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On the structure of infinitesimal automorphisms of linear Poisson manifolds II

Dedicated to Professor Masahisa Adachi on his sixtieth birthday

By

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Introduction

Let $\{x, y, z\}$ be a basis of $\mathfrak{g} = \mathfrak{so}(3, R)$ which satisfies the relations:

$$[x, y] = z,\ [z, x] = y,\ [y, z] = x. \tag{0.1}$$

We regard x, y and z as coordinate functions on $\mathfrak{g}^*$ and we may identify $\mathfrak{g}^*$ with R^3. A linear Poisson structure on $\mathfrak{g}^*$ is defined by the following antisymmetric contravariant tensor P of order 2:

$$P = z\partial_x \wedge \partial_y + y\partial_z \wedge \partial_x + x\partial_y \wedge \partial_z. \tag{0.2}$$

Using the tensor P, we can define a bracket operation $\{\cdot, \cdot\}$ on $C^\infty(\mathfrak{g}^*)$:

$$\{f, g\} = \langle df \wedge dg | P\rangle, \quad \text{for all } f, g \in C^\infty(\mathfrak{g}^*). \tag{0.3}$$

Then $C^\infty(\mathfrak{g}^*)$ becomes a Lie algebra and $\mathfrak{g}^*$ has a structure of a linear Poisson manifold. An infinitesimal automorphism of a linear Poisson manifold $\mathfrak{so}(3, R)^*$ is a smooth vector field X on $\mathfrak{so}(3, R)^*$ which satisfies $\mathscr{L}(X)P = 0$, where $\mathscr{L}(X)$ denotes the Lie derivative along X. In the present paper, we shall discuss infinitesimal automorphisms defined on a linear Poisson manifold $\mathfrak{g}^* = \mathfrak{so}(3, R)^*$. This is a sequel of the author's papers [2] and [3].

A Lie group $SO(3, R)$ acts on $\mathfrak{so}(3, R)^*$ through the coadjoint action. All coadjoint orbits except for the origin are compact, which are diffeomorphic to S^2. Contrary to the case of $\mathfrak{sl}(2, R)^*$, infinitesimal automorphisms of $\mathfrak{so}(3, R)^*$ have some restrictions. This depends on the fact that each orbit is compact in the case of $\mathfrak{so}(3, R)^*$.

In § 1, we shall prove that every infinitesimal automorphism is tangent to orbits at each point. In § 2, we consider the formal version of smooth infinitesimal automorphisms and calculate derivation algebras of the space of polynomial functions, which we call the polynomial Poisson algebra.

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§1. The Schouten bracket and infinitesimal automorphisms

First recall the definition of the Schouten bracket (Lichnerowicz [1]). We call i-tensor an antisymmetric cotravariant tensor of order i. For an i-tensor A and a j-tensor B, we shall define the Schouten bracket $[A, B]$ which is an $(i+j-1)$-tensor as follows: for every closed $(i+j-1)$-form β, we have

$$(1.1) \qquad i([A, B]\beta = (-1)^{ij+j} i(A)di(B)\beta + (-1)^{i} i(B)di(A)\beta,$$

where $i(\cdot)$ is the interior product. Then the Schouten bracket satisