}(\mathrm{II},\mathrm{I}))_{jk}=\mu^{\mathrm{I}}\Big[vP^{\mathrm{I}}\delta_{jk}+\Big(vg_{k3}^{\mathrm{I}}+\frac{2}{a_1^{\mathrm{I}}}P^{\mathrm{I}}P_k^{\mathrm{I}}\Big)P_j^{\mathrm{I}}\Big](\xi_3^{+(1)\mathrm{I}}-\xi_3^{-(1)\mathrm{I}})\times$$

$$\times(\xi_3^{+(1)\mathrm{I}}-\xi_3^{-(4)\mathrm{I}})(\xi_3^{+(1)\mathrm{I}}-\xi_3^{+(1)\mathrm{II}})^2(\xi_3^{+(1)\mathrm{I}}-\xi_3^{+(3)\mathrm{II}})(\xi_3^{+(1)\mathrm{I}}-\xi_3^{+(4)\mathrm{II}})$$

$$-\frac{1}{a_1^{\mathrm{I}}}(2\mu^{\mathrm{I}}Q^{\mathrm{I}}Q_j^{\mathrm{I}}-\mu'^{\mathrm{I}}a_3^{\mathrm{I}}vg_{j3}^{\mathrm{I}})Q_k^{\mathrm{I}}(\xi_3^{+(3)\mathrm{I}}-\xi_3^{-(1)\mathrm{I}})\times$$

$$\times(\xi_3^{+(3)\mathrm{I}}-\xi_3^{-(4)\mathrm{I}})(\xi_3^{+(3)\mathrm{I}}-\xi_3^{+(1)\mathrm{II}})^2(\xi_3^{+(3)\mathrm{I}}-\xi_3^{+(3)\mathrm{II}})(\xi_3^{+(3)\mathrm{I}}-\xi_3^{+(4)\mathrm{II}}),$$

$$P^{\mathrm{I}}=g_{j3}^{\mathrm{I}}P_j^{\mathrm{I}},\quad Q^{\mathrm{I}}=g_{j3}^{\mathrm{I}}Q_j^{\mathrm{I}};$$

$\alpha^{(8)}(\mathrm{II},\mathrm{II})$ = [the right-hand side of $\alpha^{(8)}(\mathrm{II},\mathrm{I})$ with a_r^{I}, g_{h3}^{I} and $\xi_3^{\pm(r)\mathrm{I}}$ replaced by a_r^{II}, g_{h3}^{II} and $-\xi_3^{\mp(r)\mathrm{II}}$ respectively and vice versa].

After considerably lengthy calculations we obtain

(2.6)
$$\det\alpha^{(8)}=(\alpha_{44}^{(8)}\alpha_{88}^{(8)}-\alpha_{48}^{(8)}\alpha_{84}^{(8)})i[(a_1^{\mathrm{I}})^2a_3^{\mathrm{I}}(a_4^{\mathrm{I}})^2a_5^{\mathrm{I}}(a_1^{\mathrm{II}})^2a_3^{\mathrm{II}}(a_4^{\mathrm{II}})^2a_5^{\mathrm{II}}]^{-3}\times$$
$$\times|g_{h3}^{\mathrm{I}}|^{33}|g_{h3}^{\mathrm{II}}|^{33}(\xi_3^{+(1)\mathrm{I}}-\xi_3^{+(3)\mathrm{I}})^{-3}(\xi_3^{-(1)\mathrm{II}}-\xi_3^{-(3)\mathrm{II}})^{-3}\varDelta,$$

where

(2.7)
$$\varDelta=\det\begin{pmatrix}|g_{h3}^{\mathrm{I}}|'\alpha^{(8)}(\mathrm{I},\mathrm{I}) & |g_{h3}^{\mathrm{II}}|'\alpha^{(8)}(\mathrm{I},\mathrm{II})\\ '\alpha^{(8)}(\mathrm{II},\mathrm{I}) & '\alpha^{(8)}(\mathrm{II},\mathrm{II})\end{pmatrix}$$
$$=[(\xi_3^{+(1)\mathrm{I}}-\xi_3^{-(1)\mathrm{I}})^2(\xi_3^{+(1)\mathrm{I}}-\xi_3^{-(4)\mathrm{I}})^2(\xi_3^{+(3)\mathrm{I}}-\xi_3^{-(1)\mathrm{I}})\times$$
$$\times(\xi_3^{+(3)\mathrm{I}}-\xi_3^{-(4)\mathrm{I}})(\xi_3^{+(1)\mathrm{I}}-\xi_3^{+(1)\mathrm{II}})^4(\xi_3^{+(1)\mathrm{I}}-\xi_3^{+(3)\mathrm{II}})^2\times$$

$$\times(\xi_3^{+(1)\mathrm{I}}-\xi_3^{+(4)\mathrm{II}})^2(\xi_3^{+(3)\mathrm{I}}-\xi_3^{+(1)\mathrm{II}})^2(\xi_3^{+(3)\mathrm{I}}-\xi_3^{+(3)\mathrm{II}})\times$$

$$\times(\xi_3^{+(3)\mathrm{I}}-\xi_3^{+(4)\mathrm{II}})(a_1^{\mathrm{I}})^{-2}\nu P_j^{\mathrm{I}}Q_j^{\mathrm{I}}]_A\times$$

$$\times[\xi_3^{\pm(r)\mathrm{I}}\longleftrightarrow-\xi_3^{\mp(r)\mathrm{II}},\quad(a_1^{\mathrm{I}},P_j^{\mathrm{I}},Q_j^{\mathrm{I}})\longleftrightarrow(a_1^{\mathrm{II}},P_j^{\mathrm{II}},Q_j^{\mathrm{II}})]_A\times$$

$$\times\left\{\begin{aligned}&[|g_{h3}^{\mathrm{I}}|^3(\mu^{\mathrm{II}})^3P^{\mathrm{II}}P_k^{\mathrm{I}}Q_k^{\mathrm{I}}(4P_m^{\mathrm{II}}Q_m^{\mathrm{II}}D^{\mathrm{II}}+4a_1^{\mathrm{II}}\nu D^{\mathrm{II}}+(a_1^{\mathrm{II}})^2\nu^2|g_{h3}^{\mathrm{II}}|^2)]_B-\\&\qquad-[\mathrm{I}\longleftrightarrow\mathrm{II}]_B+\\&+[|g_{h3}^{\mathrm{I}}|\,|g_{h3}^{\mathrm{II}}|^2(\mu^{\mathrm{I}})^2\mu^{\mathrm{II}}\{(P^{\mathrm{II}}-Q^{\mathrm{II}})(4P_k^{\mathrm{I}}Q_k^{\mathrm{I}}+3a_1^{\mathrm{I}}\nu)|g_{h3}^{\mathrm{II}}|^{-2}[(D_{23}^{\mathrm{I}}D_{23}^{\mathrm{II}}+\\&+D_{31}^{\mathrm{I}}D_{31}^{\mathrm{II}}+D_{12}^{\mathrm{I}}D_{12}^{\mathrm{II}})^2-D^{\mathrm{I}}D^{\mathrm{II}}]+2P^{\mathrm{I}}(2P_k^{\mathrm{I}}Q_k^{\mathrm{I}}+a_1^{\mathrm{I}}\nu)(2P_m^{\mathrm{II}}Q_m^{\mathrm{II}}+a_1^{\mathrm{II}}\nu)\times\\&\times(D_{23}^{\mathrm{I}}D_{23}^{\mathrm{II}}+D_{31}^{\mathrm{I}}D_{31}^{\mathrm{II}}+D_{12}^{\mathrm{I}}D_{12}^{\mathrm{II}})+a_1^{\mathrm{I}}a_1^{\mathrm{II}}\nu^2P^{\mathrm{I}}(P^{\mathrm{I}}Q^{\mathrm{II}}+P^{\mathrm{II}}Q^{\mathrm{I}})+\\&+P^{\mathrm{II}}P_k^{\mathrm{II}}Q_k^{\mathrm{II}}[4P_m^{\mathrm{I}}Q_m^{\mathrm{I}}D^{\mathrm{I}}+4a_1^{\mathrm{I}}\nu D^{\mathrm{I}}+(a_1^{\mathrm{I}}\nu|g_{h3}^{\mathrm{I}}|)^2]\}]_C-[\mathrm{I}\longleftrightarrow\mathrm{II}]_C.\end{aligned}\right\}_D$$

2°. In this step, we show that $\det\alpha^{(8)}\neq0$ for a suitable δ. On account of the results in [4] and the relation

(2.8)
$$\{\cdots\}_D(\xi_3^{+(1)\mathrm{I}}-\xi_3^{+(3)\mathrm{I}})^{-1}(\xi_3^{-(1)\mathrm{II}}-\xi_3^{-(3)\mathrm{II}})^{-1}=$$

$$=-[\mu^{\mathrm{I}}|g_{h3}^{\mathrm{II}}|P^{\mathrm{I}}-\mu^{\mathrm{II}}|g_{h3}^{\mathrm{I}}|P^{\mathrm{II}}](1-a^{\mathrm{I}})^{-1}(1-a^{\mathrm{II}})^{-1}\times$$

$$\times\left\{\begin{aligned}&(\mu^{\mathrm{II}})^2|g_{h3}^{\mathrm{I}}|^2[(a^{\mathrm{I}}P^{\mathrm{I}}+Q^{\mathrm{I}})\{4(1-a^{\mathrm{II}})D^{\mathrm{II}}P^{\mathrm{II}}+a_1^{\mathrm{II}}\nu|g_{h3}^{\mathrm{II}}|^2(P^{\mathrm{II}}+\\&\qquad+Q^{\mathrm{II}})]_{E,1}+(\mu^{\mathrm{I}})^2|g_{h3}^{\mathrm{II}}|^2[\mathrm{I}\longleftrightarrow\mathrm{II}]_{E,1}+\\&+\mu^{\mathrm{I}}\mu^{\mathrm{II}}|g_{h3}^{\mathrm{I}}|\,|g_{h3}^{\mathrm{II}}|[2\{(2a^{\mathrm{I}}-1)P^{\mathrm{I}}+Q^{\mathrm{I}}\}\{(2a^{\mathrm{II}}-1)P^{\mathrm{II}}+Q^{\mathrm{II}}\}\times\\&\times(D_{23}^{\mathrm{I}}D_{23}^{\mathrm{II}}+D_{31}^{\mathrm{I}}D_{31}^{\mathrm{II}}+D_{12}^{\mathrm{I}}D_{12}^{\mathrm{II}})+(P^{\mathrm{I}}+Q^{\mathrm{I}})(P^{\mathrm{II}}+Q^{\mathrm{II}})(P^{\mathrm{I}}Q^{\mathrm{II}}+\\&+P^{\mathrm{II}}Q^{\mathrm{I}})+4(1-a^{\mathrm{I}})(1-a^{\mathrm{II}})\{D^{\mathrm{I}}D^{\mathrm{II}}-(D_{23}^{\mathrm{I}}D_{23}^{\mathrm{II}}+D_{31}^{\mathrm{I}}D_{31}^{\mathrm{II}}+\\&+D_{12}^{\mathrm{I}}D_{12}^{\mathrm{II}})^2\}-(\mu^{\mathrm{I}}|g_{h3}^{\mathrm{II}}|P^{\mathrm{I}}-\mu^{\mathrm{II}}|g_{h3}^{\mathrm{I}}|P^{\mathrm{II}})^{-1}[\mu^{\mathrm{I}}|g_{h3}^{\mathrm{II}}|(1-a^{\mathrm{II}})(P^{\mathrm{I}}+\\&+Q^{\mathrm{I}})-\mu^{\mathrm{II}}|g_{h3}^{\mathrm{I}}|(1-a^{\mathrm{I}})(P^{\mathrm{II}}+Q^{\mathrm{II}})]\{D^{\mathrm{I}}D^{\mathrm{II}}-\\&-(D_{23}^{\mathrm{I}}D_{23}^{\mathrm{II}}+D_{31}^{\mathrm{I}}D_{31}^{\mathrm{II}}+D_{12}^{\mathrm{I}}D_{12}^{\mathrm{II}})^2\}]_{E,2}.\end{aligned}\right\}_E$$

It is sufficient to show that $[\cdots]_E\neq0$ for a suitable δ. To begin with, we calculate $\operatorname{Im}[\cdots]_{E,1}$ and $\operatorname{Im}[\cdots]_{E,2}$.

(2.9)
$$\operatorname{Im}[\cdots]_{E,1}=\frac{\operatorname{Im}\nu}{2}\Big\{\Big(\frac{a^{\mathrm{I}}a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}+\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}\Big)|g_{h3}^{\mathrm{I}}|^2[4(1-a^{\mathrm{II}})D^{\mathrm{II}}B_1^{\mathrm{II}}+a_1^{\mathrm{II}}|g_{h3}^{\mathrm{II}}|^2\operatorname{Re}\nu\times$$

$$\times(B_1^{\mathrm{II}}+B_3^{\mathrm{II}})]+(a^{\mathrm{I}}B_1^{\mathrm{I}}+B_3^{\mathrm{I}})\Big[4(1-a^{\mathrm{II}})\frac{D^{\mathrm{II}}}{B_1^{\mathrm{II}}}+|g_{h3}^{\mathrm{II}}|^2\operatorname{Re}\nu\Big(\frac{a_1^{\mathrm{II}}}{B_1^{\mathrm{II}}}+\frac{a_3^{\mathrm{II}}}{B_3^{\mathrm{II}}}\Big)\Big]+$$

$$+a_1^{\mathrm{II}}|g_{h3}^{\mathrm{II}}|^2\Big[2(a^{\mathrm{I}}B_1^{\mathrm{I}}+B_3^{\mathrm{I}})(B_1^{\mathrm{II}}+B_3^{\mathrm{II}})-\frac{(\operatorname{Im}\nu)^2}{2}|g_{h3}^{\mathrm{I}}|^2|g_{h3}^{\mathrm{II}}|^2\times$$

$$\times\Big(\frac{a^{\mathrm{I}}a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}+\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}\Big)\Big(\frac{a_1^{\mathrm{II}}}{B_1^{\mathrm{II}}}+\frac{a_3^{\mathrm{II}}}{B_3^{\mathrm{II}}}\Big)\Big]\Big\}_{E,1},$$

$$(2.10)\quad \operatorname{Im}[\cdots]_{E,2}=\frac{\operatorname{Im}\nu}{2}\{[\cdots]_{E,2,1}+[\cdots]_{E,2,2}+[\cdots]_{E,2,3}+[\cdots]_{E,2,4}\},$$

$$\begin{cases} [\cdots]_{E,2,1}=\Big\{\Big[(B_1^{\mathrm{I}}+B_3^{\mathrm{I}})(B_1^{\mathrm{II}}+B_3^{\mathrm{II}})-\frac{(\operatorname{Im}\nu)^2}{4}|g_{h3}^{\mathrm{I}}|^2|g_{h3}^{\mathrm{II}}|^2\times \\ \times\Big(\frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}+\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}\Big)\Big(\frac{a_1^{\mathrm{II}}}{B_1^{\mathrm{II}}}+\frac{a_3^{\mathrm{II}}}{B_3^{\mathrm{II}}}\Big)\Big]\Big(\frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}B_3^{\mathrm{II}}+\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}B_1^{\mathrm{II}}\Big)|g_{h3}^{\mathrm{I}}|^2\Big\}_{E,2,1}+\{\mathrm{I}\longleftrightarrow\mathrm{II}\}_{E,2,1}, \\ [\cdots]_{E,2,2}=\Big\{\Big[B_1^{\mathrm{I}}B_3^{\mathrm{II}}+B_3^{\mathrm{I}}B_1^{\mathrm{II}}-\frac{(\operatorname{Im}\nu)^2}{4}|g_{h3}^{\mathrm{I}}|^2|g_{h3}^{\mathrm{II}}|^2\Big(\frac{a_1^{\mathrm{I}}a_3^{\mathrm{II}}}{B_1^{\mathrm{I}}B_3^{\mathrm{II}}}+\frac{a_3^{\mathrm{I}}a_1^{\mathrm{II}}}{B_3^{\mathrm{I}}B_1^{\mathrm{II}}}\Big)\Big]\times \\ \times\Big(\frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}+\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}\Big)(B_1^{\mathrm{II}}+B_3^{\mathrm{II}})|g_{h3}^{\mathrm{I}}|^2\Big\}_{E,2,2}+\{\mathrm{I}\longleftrightarrow\mathrm{II}\}_{E,2,2}, \\ [\cdots]_{E,2,3}=\Big\{2\Big[(2a^{\mathrm{I}}-1)\frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}+\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}\Big][(2a^{\mathrm{II}}-1)B_1^{\mathrm{II}}+B_3^{\mathrm{II}}]|g_{h3}^{\mathrm{I}}|^2(D_{23}^{\mathrm{I}}D_{23}^{\mathrm{II}}+ \\ +D_{31}^{\mathrm{I}}D_{31}^{\mathrm{II}}+D_{12}^{\mathrm{I}}D_{12}^{\mathrm{II}})\Big\}_{E,2,3}+\{\mathrm{I}\longleftrightarrow\mathrm{II}\}_{E,2,3}, \\ [\cdots]_{E,2,4}=\Big\{\mu^{\mathrm{I}}|g_{h3}^{\mathrm{I}}|^2|g_{h3}^{\mathrm{II}}|\Big[\mu^{\mathrm{I}}|g_{h3}^{\mathrm{II}}|\frac{1-a^{\mathrm{II}}}{B_1^{\mathrm{I}}B_3^{\mathrm{I}}}a_1^{\mathrm{I}}((B_3^{\mathrm{I}})^2-a^{\mathrm{I}}(B_1^{\mathrm{I}})^2)+ \\ +\mu^{\mathrm{II}}|g_{h3}^{\mathrm{I}}|\Big\{(1-a^{\mathrm{I}})\frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}(B_1^{\mathrm{II}}+B_3^{\mathrm{II}})-(1-a^{\mathrm{II}})B_1^{\mathrm{I}}\Big(\frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{II}}}+\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}\Big)\Big\}\Big]\times \\ \times\Big[\frac{(\operatorname{Im}\nu)^2}{4}|g_{h3}^{\mathrm{I}}|^2|g_{h3}^{\mathrm{II}}|^2\Big(\mu^{\mathrm{I}}|g_{h3}^{\mathrm{I}}|\frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}+\mu^{\mathrm{II}}|g_{h3}^{\mathrm{II}}|\frac{a_1^{\mathrm{II}}}{B_1^{\mathrm{II}}}\Big)^2+(\mu^{\mathrm{I}}|g_{h3}^{\mathrm{II}}|B_1^{\mathrm{I}}+ \\ +\mu^{\mathrm{II}}|g_{h3}^{\mathrm{I}}|B_1^{\mathrm{II}})^2\Big]_F^{-1}[D^{\mathrm{I}}D^{\mathrm{II}}-(D_{23}^{\mathrm{I}}D_{23}^{\mathrm{II}}+D_{31}^{\mathrm{I}}D_{31}^{\mathrm{II}}+D_{12}^{\mathrm{I}}D_{12}^{\mathrm{II}})^2]\Big\}_{E,2,4}+ \\ +\{\mathrm{I}\longleftrightarrow\mathrm{II}\}_{E,2,4}. \end{cases}$$

Hereafter we consider any $\xi'\in \boldsymbol{R}^2$ and $\nu\in C^1$ satisfying

$$(2.11)\qquad \operatorname{Re}\nu\geqq-\frac{1}{6}\min\Big\{\frac{D^{\mathrm{I}}}{a_1^{\mathrm{I}}|g_{h3}^{\mathrm{I}}|^2},\ \frac{D^{\mathrm{II}}}{a_1^{\mathrm{II}}|g_{h3}^{\mathrm{II}}|^2}\Big\},\quad \xi'^4+|\nu|^2>0.$$

If $\operatorname{Re}\nu\geqq 0$, then it is obvious that $\{\cdots\}_{E,1}\geqq 0$. On the other hand if $\operatorname{Re}\nu<0$, then by Lemma 2.2 and (2.11) we obtain

$$(2.12)\qquad B_3^j\leqq 2B_1^j,\quad \frac{a_3^j}{B_3^j}\leqq\frac{a_1^j}{B_1^j}\quad (j=\mathrm{I},\ \mathrm{II}),$$

from which it follows that

$$\begin{cases} 4(1-a^{\mathrm{II}})D^{\mathrm{II}}B_1^{\mathrm{II}}+a_1^{\mathrm{II}}|g_{h3}^{\mathrm{II}}|^2\operatorname{Re}\nu(B_1^{\mathrm{II}}+B_3^{\mathrm{II}})\geqq(D^{\mathrm{II}}+3a_1^{\mathrm{II}}|g_{h3}^{\mathrm{II}}|^2\operatorname{Re}\nu)B_1^{\mathrm{II}}\geqq 0, \\ 4(1-a^{\mathrm{II}})\frac{D^{\mathrm{II}}}{B_1^{\mathrm{II}}}+|g_{h3}^{\mathrm{II}}|^2\operatorname{Re}\nu\Big(\frac{a_1^{\mathrm{II}}}{B_1^{\mathrm{II}}}+\frac{a_3^{\mathrm{II}}}{B_3^{\mathrm{II}}}\Big)\geqq(D^{\mathrm{II}}+2a_1^{\mathrm{II}}|g_{h3}^{\mathrm{II}}|^2\operatorname{Re}\nu)\frac{1}{B_1^{\mathrm{II}}}\geqq 0, \\ 2(a^{\mathrm{I}}B_1^{\mathrm{I}}+B_3^{\mathrm{I}})(B_1^{\mathrm{II}}+B_3^{\mathrm{II}})-\frac{(\operatorname{Im}\nu)^2}{2}|g_{h3}^{\mathrm{I}}|^2|g_{h3}^{\mathrm{II}}|^2\Big(\frac{a^{\mathrm{I}}a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}+\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}\Big)\Big(\frac{a_1^{\mathrm{II}}}{B_1^{\mathrm{II}}}+\frac{a_3^{\mathrm{II}}}{B_3^{\mathrm{II}}}\Big)= \end{cases}$$

$$\left\{\begin{aligned} &=a^{\mathrm{I}}\left\{\left\{\frac{1}{2B_1^{\mathrm{I}}B_1^{\mathrm{II}}}[(2B_1^{\mathrm{I}}B_1^{\mathrm{II}})^2-a_1^{\mathrm{I}}a_1^{\mathrm{II}}|g_{h3}^{\mathrm{I}}|^2|g_{h3}^{\mathrm{II}}|^2(\mathrm{Im}\,\nu)^2]\right\}\right\}_{E,1}+\\ &\quad+a^{\mathrm{I}}\{\{(B_1^{\mathrm{II}},a_1^{\mathrm{II}})\longrightarrow(B_3^{\mathrm{II}},a_3^{\mathrm{II}})\}\}_{E,1}+\{\{(B_1^{\mathrm{I}},a_1^{\mathrm{I}})\longrightarrow(B_3^{\mathrm{I}},a_3^{\mathrm{I}})\}\}_{E,1}+\\ &\quad+\{\{(B_1^{\mathrm{I}},B_1^{\mathrm{II}},a_1^{\mathrm{I}},a_1^{\mathrm{II}})\longrightarrow(B_3^{\mathrm{I}},B_3^{\mathrm{II}},a_3^{\mathrm{I}},a_3^{\mathrm{II}})\}\}_{E,1}\geqq 0.\end{aligned}\right.$$

Hence, $\{\cdots\}_{E,1}\geqq 0$; here the equality holds if and only if $\zeta'=\mathrm{Re}\,\nu=0$.

In the next place, we transform the term $\{\cdots\}_{E,2,1}$ into a more convenient form:

$$\begin{aligned}\{\cdots\}_{E,2,1}&=|g_{h3}^{\mathrm{I}}|^2\Big[\frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}B_1^{\mathrm{II}}\left\{\left\{B_3^{\mathrm{I}}B_3^{\mathrm{II}}-\frac{(\mathrm{Im}\,\nu)^2}{4}|g_{h3}^{\mathrm{I}}|^2|g_{h3}^{\mathrm{II}}|^2\frac{a_3^{\mathrm{I}}a_3^{\mathrm{II}}}{B_3^{\mathrm{I}}B_3^{\mathrm{II}}}\right\}\right\}_{E,2,1}+\\ &+\frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}B_3^{\mathrm{II}}[\{\{(B_3^{\mathrm{I}},B_3^{\mathrm{II}},a_3^{\mathrm{I}},a_3^{\mathrm{II}})\longrightarrow(B_1^{\mathrm{I}},B_1^{\mathrm{II}},a_1^{\mathrm{I}},a_1^{\mathrm{II}})\}\}_{E,2,1}+\{\{\quad\}\}_{E,2,1}+\\ &+\{\{(B_3^{\mathrm{I}},a_3^{\mathrm{I}})\longrightarrow(B_1^{\mathrm{I}},a_1^{\mathrm{I}})\}\}_{E,2,1}\Big]+\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}B_1^{\mathrm{II}}[\{\{(B_3^{\mathrm{II}},a_3^{\mathrm{II}})\longrightarrow(B_1^{\mathrm{II}},a_1^{\mathrm{II}})\}\}_{E,2,1}+\\ &+\{\{(B_1^{\mathrm{I}},B_1^{\mathrm{II}},a_3^{\mathrm{I}},a_3^{\mathrm{II}})\longrightarrow(B_1^{\mathrm{I}},B_1^{\mathrm{II}},a_1^{\mathrm{I}},a_1^{\mathrm{II}})\}\}_{E,2,1}+\{\{\quad\}\}_{E,2,1}]+\\ &+\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}B_3^{\mathrm{II}}\{\{(B_3^{\mathrm{I}},B_3^{\mathrm{II}},a_3^{\mathrm{I}},a_3^{\mathrm{II}})\longrightarrow(B_1^{\mathrm{I}},B_1^{\mathrm{II}},a_1^{\mathrm{I}},a_1^{\mathrm{II}})\}\}_{E,2,1}]\equiv\sum_{k=1}^{8}\{\cdots\}_{E,2,1,k}.\end{aligned}$$

Since $\{\cdots\}_{E,2,3}$ is estimated from below by (2.12) as follows:

$$(2.13)\quad \{\cdots\}_{E,2,3}\geqq-|g_{h3}^{\mathrm{I}}|^2\max\left\{\frac{a_1^{\mathrm{I}}}{2B_1^{\mathrm{I}}}B_1^{\mathrm{II}}+\frac{3}{2}\frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}B_3^{\mathrm{II}}+\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}B_1^{\mathrm{II}}+\frac{3}{2}\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}B_3^{\mathrm{II}},\right.$$
$$\left.2\frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}B_1^{\mathrm{II}},\ \frac{3}{2}\frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}B_1^{\mathrm{II}}+\frac{3}{2}\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}B_1^{\mathrm{II}},\ \frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}B_3^{\mathrm{II}}+2\frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}B_1^{\mathrm{II}}\right\}\sqrt{D^{\mathrm{I}}D^{\mathrm{II}}},$$

there can occur three cases according to which is the largest term in the brace of the right-hand side of (2.13):

(i) $\{\cdots\}_{E,2,2}+\{\cdots\}_{E,2,3}\geqq 0$ if either the first or the second term be the largest,

(ii) $\{\cdots\}_{E,2,2}+\{\cdots\}_{E,2,3}\geqq -a_1^{\mathrm{I}}B_3^{\mathrm{II}}|g_{h3}^{\mathrm{I}}|^2\sqrt{D^{\mathrm{I}}D^{\mathrm{II}}}/(2B_1^{\mathrm{I}})$ if the third one be the largest,

(iii) $\{\cdots\}_{E,2,2}+\{\cdots\}_{E,2,3}\geqq -a_1^{\mathrm{I}}B_1^{\mathrm{II}}|g_{h3}^{\mathrm{I}}|^2\sqrt{D^{\mathrm{I}}D^{\mathrm{II}}}/(2B_1^{\mathrm{I}})$ if the fourth one be the largest.

For (i) we need not consider any more. With regards (ii) [resp. (iii)], we only consider the sum of $\{\cdots\}_{E,2,2}+\{\cdots\}_{E,2,3}$ and $\{\cdots\}_{E,2,1,2}$ [resp. $\{\cdots\}_{E,2,1,1}$], which is non-negative by using Lemma 2.2.

Lastly, since all the other terms in $\{\cdots\}_{E,2,4}$ except

$$-\mu^{\mathrm{I}}\mu^{\mathrm{II}}|g_{h3}^{\mathrm{I}}|^3|g_{h3}^{\mathrm{II}}|(1-a^{\mathrm{II}})B_1^{\mathrm{II}}\left(\frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}+\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}\right)[\cdots]_F^{-1}D^{\mathrm{I}}D^{\mathrm{II}}$$

are non-negative, we consider the sum of this and $\{\cdots\}_{E,2,1,4}+\{\cdots\}_{E,2,1,6}$. Then noting that

$$[\cdots]_F\geqq 2\mu^{\mathrm{I}}\mu^{\mathrm{II}}|g_{h3}^{\mathrm{I}}|\,|g_{h3}^{\mathrm{II}}|B_1^{\mathrm{I}}B_1^{\mathrm{II}},$$

we can prove that the above sum is not smaller than

$$(2.14)\quad [\cdots]_F^{-1}\mu^{\mathrm{I}}\mu^{\mathrm{II}}|g^{\mathrm{I}}_{h3}|^3|g^{\mathrm{II}}_{h3}|\Big\{2B^{\mathrm{I}}_1B^{\mathrm{II}}_1\Big[\frac{a^{\mathrm{I}}_1}{B^{\mathrm{I}}_1}B^{\mathrm{II}}_3\Big(B^{\mathrm{I}}_1B^{\mathrm{II}}_3-\frac{(\mathrm{Im}\,\nu)^2}{4}|g^{\mathrm{I}}_{h3}|^2|g^{\mathrm{II}}_{h3}|^2\times$$

$$\times\frac{a^{\mathrm{I}}_1a^{\mathrm{II}}_3}{B^{\mathrm{I}}_1B^{\mathrm{II}}_3}\Big)+\frac{a^{\mathrm{I}}_3}{B^{\mathrm{I}}_3}B^{\mathrm{II}}_1\Big(B^{\mathrm{I}}_1B^{\mathrm{II}}_1-\frac{(\mathrm{Im}\,\nu)^2}{4}|g^{\mathrm{I}}_{h3}|^2|g^{\mathrm{II}}_{h3}|^2\frac{a^{\mathrm{I}}_1a^{\mathrm{II}}_1}{B^{\mathrm{I}}_1B^{\mathrm{II}}_1}\Big)\Big]-$$

$$-B^{\mathrm{II}}_1\Big(\frac{a^{\mathrm{I}}_1}{B^{\mathrm{I}}_1}+\frac{a^{\mathrm{I}}_3}{B^{\mathrm{I}}_3}\Big)D^{\mathrm{I}}D^{\mathrm{II}}\Big\}\,.$$

Furthermore the brace of (2.14) is estimated from below by

$$(2.15)\quad \frac{a^{\mathrm{I}}_1}{2B^{\mathrm{I}}_1}B^{\mathrm{II}}_1\Big[(2B^{\mathrm{I}}_1B^{\mathrm{II}}_3)^2-a^{\mathrm{I}}_1a^{\mathrm{II}}_3|g^{\mathrm{I}}_{h3}|^2|g^{\mathrm{II}}_{h3}|^2(\mathrm{Im}\,\nu)^2-\frac{12}{5}\sqrt{D^{\mathrm{I}}D^{\mathrm{II}}}B^{\mathrm{I}}_1B^{\mathrm{II}}_3\Big]+$$

$$+\frac{a^{\mathrm{I}}_3}{2B^{\mathrm{I}}_3}B^{\mathrm{II}}_1\Big[(2B^{\mathrm{I}}_1B^{\mathrm{II}}_1)^2-a^{\mathrm{I}}_1a^{\mathrm{II}}_1|g^{\mathrm{I}}_{h3}|^2|g^{\mathrm{II}}_{h3}|^2(\mathrm{Im}\,\nu)^2-\frac{12}{5}\sqrt{D^{\mathrm{I}}D^{\mathrm{II}}}B^{\mathrm{I}}_1B^{\mathrm{II}}_1\Big]\,;$$

here we utilize the inequality

$$(2.16)\qquad \Big(\frac{5}{6}D^j\Big)^{1/2}\leqq B^j_r\quad (j=\mathrm{I},\ \mathrm{II};\ r=1,\ 3),$$

which is easily deduced by use of (2.11). The non-negativity of (2.15) is due to Lemma 2.2. Also by Lemma 2.2,

$$\{\cdots\}_{E,2,1,k}\geqq 0\quad (k=1,\ 2,\ 3,\ldots,\ 8).$$

Interchanging I and II in the above arguments, we have the same results for the remaining terms.

As a result, we obtain

$$\mathrm{Im}\,[\cdots]_E=(\mathrm{Im}\,\nu)\times[\text{non-negative term}]\,.$$

$\mathrm{Im}\,[\cdots]_E=0$ holds if and only if either $\mathrm{Im}\,\nu=0$ or $\xi'=\mathrm{Re}\,\nu=0$. In the case of $\mathrm{Im}\,\nu=0$, $A^{\mathrm{I}}_r=A^{\mathrm{II}}_r=0$ $(r=1,\ 3)$, hence

$$\begin{cases} [\cdots]_{E,1}=(a^{\mathrm{I}}B^{\mathrm{I}}_1+B^{\mathrm{I}}_3)\{4(1-a^{\mathrm{II}})D^{\mathrm{II}}B^{\mathrm{II}}_1+a^{\mathrm{II}}_1|g^{\mathrm{II}}_{h3}|^2\,\mathrm{Re}\,\nu(B^{\mathrm{II}}_1+B^{\mathrm{II}}_3)\}, \\ [\cdots]_{E,2}=2[(2a^{\mathrm{I}}-1)B^{\mathrm{I}}_1+B^{\mathrm{I}}_3][(2a^{\mathrm{II}}-1)B^{\mathrm{II}}_1+B^{\mathrm{II}}_3](D^{\mathrm{I}}_{23}D^{\mathrm{II}}_{23}+D^{\mathrm{I}}_{31}D^{\mathrm{II}}_{31}+D^{\mathrm{I}}_{12}D^{\mathrm{II}}_{12})+ \\ \quad +(B^{\mathrm{I}}_1+B^{\mathrm{I}}_3)(B^{\mathrm{II}}_1+B^{\mathrm{II}}_3)(B^{\mathrm{I}}_1B^{\mathrm{II}}_3+B^{\mathrm{I}}_3B^{\mathrm{II}}_1)+4(1-a^{\mathrm{I}})(1-a^{\mathrm{II}})[D^{\mathrm{I}}D^{\mathrm{II}}-(D^{\mathrm{I}}_{23}D^{\mathrm{II}}_{23}+ \\ \quad +D^{\mathrm{I}}_{31}D^{\mathrm{II}}_{31}+D^{\mathrm{I}}_{12}D^{\mathrm{II}}_{12})^2]-[\mu^{\mathrm{I}}|g^{\mathrm{II}}_{h3}|(1-a^{\mathrm{II}})(B^{\mathrm{I}}_1+B^{\mathrm{I}}_3)+\mu^{\mathrm{II}}|g^{\mathrm{I}}_{h3}|(1-a^{\mathrm{I}})(B^{\mathrm{II}}_1+ \\ \quad +B^{\mathrm{II}}_3)][D^{\mathrm{I}}D^{\mathrm{II}}-(D^{\mathrm{I}}_{23}D^{\mathrm{II}}_{32}+D^{\mathrm{I}}_{31}D^{\mathrm{II}}_{31}+D^{\mathrm{I}}_{12}D^{\mathrm{II}}_{12})^2](\mu^{\mathrm{I}}|g^{\mathrm{II}}_{h3}|B^{\mathrm{I}}_1+\mu^{\mathrm{II}}|g^{\mathrm{I}}_{h3}|B^{\mathrm{II}}_1)^{-1}. \end{cases}$$

The hypothesis of the lemma implies, in the present case, that $\mathrm{Re}\,\nu>0$ if $\xi'=0$ and D^{I}, $D^{\mathrm{II}}>0$ if $\xi'\neq 0$. Hence for both $\xi'=0$ and $\xi'\neq 0$, the inequality $[\cdots]_{E,1}>0$ can be proved in the same manner as before.

From (2.16) it follows that

$$-\max\left\{\frac{1}{2}B_1^{\mathrm{I}}B_1^{\mathrm{II}}+B_1^{\mathrm{I}}B_3^{\mathrm{II}}+B_3^{\mathrm{I}}B_1^{\mathrm{II}}+2B_3^{\mathrm{I}}B_3^{\mathrm{II}},\ B_1^{\mathrm{I}}B_1^{\mathrm{II}}+2B_3^{\mathrm{I}}B_1^{\mathrm{II}},\ B_1^{\mathrm{I}}B_1^{\mathrm{II}}+2B_1^{\mathrm{I}}B_3^{\mathrm{II}},\ 2B_1^{\mathrm{I}}B_1^{\mathrm{II}}\right\}\times$$
$$\times\sqrt{D^{\mathrm{I}}D^{\mathrm{II}}}+(B_1^{\mathrm{I}}+B_3^{\mathrm{I}})(B_1^{\mathrm{II}}+B_3^{\mathrm{II}})(B_1^{\mathrm{I}}B_3^{\mathrm{II}}+B_3^{\mathrm{I}}B_1^{\mathrm{II}})\geqq\frac{4}{5}B_3^{\mathrm{I}}B_3^{\mathrm{II}}(B_1^{\mathrm{I}}B_3^{\mathrm{II}}+B_3^{\mathrm{I}}B_1^{\mathrm{II}}).$$

Considering this result in connection to the inequalities

$$4(1-a^{\mathrm{I}})(1-a^{\mathrm{II}})-[\mu^{\mathrm{I}}|g_{h3}^{\mathrm{II}}|(1-a^{\mathrm{II}})(B_1^{\mathrm{I}}+B_3^{\mathrm{I}})+\mu^{\mathrm{II}}|g_{h3}^{\mathrm{I}}|(1-a^{\mathrm{I}})(B_1^{\mathrm{II}}+B_3^{\mathrm{II}})]\times$$
$$\times(\mu^{\mathrm{I}}|g_{h3}^{\mathrm{II}}|B_1^{\mathrm{I}}+\mu^{\mathrm{II}}|g_{h3}^{\mathrm{I}}|B_1^{\mathrm{II}})^{-1}\geqq$$
$$\geqq-(\mu^{\mathrm{I}}|g_{h3}^{\mathrm{II}}|B_3^{\mathrm{I}}+\mu^{\mathrm{II}}|g_{h3}^{\mathrm{I}}|B_3^{\mathrm{II}})(\mu^{\mathrm{I}}|g_{h3}^{\mathrm{II}}|B_1^{\mathrm{I}}+\mu^{\mathrm{II}}|g_{h3}^{\mathrm{I}}|B_1^{\mathrm{II}})^{-1},$$

we obtain

$$[\cdots]_{E,2}\geqq\frac{2}{25}B_3^{\mathrm{I}}B_3^{\mathrm{II}}(B_1^{\mathrm{I}}B_3^{\mathrm{II}}+B_3^{\mathrm{I}}B_1^{\mathrm{II}})>0.$$

On the other hand, if $\xi'=\mathrm{Re}\,\nu=0$, then the hypothesis of the lemma implies that $\mathrm{Im}\,\nu\neq 0$. Hence we have

$$\left\{\begin{aligned}
&\mathrm{Re}\,[\cdots]_{E,1}=-\frac{(\mathrm{Im}\,\nu)^2}{4}a_1^{\mathrm{II}}|g_{h3}^{\mathrm{II}}|^2\Big\{2\Big(\frac{a^{\mathrm{I}}a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}+\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}\Big)(B_1^{\mathrm{II}}+B_3^{\mathrm{II}})|g_{h3}^{\mathrm{I}}|^2+\\
&\quad+2(a^{\mathrm{I}}B_1^{\mathrm{I}}+B_3^{\mathrm{I}})\Big(\frac{a_1^{\mathrm{II}}}{B_1^{\mathrm{II}}}+\frac{a_3^{\mathrm{II}}}{B_3^{\mathrm{II}}}\Big)|g_{h3}^{\mathrm{II}}|^2\Big\}<0,\\
&\mathrm{Re}\,[\cdots]_{E,2}=-\frac{(\mathrm{Im}\,\nu)^2}{4}\Big\{\Big(\frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}+\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}\Big)(B_1^{\mathrm{II}}+B_3^{\mathrm{II}})|g_{h3}^{\mathrm{I}}|^2+(B_1^{\mathrm{I}}+B_3^{\mathrm{I}})\Big(\frac{a_1^{\mathrm{II}}}{B_1^{\mathrm{II}}}+\frac{a_3^{\mathrm{II}}}{B_3^{\mathrm{II}}}\Big)\times\\
&\quad\times|g_{h3}^{\mathrm{II}}|^2\Big\}\Big\{\Big(\frac{a_1^{\mathrm{I}}}{B_1^{\mathrm{I}}}B_3^{\mathrm{II}}+\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}}B_1^{\mathrm{II}}\Big)|g_{h3}^{\mathrm{I}}|^2+\Big(\frac{a_1^{\mathrm{II}}}{B_1^{\mathrm{II}}}B_3^{\mathrm{I}}+\frac{a_3^{\mathrm{II}}}{B_3^{\mathrm{II}}}B_1^{\mathrm{I}}\Big)|g_{h3}^{\mathrm{II}}|^2\Big\}<0.
\end{aligned}\right.$$

After all, noting that

$$\mathrm{Im}\,[\mu^{\mathrm{I}}|g_{h3}^{\mathrm{II}}|P^{\mathrm{I}}-\mu_{\mathrm{II}}|g_{h3}^{\mathrm{I}}|P^{\mathrm{II}}]=\mu^{\mathrm{I}}|g_{h3}^{\mathrm{II}}|B_1^{\mathrm{I}}+\mu^{\mathrm{II}}|g_{h3}^{\mathrm{I}}|B_1^{\mathrm{II}}>0,$$

we obtain $\Delta\neq 0$ if δ be chosen in such a way that

$$\delta=\min\left\{\frac{1}{6a_1^{\mathrm{I}}|g_{h3}^{\mathrm{I}}|^2}\min_{\xi'^2=1}D^{\mathrm{I}},\ \frac{1}{6a_1^{\mathrm{II}}|g_{h3}^{\mathrm{II}}|^2}\min_{\xi'^2=1}D^{\mathrm{II}},\ \frac{\delta^{\mathrm{I}}\delta^{\mathrm{II}}}{1+C}\right\},\tag{2.17}$$

where C is an arbitrary number greater than $\delta^{\mathrm{I}}\delta^{\mathrm{II}}$. Q.E.D.

Remark. In the range stated in Lemma 2.3 with δ defined by (2.17) it holds that

$$B_r^j(\xi',\nu)\geqq\left[\frac{1}{2}a_r^j|g_{h3}^j|^2\delta\right]^{1/2}(\xi'^4+|\nu|^2)^{1/4}.\tag{2.18}$$

Indeed, it is obvious in the case $\mathrm{Re}\,\nu\geqq 0$. On the other hand if $\mathrm{Re}\,\nu<0$, then (2.18) is easily shown from the inequality

$$2(B_r^j(\xi',\nu))^2\geqq a_r^j|g_{h3}^j|^2|\,\mathrm{Im}\,\nu|+a_r^j|g_{h3}^j|^2|\,\mathrm{Re}\,\nu|+4a_r^j|g_{h3}^j|^2\delta\xi'^2.$$

2.2. Poisson kernel and Green matrix in R^3_+.

In the same way as in [3, 4], we can construct the Poisson kernel and the Green matrix in the half space. Indeed, let $\alpha_8=(\alpha_8^{(j,k)})$ be the inverse matrix of $\alpha^{(8)}$ and

$$(2.19)\qquad \tilde{H}_1(y,\nu)=\frac{1}{2\pi i}\int_{\gamma_+}\hat{\mathscr{A}}(x_0,t_0,w;iy',i\xi_3,\nu)\alpha_8(y',\xi_3,\nu)\frac{e^{iy_3\xi_3}}{M(y',\xi_3,\nu)}d\xi_3,$$

where γ_+ is a contour enclosing all $\xi_3^{+(r)\mathrm{I}}$ and $\xi_3^{-(r)\mathrm{II}}$ $(r=1,2,3,4)$;

$$(2.20)\qquad H_1(y,\tau)=\frac{1}{(2\pi)^3 i}\int_{R^2}e^{i(y',\xi')}d\xi'\int_{\sigma-i\infty}^{\sigma+i\infty}e^{\tau\nu}\tilde{H}_1(\xi',y_3,\nu)d\nu\quad(\sigma>-\delta\xi'^2);$$

$$(2.21)\qquad H_0(y,\tau;\xi,\tau_0)=Z_0(y-\xi,\tau-\tau_0;x_0,t_0,w)-\int_{\tau_0}^{\tau}d\tau_0'\int_{R^2}H_1(y-y_0',\tau-\tau_0')\times$$
$$\times B(x_0,t_0,w;\nabla_{y_0})Z_0(y_0-\xi,\tau_0'-\tau;x_0,t_0,w)_{|y_{0,3}=0}dy_0'$$

where Z_0 is the fundamental solution of

$$\frac{\partial}{\partial\tau}W=\mathscr{A}(x_0,t_0,w;\nabla_{y_0})W.$$

In order to evaluate H_1, we extend the domain of $\tilde{H}_1$ to the double complex space. Let q be $\nu+\delta\xi'^2$.

Lemma 2.4. *There exists a positive constant β such that*

$$(2.22)\qquad 2[B_r^j(\zeta',q-\delta\zeta'^2)]^2\geqq\frac{\delta}{4}a_r^j|g_{h3}^j|^2(\xi'^4+|q|^2)^{1/2}$$

if $\zeta'=\xi'+i\eta'\in C^2$ and $q\in C^1$ satisfy

$$\xi'^4+|q|^2>0,\quad \mathrm{Re}\,q\geqq 0,\quad |\eta'|\leqq\beta(\xi'^4+|q|^2)^{1/4}.$$

Proof. Applying the mean value theorem with respect to the variable η' in the range $\{\eta'\,|\,|\eta'|\leqq(\xi'^4+|q|^2)^{1/4}\}$ to the function $2[B_r^j(\zeta',q-\delta\zeta'^2)]^2$ and noting that

$$|2\nabla_{\eta'}[B_r^j(\zeta',q-\delta\zeta'^2)]^2(\eta'\to\varepsilon\eta')\cdot\eta'|\leqq$$
$$\leqq 6[\max_{\xi'^2=1}D^j(\xi')+a_r^j|g_{h3}^j|^2\delta](\xi'^4+|q|^2)^{1/4}|\eta'|\quad(0<\varepsilon<1),$$

we can easily prove (2.22) if we take

$$\beta=\frac{1}{24}\min_{\substack{r=1,2,3,4\\ j=\mathrm{I},\mathrm{II}}}\left\{\frac{\min[a_r^j|g_{h3}^j|^2\delta,\sqrt{a_r^j|g_{h3}^j|^2\delta}]}{\max_{\xi'^2=1}D^j(\xi')+a_r^j|g_{h3}^j|^2\delta}\right\},\qquad \text{Q.E.D.}$$

Lemma 2.5. *There exist positive constants β_5 and β_6 such that*

$$(2.23)\qquad 2[B_r^j(\zeta',q-\delta\zeta'^2)]^2\geqq\frac{\delta}{8}a_r^j|g_{h3}^j|^2(\xi'^4+|q|^2)^{1/2}$$

in the range

$$(2.24)\qquad \xi'^4+|q|^2>0,\ \mathrm{Re}\,q\geqq-\beta_5|\mathrm{Im}\,q|,\quad |\eta'|\leqq\beta_6(\xi'^4+|q|^2)^{1/4}.$$

Proof. From (2.18) it follows that

$$2[B_r^j(\xi', q-\delta\xi'^2)]^2 \geqq \frac{\delta}{2} a_r^j |g_{h3}^j|^2 (\xi'^4+|q|^2)^{1/2}$$

for any ξ' and q satisfying

$$\xi'^4+|q|^2>0, \quad \mathrm{Re}\, q \geqq -\beta_5 |\,\mathrm{Im}\, q| \quad \left(\beta_5=\frac{2\beta}{1-\beta^2}\right).$$

Indeed, it is obvious when Re $q \geqq 0$. On the other hand if Re $q<0$, we apply Lemma 2.4 to the right hand side of

$$B_r^j(\xi', qe^{2i\gamma}-\delta\xi'^2)=B_r^j(e^{-i\gamma}\xi', q-\delta e^{-2i\gamma}\xi'^2) \quad (\mathrm{Re}\, q=0).$$

Using Lemma 2.4 once more, we have (2.23) in the range (2.24) with $\beta_6=\beta/2$.

Q.E.D.

Lemma 2.6. *There exist positive constants β_7 ($\leqq \beta_5$) and β_8 ($\leqq \beta_6$) such that*

(2.25) $$|\det \alpha^{(8)}|(\zeta', q-\delta\zeta'^2) \geqq C_2(\xi'^4+|q|^2)^8$$

for any (ζ', q) belonging to

(2.26) $$\xi'^4+|q|^2>0, \quad \mathrm{Re}\, q \geqq -\beta_7 |\,\mathrm{Im}\, q|, \quad |\eta'| \leqq \beta_8(\xi'^4+|q|^2)^{1/4}.$$

Proof. At first we shall obtain estimates from below and from above of $|[\cdots]_E|(\xi', q-\delta\xi'^2)$ for any (ξ', q) satisfying

$$\xi'^4+|q|^2=1, \ \mathrm{Re}\, q \geqq 0.$$

At this stage, the real part of $[\cdots]_E$ is needed. From Lemma 2.2, (2.17) and (2.18) it follows that

$$\begin{cases} \mathrm{Re}\,[\cdots]_{E,1}(\xi', q-\delta\xi'^2) \geq 2a^{\mathrm{I}} a_1^{\mathrm{II}} |g_{h3}^{\mathrm{II}}|^2 B_1^{\mathrm{I}} B_1^{\mathrm{II}} \delta\sqrt{1-(\mathrm{Im}\, q)^2} - \\ \quad -\frac{14}{\delta^2}(\mathrm{Im}\, q)^2 B_1^{\mathrm{I}} B_1^{\mathrm{II}} J^{\mathrm{II}} \quad (J^j \equiv \max\limits_{\xi'^2=1} D^j(\xi')+a_1^j |g_{h3}^{\mathrm{II}}|^2 (j=\mathrm{I}, \mathrm{II})) \\ \mathrm{Re}\,[\cdots]_{E,1} \leqq 8\sqrt{2} B_1^{\mathrm{I}} B_1^{\mathrm{II}} J^{\mathrm{II}} \sqrt{1-|\,\mathrm{Im}\, q|^2} - \frac{(\mathrm{Im}\, q)^2}{2B_1^{\mathrm{I}} B_1^{\mathrm{II}}} a_3^{\mathrm{I}} (a_1^{\mathrm{II}})^2 |g_{h3}^{\mathrm{I}}|^2 |g_{h3}^{\mathrm{II}}|^4 \delta; \\ \mathrm{Re}\,[\cdots]_{E,2}(\xi', q-\delta\xi'^2) \geqq \left[\frac{2}{25}\sqrt{a^{\mathrm{I}} a^{\mathrm{II}}}(\sqrt{a^{\mathrm{I}}}+\sqrt{a^{\mathrm{II}}}) - \frac{64}{\delta^2}(\mathrm{Im}\, q)^2\right](B_1^{\mathrm{I}} B_1^{\mathrm{II}})^2, \\ \mathrm{Re}\,[\cdots]_{E,2} \leqq -2(\mathrm{Im}\, q)^2 \left(\frac{a_3^{\mathrm{I}}}{B_3^{\mathrm{I}}} B_3^{\mathrm{II}} |g_{h3}^{\mathrm{I}}|^2 + \frac{a_3^{\mathrm{II}}}{B_3^{\mathrm{II}}} B_3^{\mathrm{I}} |g_{h3}^{\mathrm{II}}|^2\right)^2 + \\ \quad +8(B_3^{\mathrm{I}} B_3^{\mathrm{II}})^{-2}(1-|\,\mathrm{Im}\, q|^2)(J^{\mathrm{I}} J^{\mathrm{II}})^2 + \frac{3}{5} 2^6 \sqrt{1-|\,\mathrm{Im}\, q|^2} J^{\mathrm{I}} J^{\mathrm{II}} + \\ \quad +4 \max\limits_{\xi'^2=1} D^{\mathrm{I}}(\xi') \cdot \max\limits_{\xi'^2=1} D^{\mathrm{II}}(\xi') \cdot \xi'^4; \end{cases}$$

here we use the inequalities

$$B_3^j \leqq B_1^j \quad \text{if} \quad \mathrm{Re}\, q \geqq 0,$$

$$2[B_r^j(\xi',\, q-\delta\xi'^2)]^2 - a_r^j|g_{h3}^j|^2|\,\mathrm{Im}\, q| \leqq 2\sqrt{2J^j}\sqrt{\xi'^4+(\mathrm{Re}\, q)^2} \quad (j=\mathrm{I},\ \mathrm{II}).$$

Then we have

$$\begin{cases} \mathrm{Re}\,[\cdots]_{E,1}(\xi',\, q-\delta\xi'^2) \geqq \frac{1}{4} a^{\mathrm{I}} a_1^{\mathrm{II}} \sqrt{a_1^{\mathrm{I}} a_1^{\mathrm{II}}}\, |g_{h3}^{\mathrm{I}}|\, |g_{h3}^{\mathrm{II}}|^3 \delta^2 \{2 - C_4(C_{3,1}^{\mathrm{I},\mathrm{II}})\} \equiv C_{5,1}^{\mathrm{I},\mathrm{II}} \\ \qquad \text{if} \quad (\mathrm{Im}\, q)^2 \leqq C_4(C_{3,1}^{\mathrm{I},\mathrm{II}})/2 \\ (C_{3,1}^{\mathrm{I},\mathrm{II}} \equiv [a^{\mathrm{I}} a_1^{\mathrm{II}} |g_{h3}^{\mathrm{II}}|^2 \delta^3/(14 J^{\mathrm{II}})]^2, \quad C_4(C_{3,1}^{\mathrm{I},\mathrm{II}}) \equiv C_{3,1}^{\mathrm{I},\mathrm{II}}[\sqrt{(C_{3,1}^{\mathrm{I},\mathrm{II}})^2+4} - C_{3,1}^{\mathrm{I},\mathrm{II}}]), \\ \mathrm{Re}\,[\cdots]_{E,1} \leqq -a_3^{\mathrm{I}}(a_1^{\mathrm{II}})^2 |g_{h3}^{\mathrm{I}}|^2 |g_{h3}^{\mathrm{II}}|^4 \delta C_4(C_{3,2}^{\mathrm{I},\mathrm{II}})/(16\sqrt{J^{\mathrm{I}} J^{\mathrm{II}}}) \equiv -C_{5,1}^{\mathrm{I},\mathrm{II}} \\ \qquad \text{if} \quad 1 \geqq (\mathrm{Im}\, q)^2 \geqq C_4(C_{3,2}^{\mathrm{I},\mathrm{II}})/2 \\ \quad (C_{3,2}^{\mathrm{I},\mathrm{II}} \equiv 2^{15}\{J^{\mathrm{I}} J^{\mathrm{II}}/[a_3^{\mathrm{I}}(a_1^{\mathrm{II}})^2 |g_{h3}^{\mathrm{I}}|^2 |g_{h3}^{\mathrm{II}}|^4 \delta]\}^2); \\ \mathrm{Re}\,[\cdots]_{E,2}(\xi',\, q-\delta\xi'^2) \geqq \frac{\delta^2}{100}\sqrt{a^{\mathrm{I}} a^{\mathrm{II}}}(\sqrt{a^{\mathrm{I}}}+\sqrt{a^{\mathrm{II}}}) a_1^{\mathrm{I}} a_1^{\mathrm{II}} |g_{h3}^{\mathrm{I}}|^2 |g_{h3}^{\mathrm{II}}|^2 \equiv C_{5,3} \\ \qquad \text{if} \quad (\mathrm{Im}\, q)^2 \leqq 40^{-2}\delta^2\sqrt{a^{\mathrm{I}} a^{\mathrm{II}}}(\sqrt{a^{\mathrm{I}}}+\sqrt{a^{\mathrm{II}}}) \equiv C_{4,3}, \\ \mathrm{Re}\,[\cdots]_{E,2} \leqq -(a_3^{\mathrm{I}} a_3^{\mathrm{II}} |g_{h3}^{\mathrm{I}}|^2 |g_{h3}^{\mathrm{II}}|^2 \delta)^2 J^{\mathrm{I}} J^{\mathrm{II}} C_4(C_{3,4})/16 \equiv -C_{5,4} \\ \qquad \text{if} \quad 1 \geqq (\mathrm{Im}\, q)^2 \geqq C_4(C_{3,4})/2 \\ \quad (C_{3,4} \equiv 4 J^{\mathrm{I}} J^{\mathrm{II}}(8 J^{\mathrm{I}} J^{\mathrm{II}} + 43)(a_3^{\mathrm{I}} a_3^{\mathrm{II}} |g_{h3}^{\mathrm{I}}|^2 |g_{h3}^{\mathrm{II}}|^2 \delta)^{-4}). \end{cases}$$

As a result, if

$$(\mathrm{Im}\, q)^2 \leqq \frac{1}{2}\min\{C_4(C_{3,1}^{\mathrm{I},\mathrm{II}}),\ C_4(C_{3,1}^{\mathrm{II},\mathrm{I}}),\ 2C_{4,3}\} \equiv C_6,$$

then

$$\mathrm{Re}\,[\cdots]_E \geqq (\mu^{\mathrm{II}})^2 |g_{h3}^{\mathrm{I}}|^2 C_{5,1}^{\mathrm{I},\mathrm{II}} + (\mu^{\mathrm{I}})^2 |g_{h3}^{\mathrm{II}}|^2 C_{5,1}^{\mathrm{II},\mathrm{I}} + \mu^{\mathrm{I}}\mu^{\mathrm{II}} |g_{h3}^{\mathrm{I}}|\, |g_{h3}^{\mathrm{II}}| C_{5,3},$$

and if

$$1 \geqq (\mathrm{Im}\, q)^2 \geqq \frac{1}{2}\max\{C_4(C_{3,2}^{\mathrm{I},\mathrm{II}}),\ C_4(C_{3,2}^{\mathrm{II},\mathrm{I}}),\ C_4(C_{3,4})\} \equiv C_7,$$

then

$$\mathrm{Re}\,[\cdots]_E \leqq -(\mu^{\mathrm{II}})^2 |g_{h3}^{\mathrm{I}}|^2 C_{5,2}^{\mathrm{I},\mathrm{II}} - (\mu^{\mathrm{I}})^2 |g_{h3}^{\mathrm{II}}|^2 C_{5,2}^{\mathrm{II},\mathrm{I}} - \mu^{\mathrm{I}}\mu^{\mathrm{II}} |g_{h3}^{\mathrm{I}}|\, |g_{h3}^{\mathrm{II}}| C_{5,4}.$$

Referring to the proof of Lemma 2.3, we can easily show that

$$|\,\mathrm{Im}\,[\cdots]_E|\,(\xi',\, q-\delta\xi'^2) \geqq C_8$$

for any ξ', q belonging to

$$\xi'^4 + |q|^2 = 1, \quad \mathrm{Re}\, q \geqq 0, \quad C_6 \leqq (\mathrm{Im}\, q)^2 \leqq C_7.$$

Therefore we deduce from the homogeneity property of $|\det \alpha^{(8)}|$ that

$$|\det \alpha^{(8)}|(\xi', q-\delta\xi'^3) \geqq C_9(\xi'^4+|q|^2)^8$$

for any ξ' and q satisfying

$$\xi'^4+|q|^2>0, \quad \mathrm{Re}\, q \geqq 0.$$

Since

$$\sup\{|\nabla_{\eta'}|\det \alpha^{(8)}|(\zeta', q-\delta\zeta'^2)|;\ |\eta'| \leqq \beta_6(\xi'^4+|q|^2)^{1/4}\} \leqq C_{10}(\xi'^4+|q|^2)^{31/4}$$

holds, we obtain (2.25) with $C_2=C_9/2$, $\beta_7=2\beta_9 C/(1-\beta_9^2)$, $\beta_8=\beta_9 C'/2$ in the same way as in the proof of Lemma 2.5 where

$$\beta_9=C_9/(2C_{10}), \quad 0<\forall C<(1-C_9^2)\beta_5/(2\beta_9), \quad 0<\forall C'<2\beta_6/\beta_9. \qquad \text{Q.E.D.}$$

As in [4], we can easily prove the following basic estimates for H_0 and H_1.

Lemma 2.7.

$$\begin{cases} |D_\tau^r D_y^s H_0(y, \tau;\ \xi, \tau_0)| \leqq C_{11}(\tau-\tau_0)^{-(2r+|s|+3)/2} \exp\left[-d\dfrac{|y-\xi|^2}{\tau-\tau_0}\right], \\ |D_\tau^r D_y^s H_1(y, \tau)| \leqq C_{12}\tau^{-(2r+|s|+3)/2} \exp\left[-d\dfrac{|y|^2}{\tau}\right]. \end{cases}$$

2.3. Solution of (2.1).

For the present problem, unlike that in [3, 4], we need to introduce the systems of coverings $\{\omega_k^{\mathrm{I}}\}$, $\{\Omega_k^{\mathrm{I}}\}$ and $\{\omega_l^{\mathrm{II}}\}$, $\{\Omega_l^{\mathrm{II}}\}$ of $\overline{\Omega^{\mathrm{I}}}$ and $\overline{\Omega^{\mathrm{II}}}$ respectively. They are constructed in much the same manner as that in [3] except $\{\omega_l^{\mathrm{II}}\}$ and $\{\Omega_l^{\mathrm{II}}\}$ which intersect Γ^{I}. Since $\{\omega_{k''}^{\mathrm{I}}\}$ and $\{\Omega_{k''}^{\mathrm{I}}\}$ are constructed by using the local rectangular coordinate system $\{\hat{x}_0\}=\{(\hat{x}_{0,1}, \hat{x}_{0,2}, \hat{x}_{0,3})\}$ with the center at $\xi_{k''} \in \Gamma^{\mathrm{I}}$ as follows:

$$\begin{cases} \omega_{k''}^{\mathrm{I}}=\left\{\hat{x}_0 \mid |\hat{x}_{0,j}| \leqq \dfrac{1}{2}\beta_4\lambda (j=1, 2), \quad 0 \leqq \hat{x}_{0,3}-\hat{F}_0^{\mathrm{I}}(\hat{x}_0';\ \xi_{k''}) \leqq \beta_4\lambda\right\}, \\ \Omega_{k''}^{\mathrm{I}}=\{\hat{x}_0 \mid |\hat{x}_{0,j}| \leqq \beta_4\lambda (j=1, 2), \quad 0 \leqq \hat{x}_{0,3}-\hat{F}_0^{\mathrm{I}}(\hat{x}_0';\ \xi_{k''}) \leqq 2\beta_4\lambda\}, \end{cases}$$

where $\Pi_{x_0}^{\hat{x}_0} F_0^{\mathrm{I}}(x_0)=\hat{x}_{0,3}-\hat{F}_0^{\mathrm{I}}(\hat{x}_0';\ \xi_{k''})$, we define $\{\omega_{l''=k''}^{\mathrm{II}}\}$ and $\{\Omega_{l''=k''}^{\mathrm{II}}\}$ by

$$\begin{cases} \omega_{l''=k''}^{\mathrm{II}}=\left\{\hat{x}_0 \mid |\hat{x}_{0,j}| \leqq \dfrac{1}{2}\beta_4\lambda (j=1, 2), \quad 0 \geqq \hat{x}_{0,3}-\hat{F}_0^{\mathrm{I}}(\hat{x}_0';\ \xi_{l''=k''}) \geqq -\beta_4\lambda\right\}, \\ \Omega_{l''=k''}^{\mathrm{II}}=\{\hat{x}_0 \mid |\hat{x}_{0,j}| \leqq \beta_4\lambda (j=1, 2), \quad 0 \geqq \hat{x}_{0,3}-\hat{F}_0^{\mathrm{I}}(\hat{x}_0';\ \xi_{l''=k''}) \geqq -2\beta_4\lambda\}. \end{cases}$$

When $\{\omega_l^{\mathrm{II}}\}$ and $\{\Omega_l^{\mathrm{II}}\}$ intersect Γ, we use the notation $l=l'''$. In accordance with these coverings, the functions $\{\zeta_k^{\mathrm{I}}(x_0), \eta_k^{\mathrm{I}}(x_0)\}$, $\{\zeta_l^{\mathrm{II}}(x_0), \eta_l^{\mathrm{II}}(x_0)\}$ are defined as in [3].

Then the regularizers R^{I} and R^{II} of the problem $(2.1)_\tau$, which is the problem on the time interval $(\tau, \tau+h]$ $(\forall\tau \geqq 0, \forall h \leqq T-\tau)$, are defined by

$$\begin{cases} R^{\mathrm{I}}(\mathscr{B}^{\mathrm{I}}, \mathscr{B}^{\mathrm{II}}, \psi^{\mathrm{I}}, \psi^{\mathrm{II}})=\sum_k \eta_k^{\mathrm{I}}(x_0) W^{\mathrm{I}}(x_0, t_0), \\ R^{\mathrm{II}}(\mathscr{B}^{\mathrm{I}}, \mathscr{B}^{\mathrm{II}}, \psi^{\mathrm{I}}, \psi^{\mathrm{II}}, \phi_e)=\sum_l \eta_l^{\mathrm{II}}(x_0) W^{\mathrm{II}}(x_0, t_0) \end{cases} \tag{2.28}$$

where

$$W^{\mathrm{I}}_{k'}(x_0, t_0) = R^{\mathrm{I}}_{k'}(\mathscr{B}^{\mathrm{I}}) \equiv$$

$$\equiv \int_{\tau}^{t_0} d\tau_0 \int_{\Omega^{\mathrm{I}}_{k'}} Z_0(x_0 - y, t_0 - \tau_0; \xi_{k'}, \tau_0; w^{\mathrm{I}}) \zeta^{\mathrm{I}}_{k'}(y) \mathscr{B}^{\mathrm{I}}(y, \tau_0, w^{\mathrm{I}}) dy,$$

$$W^{\mathrm{II}}_{l'}(x_0, t_0) = R^{\mathrm{II}}_{l'}(\mathscr{B}^{\mathrm{II}}) \equiv$$

$$\equiv \int_{\tau}^{t_0} d\tau_0 \int_{\Omega^{\mathrm{II}}_{l'}} Z_0(x_0 - y, t_0 - \tau_0; \xi_{l'}, \tau_0; w^{\mathrm{II}}) \zeta^{\mathrm{II}}_{l'}(y) \mathscr{B}^{\mathrm{II}}(y, \tau_0, w^{\mathrm{II}}) dy,$$

$$W^{\mathrm{I}}_{k''}(x_0, t_0) = \Pi^{\bar{x}_0}_{x_0} \overline{W}^{\mathrm{I}}_{k''}(\bar{x}_0, t_0), \quad W^{\mathrm{II}}_{l''}(x_0, t_0) = \Pi^{\bar{x}_0}_{x_0} \overline{W}^{\mathrm{II}}_{l''}(\bar{x}_0, t_0);$$

$$W^{\mathrm{II}}_{l'''}(x_0, t_0) = \Pi^{\bar{x}_0}_{x_0} \overline{W}^{\mathrm{II}}_{l'''}(\bar{x}, t_0),$$

$$\overline{W}^{\mathrm{I}}_{k''}(\bar{x}_0, t_0) = \bar{R}^{\mathrm{I}}_{k''}(\mathscr{B}^{\mathrm{I}}, \mathscr{B}^{\mathrm{II}}) + {}'\bar{R}^{\mathrm{I}}_{k''}(\psi^{\mathrm{I}}, \psi^{\mathrm{II}}),$$

$$\overline{W}^{\mathrm{II}}_{l''}(\bar{x}_0, t_0) = \bar{R}^{\mathrm{II}}_{l''}(\mathscr{B}^{\mathrm{I}}, \mathscr{B}^{\mathrm{II}}) + {}'\bar{R}^{\mathrm{II}}_{l''}(\psi^{\mathrm{I}}, \psi^{\mathrm{II}}),$$

$$\overline{W}^{\mathrm{II}}_{l'''}(\bar{x}_0, t_0) = \bar{R}^{\mathrm{II}}_{l'''}(\mathscr{B}^{\mathrm{II}}) + {}'\bar{R}^{\mathrm{II}}_{l'''}(\phi_e),$$

$$\begin{pmatrix} \bar{R}^{\mathrm{I}}_{k''}(\mathscr{B}^{\mathrm{I}}, \mathscr{B}^{\mathrm{II}}) \\ \bar{S}_{l''=k''}(\mathscr{B}^{\mathrm{I}}, \mathscr{B}^{\mathrm{II}}) \end{pmatrix} (\bar{x}_0, t_0) =$$

$$= \int_{\tau}^{t_0} d\tau_0 \int_{K_{k''}} H_0^{(k'')}(\bar{x}_0, t_0; \bar{y}, \tau_0) \begin{pmatrix} \bar{\zeta}^{\mathrm{I}}_{k''}(\bar{y}) \overline{\mathscr{B}}^{\mathrm{I}}(\bar{y}, \tau_0, w^{\mathrm{I}}) \\ \bar{\zeta}^{\mathrm{II}}_{l''=k''}(\bar{y}', -\bar{y}_3) \overline{\mathscr{B}}^{\mathrm{II}}(\bar{y}', -\bar{y}_3, \tau_0, w^{\mathrm{II}}) \end{pmatrix} d\bar{y},$$

$$\begin{pmatrix} {}'\bar{R}^{\mathrm{I}}_{k''}(\psi^{\mathrm{I}}, \psi^{\mathrm{II}}) \\ {}'\bar{S}_{l''=k''}(\psi^{\mathrm{I}}, \psi^{\mathrm{II}}) \end{pmatrix} (\bar{x}_0, t_0) =$$

$$= \int_{\tau}^{t_0} d\tau_0 \int_{{}'K_{k''}} H_1^{(k'')}(\bar{x}_0 - \bar{y}', t_0 - \tau_0) [\bar{\zeta}^{\mathrm{II}}_{l''=k''}(\bar{y}') \overline{\psi}^{\mathrm{II}}(\bar{y}', \tau_0) -$$

$$- \bar{\zeta}^{\mathrm{I}}_{k''}(\bar{y}') \overline{\psi}^{\mathrm{I}}(\bar{y}', \tau_0)] d\bar{y}',$$

$$\bar{R}^{\mathrm{II}}_{l''}(\mathscr{B}^{\mathrm{I}}, \mathscr{B}^{\mathrm{II}})(\bar{x}_0, t_0) = \bar{S}_{l''}(\mathscr{B}^{\mathrm{I}}, \mathscr{B}^{\mathrm{II}})(\bar{x}_0', -\bar{x}_{0,3}, t_0),$$

$${}'\bar{R}^{\mathrm{II}}_{l''}(\psi^{\mathrm{I}}, \psi^{\mathrm{II}})(\bar{x}_0, t_0) = {}'\bar{S}_{l''}(\psi^{\mathrm{I}}, \psi^{\mathrm{II}})(\bar{x}_0', -\bar{x}_{0,3}, t_0);$$

$$\bar{R}^{\mathrm{II}}_{l'''}(\mathscr{B}^{\mathrm{II}})(\bar{x}_0, t_0) = \int_{\tau}^{t_0} d\tau_0 \int_{K_{l'''}} H_0^{(l''')}(\bar{x}_0, t_0; \bar{y}, \tau_0) \bar{\zeta}^{\mathrm{II}}_{l'''}(\bar{y}) \overline{\mathscr{B}}^{\mathrm{II}}(\bar{y}, \tau_0, w^{\mathrm{II}})\, d\bar{y},$$

$${}'\bar{R}^{\mathrm{II}}_{l'''}(\phi_e)(\bar{x}_0, t_0) = \int_{\tau}^{t_0} d\tau_0 \int_{{}'K_{l'''}} H_1^{(l''')}(\bar{x}_0 - \bar{y}', t_0 - \tau_0) \bar{\zeta}^{\mathrm{II}}_{l'''}(\bar{y}') \overline{\phi}_e(\bar{y}', \tau_0) d\bar{y}';$$

$$\begin{cases} K_{k''} = \Pi^{\hat{x}_0}_{\bar{x}_0} \Omega^{\mathrm{I}}_{k''} = \{|\bar{x}_{0,j}| \leqq \beta_4 \lambda \quad (j = 1, 2),\ 0 \leqq \bar{x}_{0,3} \leqq 2\beta_4 \lambda\}, \\ {}'K_{k''} = \Pi^{\hat{x}_0}_{\bar{x}_0}(\Gamma \cap \Omega^{\mathrm{I}}_{k''}) = \{|\bar{x}_{0,j}| \leqq \beta_4 \lambda \quad (j = 1, 2),\ \bar{x}_{0,3} = 0\}, \\ \qquad (\bar{x}_{0,j} = \hat{x}_{0,j} \quad (j = 1, 2),\ \bar{x}_{0,3} = \hat{x}_{0,3} - \hat{F}^{\mathrm{I}}_0(\hat{x}_0'; \xi_{k''})), \\ K_{l'''} = \Pi^{\hat{x}_0}_{\bar{x}_0} \Omega^{\mathrm{II}}_{l'''} = \{|\bar{x}_{0,j}| \leqq \beta_4' \lambda \quad (j = 1, 2),\ 0 \leqq \bar{x}_{0,3} \leqq 2\beta_4' \lambda\}, \\ {}'K_{l'''} = \Pi^{\hat{x}_0}_{\bar{x}_0}(\Gamma \cap \Omega^{\mathrm{II}}_{l'''}) = \{|\bar{x}_{0,j}| \leqq \beta_4' \lambda \quad (j = 1, 2),\ \bar{x}_{0,3} = 0\} \\ \qquad \begin{pmatrix} \bar{x}_{0,j} = \hat{x}_{0,j} \quad (j = 1, 2),\ \bar{x}_{0,3} = \hat{x}_{0,3} - \hat{F}_0(\hat{x}_0'; \xi_{l'''}) \\ \Pi^{x_0}_{\hat{x}_0} F_0(x_0) = \hat{x}_{0,3} - \hat{F}_0(\hat{x}_0'; \xi_{l'''}) \end{pmatrix} \end{cases}$$

$Z_0(x_0-y,\ t_0-\tau_0;\ \xi_{k'},\ \tau;\ w^{\rm I})$ and $Z_0(x_0-y,\ t_0-\tau_0;\ \xi_{l'},\ \tau;\ w^{\rm II})$ are the fundamental solutions of

$$\frac{\partial}{\partial t_0}W=\mathscr{A}^{\rm I}(\xi_{k'},\ \tau,\ w^{\rm I};\ \mathring{\nabla})W \quad \text{and} \quad \frac{\partial}{\partial t_0}W=\mathscr{A}^{\rm II}(\xi_{l'},\ \tau,\ w^{\rm II};\ \mathring{\nabla})W$$

respectively;

$H_0^{(k'')}$, $H_1^{(k'')}$ and $Z_0^{(k'')}$ are the Green matrix, the Poisson kernel and the fundamental solution defined by the formulae analogous to (2.19)–(2.21) for the problem:

$$(2.6)_{k''}\quad \begin{cases} \dfrac{\partial}{\partial t_0}\overline{W}=\mathscr{A}^{(k'')}(\xi_{k''},\ \tau,\ w;\ \overline{\nabla})\overline{W}+\begin{pmatrix} \bar{\zeta}^{\rm I}_{k''}(\bar{x}_0)\bar{\mathscr{B}}^{\rm I}(\bar{x}_0,\ t_0,\ w^{\rm I}) \\ \bar{\zeta}^{\rm II}_{l''}(\bar{x}_0',\ -\bar{x}_{0,3})\bar{\mathscr{B}}^{\rm II}(\bar{x}_0',\ -\bar{x}_{0,3},\ t_0,\ w^{\rm II}) \end{pmatrix}, \\ \overline{W}_{|t_0=0}=0, \\ B^{(k'')}(\xi_{k''},\ \tau,\ w;\ \overline{\nabla})\overline{W}_{|\bar{x}_{0,3}=0}=[\bar{\zeta}^{\rm II}_{k''}\bar{\psi}^{\rm II}-\bar{\zeta}^{\rm I}_{k''}\bar{\psi}^{\rm I}](\bar{x}_0',\ t_0) \end{cases}$$

[For the definitions of $\mathscr{A}^{(k'')}$ and $B^{(k'')}$, see [3, 4]].

As in [3, 4], we shall find out the solution $\tilde{w}$ of (2.1) in the form of $\tilde{w}={}^t(\tilde{w}^{\rm I},\ \tilde{w}^{\rm II})={}^t(R^{\rm I}(u^{\rm I},\ u^{\rm II},\ 'u^{\rm I},\ 'u^{\rm II}),\ R^{\rm II}(u^{\rm I},\ u^{\rm II},\ 'u^{\rm I},\ u^{\rm II},\ u^e))$, where

$$(2.29)\quad (u^{\rm I},\ u^{\rm II},\ 'u^{\rm I},\ 'u^{\rm II},\ u^e)=\sum_{\nu=0}^{\infty}(u^{\rm I}_\nu,\ u^{\rm II}_\nu,\ 'u^{\rm I}_\nu,\ 'u^{\rm II}_\nu,\ u^e_\nu),$$

$$(u^{\rm I}_\nu,\ u^{\rm II}_\nu,\ 'u^{\rm I}_\nu,\ u^{\rm II}_\nu,\ u^e_\nu)=P(u^{\rm I}_{\nu-1},\ u^{\rm II}_{\nu-1},\ 'u^{\rm I}_{\nu-1},\ u^{\rm II}_{\nu-1},\ u^e_{\nu-1}),$$

$$P=(P^{\rm I},\ P^{\rm II},\ P',\ P''),\quad (u^{\rm I}_0,\ u^{\rm II}_0,\ 'u^{\rm I}_0,\ 'u^{\rm II}_0,\ u^e_0)=(\mathscr{B}^{\rm I},\ \mathscr{B}^{\rm II},\ \psi^{\rm I},\ \psi^{\rm II},\ \phi_e);$$

$$P^{\rm I}(u^{\rm I},\ u^{\rm II},\ 'u^{\rm I},\ 'u^{\rm II},\ u^e)=\sum_k \eta^{\rm I}_k(x_0)[\mathscr{A}^{\rm I}(\xi_k,\ \tau,\ w^{\rm I};\ \mathring{\nabla})-$$
$$-\mathscr{A}^{\rm I}(x_0,\ t_0,\ w^{\rm I};\ \mathring{\nabla})]W^{\rm I}_k+\sum_{k''}\eta^{\rm I}_{k''}(x_0)\Pi^{\bar{x}_0}_{x_0}[\mathscr{A}^{\rm I}_{k''}(\xi_{k''},\ \tau,\ w^{\rm I};\ \overline{\nabla})-$$
$$-\mathscr{A}^{\rm I}_{k''}(\xi_{k''},\ \tau,\ w^{\rm I};\ \overline{\nabla}-\overline{\nabla}\hat{F}^{\rm I}_0\overline{\nabla}_3)]\overline{W}^{\rm I}_{k''}+\sum_k[\eta^{\rm I}_k(x_0)\mathscr{A}^{\rm I}(x_0,\ t_0,\ w^{\rm I};\ \mathring{\nabla})-$$
$$-\mathscr{A}^{\rm I}(x_0,\ t_0,\ w^{\rm I};\ \mathring{\nabla})\eta^{\rm I}_k(x_0)]W^{\rm I}_k,$$

$$P^{\rm II}(u^{\rm I},\ u^{\rm II},\ 'u^{\rm I},\ 'u^{\rm II},\ u^e)=\sum_l \eta^{\rm II}_l(x_0)[\mathscr{A}^{\rm II}(\xi_l,\ \tau,\ w^{\rm II};\ \mathring{\nabla})-$$
$$-\mathscr{A}^{\rm II}(x_0,\ t_0,\ w^{\rm II};\ \mathring{\nabla})]W^{\rm II}_l+\sum_{l''}\eta^{\rm II}_{l''}(x_0)\Pi^{\bar{x}_0}_{x_0}[\mathscr{A}^{\rm II}_{l''}(\xi_{l''},\ \tau,\ w^{\rm II};\ \overline{\nabla})-$$
$$-\mathscr{A}^{\rm II}_{l''}(\xi_{l''},\ w^{\rm II};\ \overline{\nabla}-\overline{\nabla}\hat{F}^{\rm I}_0\overline{\nabla}_3)\overline{W}^{\rm II}_{l''}+\sum_{l'''}\eta^{\rm II}_{l'''}(x_0)\Pi^{\bar{x}_0}_{x_0}[\mathscr{A}^{\rm II}_{l'''}(\xi_{l'''},\ \tau,\ w^{\rm II};\ \overline{\nabla})-$$
$$-\mathscr{A}^{\rm II}_{l'''}(\xi_{l'''},\ \tau,\ w^{\rm II};\ \overline{\nabla}-\overline{\nabla}\hat{F}_0\overline{\nabla}_3)]\overline{W}^{\rm II}_{l'''}+\sum_l[\eta^{\rm II}_l(x_0)\mathscr{A}^{\rm II}(x_0,\ t_0,\ w^{\rm II};\ \mathring{\nabla})-$$
$$-\mathscr{A}^{\rm II}(x_0,\ t_0,\ w^{\rm II};\ \mathring{\nabla})\eta^{\rm II}_l(x_0)]W^{\rm II}_l,$$

$$P'(u^{\rm I},\ u^{\rm II},\ 'u^{\rm I},\ 'u^{\rm II},\ u^e)=$$
$$=\{\sum_{k''}\eta^{\rm I}_{k''}(x_0)[\tilde{B}^{\rm I}(\xi_{k''},\ \tau,\ w^{\rm I};\ \mathring{\nabla})-\tilde{B}^{\rm I}(x_0,\ t_0,\ w^{\rm I};\ \mathring{\nabla})]W^{\rm I}_{k''|\Gamma_{\tau,\tau+h}}+$$
$$+\sum_{k''}\eta^{\rm I}_{k''}(x_0)\Pi^{\bar{x}_0}_{x_0}[\tilde{B}^{\rm I}_{k''}(\xi_{k''},\ \tau,\ w^{\rm I};\ \overline{\nabla})-\tilde{B}^{\rm I}_{k''}(\xi_{k''},\ \tau,\ w^{\rm I};\ \overline{\nabla}-$$

$$-\bar{V}\hat{F}_0^{\mathrm{I}}\bar{V}_3)]\,\overline{W}^{\mathrm{I}}_{k''|\bar{x}_{0,3}=0}+\sum_{k''}[\eta^{\mathrm{I}}_{k''}(x_0)\tilde{B}^{\mathrm{I}}(x_0,t_0,w^{\mathrm{I}};\mathring{V})-$$

$$-\tilde{B}^{\mathrm{I}}(x_0,t_0,w^{\mathrm{I}};\mathring{V})\eta^{\mathrm{I}}_{k''}(x_0)]\,W^{\mathrm{I}}_{k''|\Gamma_{\tau,\tau+h}}\}_G-\{\mathrm{I}\longrightarrow\mathrm{II}\}_G,$$

$P''=0$;

$$(2.30)\qquad\begin{cases} W^{\mathrm{I}}_{k'}=R^{\mathrm{I}}_{k'}(u^{\mathrm{I}}), \quad W^{\mathrm{II}}_{l'}=R^{\mathrm{II}}_{l'}(u^{\mathrm{II}}),\\ W^{\mathrm{I}}_{k''}=\Pi^{\bar{x}_0}_{x_0}\overline{W}^{\mathrm{I}}_{k''}, \quad W^{\mathrm{II}}_{l''}=\Pi^{\bar{x}_0}_{x_0}\overline{W}^{\mathrm{II}}_{l''}, \quad W^{\mathrm{II}}_{l'''}=\Pi^{\bar{x}_0}_{x_0}\overline{W}^{\mathrm{II}}_{l'''},\\ \overline{W}^{\mathrm{I}}_{k''}=\overline{R}^{\mathrm{I}}_{k''}(u^{\mathrm{I}},u^{\mathrm{II}})+{}'\overline{R}^{\mathrm{I}}_{k''}('u^{\mathrm{I}},'u^{\mathrm{II}}),\\ \overline{W}^{\mathrm{II}}_{l''}=\overline{R}^{\mathrm{II}}_{l''}(u^{\mathrm{I}},u^{\mathrm{II}})+{}'\overline{R}^{\mathrm{II}}_{l''}('u^{\mathrm{I}},'u^{\mathrm{II}}),\\ \overline{W}^{\mathrm{II}}_{l'''}=\overline{R}^{\mathrm{II}}_{l'''}(u^{\mathrm{II}})+{}'\overline{R}^{\mathrm{II}}_{l'''}(u^{e}). \end{cases}$$

[N. B. For P'' in the case of the free exterior boundary, see [4].]

As seen from [3, 4], there remains to estimate the terms $R^{\mathrm{I}}_{k''}(u^{\mathrm{I}},u^{\mathrm{II}})$, $R^{\mathrm{II}}_{l''}(u^{\mathrm{I}},u^{\mathrm{II}})$, ${}'R^{\mathrm{I}}_{k''}('u^{\mathrm{I}},'u^{\mathrm{II}})$ and ${}'R^{\mathrm{II}}_{l''}('u^{\mathrm{I}},'u^{\mathrm{II}})$ in (2.30). However, tracing the proof given in [3, 4], we can easily prove:

Lemma 2.8. *Suppose* (1.12) *and* $\tau_1-\tau=\chi\lambda^2$ $(0<\chi\leqq 1)$. *Then*

(i) $$R^j_m(u^{\mathrm{I}},u^{\mathrm{II}})\in\mathring{C}^{2+\alpha,1+\alpha/2}_{x_0,t_0}(\overline{Q^j_{m,\tau,\tau_1}})$$

$$(\overline{Q^j_{m,\tau,\tau_1}}\equiv\Omega^j_m\times[\tau,\tau_1];\ j=\mathrm{I},\mathrm{II};\ m=\begin{cases}k''\ (j=\mathrm{I})\\ l''\ (j=\mathrm{II})\end{cases},$$

provided $(u^{\mathrm{I}},u^{\mathrm{II}})\in\mathring{C}^{\alpha,\alpha/2}_{x_0,t_0}(\overline{Q^{\mathrm{I}}_{\tau,\tau_1}})\times\mathring{C}^{\alpha,\alpha/2}_{x_0,t_0}(\overline{Q^{\mathrm{II}}_{\tau,\tau_1}})$ and the following estimates hold: for the above defined j and m,

$$\begin{cases}|D^r_{t_0}D^s_{x_0}R^j_m(u^{\mathrm{I}},u^{\mathrm{II}})|\leqq C^j_{13}(t_0-\tau)^{\frac{2-2r-|s|+\alpha}{2}}(\|u^{\mathrm{I}}\|^{(\alpha)}_{Q^{\mathrm{I}}_{\tau,\tau_1}}+\|u^{\mathrm{II}}\|^{(\alpha)}_{Q^{\mathrm{II}}_{\tau,\tau_1}})_H\\ \qquad\qquad\qquad(2r+|s|\leqq 2),\\ |\Delta^{y_0}_{x_0}D^r_{t_0}D^s_{x_0}R^j_m(u^{\mathrm{I}},u^{\mathrm{II}})|\leqq C^j_{13}|x_0-y_0|^{\alpha}(\cdots)_H \quad (2r+|s|=2),\\ |\Delta^{t'_0}_{t_0}D^r_{t_0}D^s_{x_0}R^j_m(u^{\mathrm{I}},u^{\mathrm{II}})|\leqq C^j_{13}|t_0-t'_0|^{\frac{2-2r-|s|+\alpha}{2}}(\cdots)_H \quad (0<2r+|s|\leqq 2);\end{cases}$$

(ii) $${}'R^j_m('u^{\mathrm{I}},'u^{\mathrm{II}})\in\mathring{C}^{2+\alpha,1+\alpha/2}_{x_0,t_0}(\overline{Q^j_{m,\tau,\tau_1}})\quad (\textit{for } m, \textit{see } \text{(i)})$$

if ${}'u^j\in C^{1+\alpha,(1+\alpha)/2}_{x_0,t_0}(\Gamma^{\mathrm{I}}_{\tau,\tau_1})$ $(j=\mathrm{I},\mathrm{II})$ with ${}'u^{\mathrm{I}}-{}'u^{\mathrm{II}}{}_{|t_0=\tau}=0$ and moreover for j and m as in (i),

$$\begin{cases}|D^r_{t_0}D^s_{x_0}{}'R^j_m('u^{\mathrm{I}},'u^{\mathrm{II}})|\leqq C^j_{14}(t_0-\tau)^{\frac{2-2r-|s|+\alpha}{2}}(\|'u^{\mathrm{I}}\|^{(1+\alpha)}_{\Gamma_{\tau,\tau_1}}+\|'u^{\mathrm{II}}\|^{(1+\alpha)}_{\Gamma_{\tau,\tau_1}})_{\mathrm{I}},\\ \qquad\qquad\qquad(2r+|s|\leqq 2)\\ |\Delta^{y_0}_{x_0}D^r_{t_0}D^s_{x_0}{}'R^j_m('u^{\mathrm{I}},'u^{\mathrm{II}})|\leqq C^j_{14}|x_0-y_0|^{\alpha}(\cdots)_{\mathrm{I}} \quad (2r+|s|=2),\\ |\Delta^{t'_0}_{t_0}D^r_{t_0}D^s_{x_0}{}'R^j_m('u^{\mathrm{I}},'u^{\mathrm{II}})|\leqq C^j_{14}|t_0-t'_0|^{\frac{2-2r-s+\alpha}{2}}(\cdots)_{\mathrm{I}} \quad (0<2r+|s|\leqq 2);\end{cases}$$

here ${}'R^j_m('u^{\mathrm{I}},'u^{\mathrm{II}})=\Pi^{\bar{x}_0}_{x_0}{}'\overline{R}^j_m('u^{\mathrm{I}},'u^{\mathrm{II}})$ *is used.*

If we denote

$$\begin{cases} \mathscr{A}^j(x_0, t_0, w^j; \mathring{V}) = ((\mathscr{A}^j(x_0, t_0, w^j))_{ik}^{mn} \mathring{V}_m \mathring{V}_n), \\ \tilde{B}^j(x_0, t_0, w^j; \mathring{V}) = ((\tilde{B}^j(x_0, t_0, w^j)_{ik}^m \mathring{V}_m), \end{cases}$$

then the estimates

$$(2.31) \quad \begin{cases} \|(A^j(x_0, t_0, w^j))_{ik}^{mn}\|_{Q_T^j}^{(\alpha)} \leqq C_{15}^j(T, M_1^j), \\ \|(\tilde{B}^j(x_0, t_0, w^j)_{ik}^m\|_{\Gamma_T^1}^{(1+\alpha)} \leqq C_{16}^j(T, M_1^j) + C_{17}^j(T, M_1^j) M_2^j \end{cases}$$

follow from the similar arguments in [4], where all the constants $C_{15}^{\mathrm{I}} \sim C_{17}^{\mathrm{II}}$ are monotonically increasing in each argument and especially C_{17}^{I} and C_{17}^{II} tend to zero as T tends to zero.

Almost along the line of the proof of Lemma 2.12 in [4], we obtain:

Lemma 2.9. *Suppose that* $(\mathscr{B}^{\mathrm{I}}, \mathscr{B}^{\mathrm{II}}) \in C_{x_0,t_0}^{\alpha,\alpha/2}(Q_T^{\mathrm{I}}) \times C_{x_0,t_0}^{\alpha,\alpha/2}(Q_T^{\mathrm{II}})$, $(\psi^{\mathrm{I}}, \psi^{\mathrm{II}}) \in C_{x_0,t_0}^{1+\alpha,(1+\alpha)/2}(\Gamma_T^{\mathrm{I}}) \times C_{x_0,t_0}^{1+\alpha,(1+\alpha)/2}(\Gamma_T^{\mathrm{I}})$ *and* $\phi_e \in C_{x_0,t_0}^{2+\alpha,1+\alpha/2}(\Gamma_T)$ *are given in* (2.1) *and that* (1.12) *is valid. Then there exists a unique solution* $(\tilde{w}^{\mathrm{I}}, \tilde{w}^{\mathrm{II}}) \in C_{x_0,t_0}^{2+\alpha,1+\alpha/2}(\overline{Q_T^{\mathrm{I}}}) \times C_{x_0,t_0}^{2+\alpha,1+\alpha/2}(\overline{Q_T^{\mathrm{II}}})$ *of* (2.1) *satiisfying*

$$\begin{cases} |D_{t_0}^r D_{x_0}^s \tilde{w}^j| \leqq [C_{18}^j(T, M_1^{\mathrm{I}}, M_1^{\mathrm{II}}) + C_{19}^j(T, M_1^{\mathrm{I}}, M_1^{\mathrm{II}})(M_2^{\mathrm{I}} + M_2^{\mathrm{II}})]^N t_0^{\frac{2-2r-|s|+\alpha}{2}} \times \\ \quad \times (\|B^{\mathrm{I}}\|_{Q_T^{\mathrm{I}}}^{(\alpha)} + \|B^{\mathrm{II}}\|_{Q_T^{\mathrm{II}}}^{(\alpha)} + \|\psi^{\mathrm{I}}\|_{\Gamma_T^1}^{(1+\alpha)} + \|\psi^{\mathrm{II}}\|_{\Gamma_T^1}^{(1+\alpha)} + \|\phi_e\|_{\Gamma_T}^{(2+\alpha)})_J \quad (2r+|s| \leqq 2), \\ |\Delta_{x_0}^{y_0} D_{t_0}^r D_{x_0}^s \tilde{w}^j| \leqq [C_{18}^j + C_{19}^j (M_2^{\mathrm{I}} + M_2^{\mathrm{II}})]^N |x_0 - y_0|^\alpha (\cdots)_J \quad (2r+|s|=2), \\ |\Delta_{t_0}^{t_0'} D_{t_0}^r D_{x_0}^s \tilde{w}^j| \leqq [C_{18}^j + C_{19}^j (M_2^{\mathrm{I}} + M_2^{\mathrm{II}})]^N |t_0 - t_0'|^{\frac{2-2r-|s|+\alpha}{2}} (\cdots)_J \\ \qquad\qquad (0 < 2r+|s| \leqq 2), \end{cases}$$

where C_{18}^j *and* C_{19}^j *have the same properties as* C_{16} *and* C_{17} *respectively; for* N, *see the process of the proof;* $j = \mathrm{I}, \mathrm{II}$.

Proof. The proof is devided into two steps.

1°. If χ and λ $(0 < \chi, \lambda \leqq 1)$ be suitably small, then there exists a unique solution $(\tilde{w}^{\mathrm{I}}, \tilde{w}^{\mathrm{II}}) \in C_{x_0,t_0}^{2+\alpha,1+\alpha/2}(\overline{Q_h^{\mathrm{I}}}) \times C_{x_0,t_0}^{2+\alpha,1+\alpha/2}(\overline{Q_h^{\mathrm{II}}}) (h = \chi\lambda^2)$ of (2.1) which satisfies the estimates stated in the lemma with $N=1$.

Indeed, on account of Lemma 2.8 it is sufficient to show the unique existence of $(u^{\mathrm{I}}, u^{\mathrm{II}}, 'u^{\mathrm{I}}, 'u^{\mathrm{II}}, u^e) \in \mathring{C}_{x_0,t_0}^{\alpha,\alpha/2}(\overline{Q_h^{\mathrm{I}}}) \times \mathring{C}_{x_0,t_0}^{\alpha,\alpha/2}(\overline{Q_h^{\mathrm{II}}}) \times C_{x_0,t_0}^{1+\alpha,(1+\alpha)/2}(\Gamma_h^{\mathrm{I}}) \times C_{x_0,t_0}^{1+\alpha,(1+\alpha)/2}(\Gamma_h^{\mathrm{I}}) \times \mathring{C}_{x_0,t_0}^{2+\alpha,1+\alpha/2}(\Gamma_h)$, $'u^{\mathrm{I}} - 'u^{\mathrm{II}}_{|t_0=0} = 0$. The uniqueness is clear. By Lemma 2.8 and (2.29) it is easily seen that

$$(u_0^{\mathrm{I}}, u_0^{\mathrm{II}}, 'u_0^{\mathrm{I}}, 'u_0^{\mathrm{II}}, u_0^e) = (\mathscr{B}^{\mathrm{I}}, \mathscr{B}^{\mathrm{II}}, \psi^{\mathrm{I}}, \psi^{\mathrm{II}}, \phi_e) \in \mathring{C}_{x_0,t_0}^{\alpha,\alpha/2}(\overline{Q_T^{\mathrm{I}}}) \times \mathring{C}_{x_0,t_0}^{\alpha,\alpha/2}(\overline{Q_T^{\mathrm{II}}}) \times$$

$$\times C_{x_0,t_0}^{1+\alpha,(1+\alpha)/2}(\Gamma_T^{\mathrm{I}}) \times C_{x_0,t_0}^{1+\alpha,(1+\alpha)/2}(\Gamma_T^{\mathrm{I}}) \times \mathring{C}_{x_0,t_0}^{2+\alpha,1+\alpha/2}(\Gamma_T), \quad 'u_0^{\mathrm{I}} - 'u^{\mathrm{II}}_{0|t_0=0} = 0;$$

$$(u_1^{\mathrm{I}}, u_1^{\mathrm{II}}, 'u_1^{\mathrm{I}}, 'u_1^{\mathrm{II}}, u_1^e)_{|t_0=0} = (P^{\mathrm{I}}(u_0^{\mathrm{I}}, u_0^{\mathrm{II}}, 'u_1^{\mathrm{I}}, 'u_0^{\mathrm{II}}, u_0^e), \quad P^{\mathrm{II}}(u_0^{\mathrm{I}}, u_0^{\mathrm{II}}, 'u_0^{\mathrm{I}}, 'u_0^{\mathrm{II}}, u_0^e),$$

$$P'(u_0^{\mathrm{I}}, u_0^{\mathrm{II}}, 'u_0^{\mathrm{I}}, 'u_0^{\mathrm{II}}, u_0^e), \quad P''(u_0^{\mathrm{I}}, u_0^{\mathrm{II}}, 'u_0^{\mathrm{I}}, 'u_0^{\mathrm{II}}, u_0^e))_{|t_0=0} = (0, 0, 0, 0, 0),$$

$$\begin{cases} \|u_1^{\rm I}\|_{Q_h^{\rm I}}^{(\alpha)} \leqq C_{20}^{\rm I}\Lambda(\|\mathscr{B}^{\rm I}\|_{Q_T^{\rm I}}^{(\alpha)}+\|\mathscr{B}^{\rm II}\|_{Q_T^{\rm II}}^{(\alpha)}+\|\psi^{\rm I}\|_{\Gamma_T^{\rm I}}^{(1+\alpha)}+\|\psi^{\rm II}\|_{\Gamma_T^{\rm I}}^{(1+\alpha)})_K \\ \|'u_1^{\rm I}\|_{\Gamma_T^{\rm I}}^{(1+\alpha)} \leqq (C_{20}^{\rm I}+C_{21}^{\rm I}M^{\rm I})\Lambda(\cdots)_K, \\ \|u_1^{\rm II}\|_{Q_h^{\rm II}}^{(\alpha)} \leqq C_{20}^{\rm II}\Lambda(\cdots)_J, \quad \|'u_1^{\rm II}\|_{\Gamma_h^{\rm I}}^{(1+\alpha)} \leqq (C_{20}^{\rm II}+C_{21}^{\rm II}M_2^{\rm II})\Lambda(\cdots)_J; \end{cases}$$

$u_n^e=0 \quad (n=1, 2,3,\ldots); \ \Lambda=\lambda^\alpha+\chi^{\alpha/2}.$

By induction, we obtain

$$(u_n^{\rm I}, u_n^{\rm II}, 'u_n^{\rm I}, 'u_n^{\rm II}, u_n^e) \in \mathring{C}_{x_0,t_0}^{\alpha,\alpha/2}(\overline{Q_h^{\rm I}}) \times \mathring{C}_{x_0,t_0}^{\alpha,\alpha/2}(\overline{Q_h^{\rm II}}) \times \mathring{C}_{x_0,t_0}^{1+\alpha,(1+\alpha)/2}(\Gamma_h^{\rm I}) \times$$
$$\times \mathring{C}_{x_0,t_0}^{1+\alpha,(1+\alpha)/2}(\Gamma_h^{\rm I}) \times \mathring{C}_{x_0,t_0}^{2+\alpha,1+\alpha/2}(\Gamma_h)$$

and

$$\|u_n^{\rm I}\|_{Q_h^{\rm I}}^{(\alpha)}+\|u_n^{\rm II}\|_{Q_h^{\rm II}}^{(\alpha)}+\|'u_n^{\rm I}\|_{\Gamma_h^{\rm I}}^{(1+\alpha)}+\|'u_n^{\rm II}\|_{\Gamma_h^{\rm II}}^{(1+\alpha)}+\|u_n^e\|_{\Gamma_h}^{(2+\alpha)} \leqq$$
$$\leqq 2^n[C_{20}+C_{21}(M_2^{\rm I}+M_2^{\rm II})]^n\Lambda^n(\cdots)_J$$

$(n=1, 2, 3,\ldots)$ where $C_{20}=C_{20}^{\rm I}+C_{21}^{\rm II}$ and $C_{21}=C_{21}^{\rm I}+C_{21}^{\rm II}$.

When λ and χ are chosen in such a way that

$$4[C_{20}+C_{21}(M_2^{\rm I}+M_2^{\rm II})]\Lambda=1, \tag{2.32}$$

$(u^{\rm I}, u^{\rm II}, 'u^{\rm I}, 'u^{\rm II}, u^e)=\sum_{n=0}^{\infty}(u_n^{\rm I}, u_n^{\rm II}, 'u_n^{\rm I}, 'u_n^{\rm II}, u_n^e)$ converges uniformly. Hence the above assertion concerning $(u^{\rm I}, u^{\rm II}, 'u^{\rm I}, 'u^{\rm II}, u^e)$ is ascertained. Moreover the following estimate holds:

$$\|u^{\rm I}\|_{Q_h^{\rm I}}^{(\alpha)}+\|u^{\rm II}\|_{Q_h^{\rm II}}^{(\alpha)}+\|'u^{\rm I}\|_{\Gamma_h^{\rm I}}^{(1+\alpha)}+\|'u^{\rm II}\|_{\Gamma_h^{\rm I}}^{(1+\alpha)}+\|u^e\|_{\Gamma_h}^{(2+\alpha)} \leqq 2(\cdots)_J.$$

2°. Since $\tilde{w}(x_0, t_0; h)=\tilde{w}(x_0, t_0)-\tilde{w}(x_0, h)$ $(T\geqq t_0\geqq h)$ satisfies $(2.1)_h$ with $(\mathscr{B}^{\rm I}, \mathscr{B}^{\rm II}, \psi^{\rm I}, \psi^{\rm II}, \phi_e)$ replaced by $(\mathscr{B}^{\rm I}+\mathscr{A}^{\rm I}\tilde{w}^{\rm I}(x_0, h), \mathscr{B}^{\rm II}+\mathscr{A}^{\rm II}\tilde{w}^{\rm II}(x_0, h), \psi^{\rm I}-\tilde{B}^{\rm I}\tilde{w}^{\rm I}(x_0, h)_{|\Gamma_{h,T}^{\rm I}}, \psi^{\rm II}+\tilde{B}^{\rm II}\tilde{w}^{\rm II}(x_0, h)_{|\Gamma_{h,T}^{\rm I}}, \phi_e-\tilde{B}^{\rm II}\tilde{w}^{\rm II}(x_0, h)_{|\Gamma_{h,T}})$, the result in 1° and the relation (2.31) imply that there exists a unique solution $(\tilde{w}^{\rm I}, \tilde{w}^{\rm II})(x_0, t_0; h) \in C_{x_0,t_0}^{2+\alpha,1+\alpha/2}(\overline{Q_{h,2h}^{\rm I}}) \times C_{x_0,t_0}^{2+\alpha,1+\alpha/2}(\overline{Q_{h,2h}^{\rm II}})$, hence $(\tilde{w}^{\rm I}, \tilde{w}^{\rm II}) \in C_{x_0,t_0}^{2+\alpha,1+\alpha/2}(\overline{Q_{2h}^{\rm I}}) \times C_{x_0,t_0}^{2+\alpha,1+\alpha/2}(\overline{Q_{2h}^{\rm II}})$, for which the estimates in the lemma with $N=3$ hold. We only repeat this procedure $N=2\left[\frac{T}{h}\right]+1$ times. Q.E.D.

It is to be noted that C_{18}^j and $C_{19}^j (j={\rm I, II})$ are positive constants depending on T, M_1^j, $\|v_0^j, \theta_0^j\|_{\Omega^j}^{(2+\alpha)}$, $\|\rho_0^j\|_{\Omega^j}^{(1+\alpha)}$, $(\bar{\rho}_0^j)^{-1}$, $(\bar{\theta}_0^j)^{-1}$, $(\bar{\mu}^j)^{-1}$, $(\bar{\kappa}^j)^{-1}$, $(\bar{S}_{\theta^j}^j)^{-1}$, $\|\mu^j, \mu'^j, \kappa^j, S^j\|_{\mathscr{D}_{\rho,\theta}^j}^{(2)}$, $\|F_0^{\rm I}\|_{\Gamma^{\rm I}}^{(2+\alpha)}$, $\|F_0\|_{\Gamma}^{(2+\alpha)}$, and monotonically increasing in each argument $(j={\rm I}, {\rm II})(\|\sigma^j\|_{\mathscr{D}_{\rho,\theta}^j}^{(2)}=\sum_{r+s=0}^{2}|D_\rho^r D_\theta^s\sigma^j|_{\mathscr{D}_{\rho,\theta}^j}^{(0)})$.

By virtue of (2.32), N in Lemma 2.9 is estimated from above as follows:

$$N \leqq \frac{2T}{h}+1<2T^{-\alpha/2}+1=2^{1+4/\alpha}[C_{20}+C_{21}(M_2^{\rm I}+M_2^{\rm II})]^{2/\alpha}+1$$
$$\equiv N((T, M_1^{\rm I}, M_1^{\rm II}, M_2^{\rm I}+M_2^{\rm II}).$$

For the original terms $\mathscr{B}^{\mathrm{I}}$, $\mathscr{B}^{\mathrm{II}}$, ψ^{I}, ψ^{II} and ϕ_e in (2.1), we obtain, similarly to (2.31),

$$\|\mathscr{B}^{\mathrm{I}}\|^{(\alpha)}_{Q^{\mathrm{I}}_T}+\|\mathscr{B}^{\mathrm{II}}\|^{(\alpha)}_{Q^{\mathrm{II}}_T}+\|\psi^{\mathrm{I}}\|^{(1+\alpha)}_{\Gamma^{\mathrm{I}}_T}+\|\psi^{\mathrm{II}}\|^{(1+\alpha)}_{\Gamma^{\mathrm{I}}_T}+\|\phi_e\|^{(2+\alpha)}_{\Gamma_T}\leqq$$

$$\leqq C_{22}(T,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)+C_{23}(T,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)(M^{\mathrm{I}}_2+M^{\mathrm{II}}_2),$$

where C_{22} and C_{23} have the same properties as $C_{18}=C^{\mathrm{I}}_{18}+C^{\mathrm{II}}_{18}$ and $C_{19}=C^{\mathrm{I}}_{19}+C^{\mathrm{II}}_{19}$ respectively, but moreover depend on $\|f^{\mathrm{I}},f^{\mathrm{II}}\|^{(1)}_{R^3_T}$ monotonically increasingly.

We derive from Lemma 2.9 relations

$$\begin{cases}\|\tilde{w}^{\mathrm{I}}\|^{(2)}_{Q^{\mathrm{I}}_T}+\|\tilde{w}^{\mathrm{II}}\|^{(2)}_{Q^{\mathrm{II}}_T}\leqq C[C_{22}(T,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)+C_{23}(T,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)(M^{\mathrm{I}}_2+M^{\mathrm{II}}_2)]\times\\ \times(T^{\alpha}+T^{\frac{2+\alpha}{2}})[C_{18}(T,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)+C_{19}(T,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)(M^{\mathrm{I}}_2+\\ +M^{\mathrm{II}}_2)]^{N(T,M^{\mathrm{I}}_1,M^{\mathrm{II}}_1,M^{\mathrm{I}}_2+M^{\mathrm{II}}_2)},\\ |\mathring{\nabla}\mathring{\nabla}\tilde{w}^{\mathrm{I}}|^{(\alpha)}_{x_0,\overline{Q^{\mathrm{I}}_T}}+|\mathring{\nabla}\mathring{\nabla}\tilde{w}^{\mathrm{II}}|^{(\alpha)}_{x_0,\overline{Q^{\mathrm{II}}_T}}\leqq C[C_{22}(T,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)+C_{23}(T,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)(M^{\mathrm{I}}_2+M^{\mathrm{II}}_2)]\times\\ \times[C_{18}(T,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)+C_{19}(T,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)(M^{\mathrm{I}}_2+M^{\mathrm{II}}_2)]^{N(T,M^{\mathrm{I}}_1,M^{\mathrm{II}}_1,M^{\mathrm{I}}_2+M^{\mathrm{II}}_2)}.\end{cases}$$

Since $C[C_{18}+C_{19}(M^{\mathrm{I}}_2+M^{\mathrm{II}}_2)]^N[C_{22}+C_{23}(M^{\mathrm{I}}_2+M^{\mathrm{II}}_2)]$ is increasing in $M^{\mathrm{I}}_2+M^{\mathrm{II}}_2$, we can, first of all, choose the constants M^{I}_2 and M^{II}_2 in such a way that they are larger than $C[C_{18}(T,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)+M]^{N(T,M^{\mathrm{I}}_1,M^{\mathrm{II}}_1,M)}[C_{22}(T,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)+M]$ for any positive number M, and nextly $T_0\in(0,\,T]$ such that

$$\begin{cases}C[C_{18}(T_0,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)+M]^{N(T_0,M^{\mathrm{I}}_1,M^{\mathrm{II}}_1,M)}(T^{\alpha}_0+T_0^{\frac{2+\alpha}{2}}[C_{22}(T_0,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)+M]\leqq\\ \leqq M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1,\\ C_{19}(T_0,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)M^{j}_2,\quad C_{21}(T_0,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)\,M^{j}_2,\quad C_{23}(T_0,\,M^{\mathrm{I}}_1,\,M^{\mathrm{II}}_1)\,M^{j}_2\leqq M/2\\ (j=\mathrm{I},\,\mathrm{II}).\end{cases}$$

There $(\tilde{w}^{\mathrm{I}},\,\tilde{w}^{\mathrm{II}})\in\mathfrak{S}_{T_0}$.

For simplicity, we again choose $T=T_0$ from the beginning.

§3. Nonlinear problem (1.11)

We construct the sequence $\{w^{\mathrm{I}}_n,\,w^{\mathrm{II}}_n\}(x_0,\,t_0)$ of the successive approximate solutions as follows:

$$\begin{cases}(w^{\mathrm{I}}_0,\,w^{\mathrm{II}}_0)(x_0,\,t_0)=(0,\,0)\in\mathfrak{S}_T,\\ (w^{\mathrm{I}}_n,\,w^{\mathrm{II}}_n)(x_0,\,t_0)\text{ is defined as a solution }\tilde{w}\text{ of (2.1)}\\ \text{assuming }(w^{\mathrm{I}},\,w^{\mathrm{II}})=(w^{\mathrm{I}}_{n-1},\,w^{\mathrm{II}}_{n-1})\in\mathfrak{S}_T.\end{cases}$$

Then the results in §2 say that $(w^{\mathrm{I}}_n,\,w^{\mathrm{II}}_n)(x_0,\,t_0)$ uniquely exists and belongs to $\mathfrak{S}_T$ $(n=0,\,1,\,2,\ldots)$.

Thereupon, we verify the convergence of $\{w^{\mathrm{I}}_n,\,w^{\mathrm{II}}_n\}(x_0,\,t_0)$. The difference

$\{w_n^{\mathrm{I}}-w_{n-1}^{\mathrm{I}},\ w_n^{\mathrm{II}}-w_{n-1}^{\mathrm{II}}\}(x_0,\ t_0)$ satisfies the system of equations

$$
(3.1)\quad
\begin{cases}
\dfrac{\partial}{\partial t_0}(w_n^j-w_{n-1}^j)=\mathscr{A}^j(x_0,\ t_0,\ w_{n-1}^j;\ \mathring{\mathcal{V}})(w_n^j-w_{n-1}^j)+\{\mathscr{A}^j(x_0,\ t_0,\ w_{n-1}^j;\ \mathring{\mathcal{V}})-\\
\quad -\mathscr{A}^j(x_0,\ t_0,\ w_{n-2}^j;\ \mathring{\mathcal{V}})\}w_{n-1}^j+\mathscr{B}^j(x_0,\ t_0,\ w_{n-1}^j)-\mathscr{B}^j(x_0,\ t_0,\ w_{n-2}^j)\\
\qquad\qquad\qquad\qquad\qquad\qquad\text{in}\quad Q_T^j\ (j=\mathrm{I},\ \mathrm{II}),\\
w_n^j-w_{n-1}^j{}_{|t_0=0}=0\quad (j=\mathrm{I},\ \mathrm{II}),\\
\begin{pmatrix} \mathrm{I}_3\\ \dfrac{B^{\mathrm{I}}(x_0,\ t_0,\ w_{n-1}^{\mathrm{I}};\ \mathring{\mathcal{V}})}{|\mathscr{G}^{\mathrm{I}}(w_{n-1}^{\mathrm{I}})\mathring{\mathcal{V}}F_0^{\mathrm{I}}|}\end{pmatrix}(w_n-w_{n-1})-\begin{pmatrix} \mathrm{I}_3\\ \dfrac{B^{\mathrm{II}}(x_0,\ t_0,\ w_{n-1}^{\mathrm{II}};\ \mathring{\mathcal{V}})}{|\mathscr{G}^{\mathrm{II}}(w_{n-1}^{\mathrm{II}})\mathring{\mathcal{V}}F_0^{\mathrm{I}}|}\end{pmatrix}(w_n^{\mathrm{II}}-w_{n-1}^{\mathrm{II}})=\\
=\begin{pmatrix} 0\\ \dfrac{\phi^{\mathrm{II}}(x_0,\ t_0,\ w_{n-1}^{\mathrm{II}})}{|\mathscr{G}^{\mathrm{II}}(w_{n-1}^{\mathrm{II}})\mathring{\mathcal{V}}F_0^{\mathrm{I}}|}-\dfrac{\phi^{\mathrm{II}}(x_0,\ t_0,\ w_{n-2}^{\mathrm{II}})}{|\mathscr{G}^{\mathrm{II}}(w_{n-2}^{\mathrm{II}})\mathring{\mathcal{V}}F_0^{\mathrm{I}}|}\end{pmatrix}-\\
-\begin{pmatrix} 0\\ \dfrac{\phi^{\mathrm{I}}(x_0,\ t_0,\ w_{n-1}^{\mathrm{I}})}{|\mathscr{G}^{\mathrm{I}}(w_{n-1}^{\mathrm{I}})\mathring{\mathcal{V}}F_0^{\mathrm{I}}|}-\dfrac{\phi^{\mathrm{I}}(x_0,\ t_0,\ w_{n-2}^{\mathrm{I}})}{|\mathscr{G}^{\mathrm{I}}(w_{n-2}^{\mathrm{I}})\mathring{\mathcal{V}}F_0^{\mathrm{I}}|}\end{pmatrix}-\\
-\begin{pmatrix} 0\\ \dfrac{B^{\mathrm{I}}(x_0,\ t_0,\ w_{n-1}^{\mathrm{I}};\ \mathring{\mathcal{V}})}{|\mathscr{G}^{\mathrm{I}}(w_{n-1}^{\mathrm{I}})\mathring{\mathcal{V}}F_0^{\mathrm{I}}|}-\dfrac{B^{\mathrm{I}}(x_0,\ t_0,\ w_{n-2}^{\mathrm{I}};\ \mathring{\mathcal{V}})}{|\mathscr{G}^{\mathrm{I}}(w_{n-2}^{\mathrm{I}})\mathring{\mathcal{V}}F_0^{\mathrm{I}}|}\end{pmatrix}w_{n-1}^{\mathrm{I}}+\\
+\begin{pmatrix} 0\\ \dfrac{B^{\mathrm{II}}(x_0,\ t_0,\ w_{n-1}^{\mathrm{II}};\ \mathring{\mathcal{V}})}{|\mathscr{G}^{\mathrm{II}}(w_{n-1}^{\mathrm{II}})\mathring{\mathcal{V}}F_0^{\mathrm{I}}|}-\dfrac{B^{\mathrm{II}}(x_0,\ t_0,\ w_{n-2}^{\mathrm{II}};\ \mathring{\mathcal{V}})}{|\mathscr{G}^{\mathrm{II}}(w_{n-2}^{\mathrm{II}})\mathring{\mathcal{V}}F_0^{\mathrm{I}}|}\end{pmatrix}w_{n-1}^{\mathrm{II}}\quad\text{on}\quad \Gamma_T^{\mathrm{I}},\\
w_n^{\mathrm{II}}-w_{n-1}^{\mathrm{II}}=0\quad\text{on}\quad \Gamma_T.
\end{cases}
$$

In order to apply Lemma 2.9 to the solution $(w_n^{\mathrm{I}}-w_{n-1}^{\mathrm{I}},\ w_n^{\mathrm{II}}-w_{n-1}^{\mathrm{II}})$ of (3.1) as a linear system of equations in $(w_n^{\mathrm{I}}-w_{n-1}^{\mathrm{I}},\ w_n^{\mathrm{II}}-w_{n-1}^{\mathrm{II}})$, we must evaluate the terms corresponding to $\mathscr{B}^{\mathrm{I}}$, $\mathscr{B}^{\mathrm{II}}$, ψ^{I}, ψ^{II}, ϕ_e in (2.1). As in [4], on account of the inequality

$$|w_{n-1}^j-w_{n-2}^j|_{Q_T^j}^{(0)}\leqq t_0\left|\frac{\partial}{\partial t_0}(w_{n-1}^j-w_{n-2}^j)\right|_{Q_T^j}^{(0)}\quad (j=\mathrm{I},\ \mathrm{II})$$

and Lemma 3.1 in [4], we have

$$\|[\mathscr{A}^j(x_0,\ t_0,\ w_{n-1}^j;\ \mathring{\mathcal{V}})-\mathscr{A}^j(x_0,\ t_0,\ w_{n-2}^j;\ \mathring{\mathcal{V}})]w_{n-1}^j+\mathscr{B}^j(x_0,\ t_0,\ w_{n-1}^j)-$$
$$-\mathscr{B}^j(x_0,\ t_0,\ w_{n-2}^j)\|_{Q_T^j}^{(\alpha)},\quad \left\|\frac{\phi^j(x_0,\ t_0,\ w_{n-1}^j)}{|\mathscr{G}^j(w_{n-1}^j)\mathring{\mathcal{V}}F_0^{\mathrm{I}}|}-\frac{\phi^j(x_0,\ t_0,\ w_{n-2}^j)}{|\mathscr{G}^j(w_{n-2}^j)\mathring{\mathcal{V}}F_0^{\mathrm{I}}|}+\right.$$
$$\left.+\left[\frac{B^j(x_0,\ t_0,\ w_{n-1}^j;\ \mathring{\mathcal{V}})}{|\mathscr{G}^j(w_{n-1}^j)\mathring{\mathcal{V}}F_0^{\mathrm{I}}|}-\frac{B^j(x_0,\ t_0,\ w_{n-2}^j;\ \mathring{\mathcal{V}})}{|\mathscr{G}^j(w_{n-2}^j)\mathring{\mathcal{V}}F_0^{\mathrm{I}}|}\right]w_{n-1}^j\right\|_{\Gamma_T^{\mathrm{I}}}^{(1+\alpha)}\leqq$$

$$\leqq C_{24}(T, M_1^{\mathrm{I}}, M_1^{\mathrm{II}})|\mathring{\nabla}\mathring{\nabla}(w_{n-1}^j - w_{n-2}^j)|_{x_0, \overline{Q_T^j}}^{(\alpha)} + C_{25}(T, M_1^{\mathrm{I}}, M_1^{\mathrm{II}}) \times$$

$$\times \|w_{n-1}^j - w_{n-2}^j\|_{Q_T^j}^{(2)}$$

($j=\mathrm{I}, \mathrm{II}$) where both C_{24} and C_{25} increase monotonically in each argument and tend to zero as $T \to 0$. Therefore, again from Lemma 2.9 it follows that

$$\|w_n^{\mathrm{I}} - w_{n-1}^{\mathrm{I}}\|_{Q_T^{\mathrm{I}}}^{(2+\alpha)} + \|w_n^{\mathrm{II}} - w_{n-1}^{\mathrm{II}}\|_{Q_T^{\mathrm{II}}}^{(2+\alpha)} \leqq$$

$$\leqq C_{26}(T, M_1^{\mathrm{I}}, M_1^{\mathrm{II}}, M_2^{\mathrm{I}}, M_2^{\mathrm{II}})[\|w_{n-1}^{\mathrm{I}} - w_{n-2}^{\mathrm{I}}\|_{Q_T^{\mathrm{I}}}^{(2+\alpha)} + \|w_{n-1}^{\mathrm{II}} - w_{n-2}^{\mathrm{II}}\|_{Q_T^{\mathrm{II}}}^{(2+\alpha)}]$$

where C_{26} has the same property as C_{25}.

By induction we obtain

$$\|w_n^{\mathrm{I}} - w_{n-1}^{\mathrm{I}}\|_{Q_T^{\mathrm{I}}}^{(2+\alpha)} + \|w_n^{\mathrm{II}} - w_{n-1}^{\mathrm{II}}\|_{Q_T^{\mathrm{II}}}^{(2+\alpha)} \leqq C_{26}^{n-1}[\|w_1^{\mathrm{I}}\|_{Q_T^{\mathrm{I}}}^{(2+\alpha)} + \|w_1^{\mathrm{II}}\|_{Q_T^{\mathrm{II}}}^{(2+\alpha)}]$$

By virtue of the property of C_{26}, we can find $T' \in (0, T]$ such that

$$C_{26}(T', M_1^{\mathrm{I}}, M_1^{\mathrm{II}}, M_2^{\mathrm{I}}, M_2^{\mathrm{II}}) < 1.$$

we reach a conclusion:

$\{(w_n^{\mathrm{I}}, w_n^{\mathrm{II}})\}$ is uniformly convergent to $(w^{\mathrm{I}}, w^{\mathrm{II}}) \in C_{x_0, t_0}^{2+\alpha, 1+\alpha/2}(\overline{Q_{T'}^{\mathrm{I}}}) \times C_{x_0, t_0}^{2+\alpha, 1+\alpha/2}(\overline{Q_{T'}^{\mathrm{II}}})$ as $n \to \infty$.

$$\left\{\hat{\rho}_n^j = \rho_0^j(x_0) \exp\left[-\int_0^{t_0} \nabla_{w_{n'}^j + v_0^j}(w_{n'}^j + v_0^j)(x_0, \tau)\, d\tau\right]\right\}$$

$(w^{j'} \equiv (w_1^j, w_2^j, w_3^j))$ also uniformly converges to $\hat{\rho}^j \in B^{1+\alpha}(Q_{T'}^j)$ ($j=\mathrm{I}, \mathrm{II}$) as $n \to \infty$. These functions $(w^{\mathrm{I}}, \hat{\rho}^{\mathrm{I}})$, $(w^{\mathrm{II}}, \hat{\rho}^{\mathrm{II}})$ are our desired solution of (1.11).

The uniqueness of such a solution is proved as in [3, 4].

Thus the proof of Theorem A is completed.

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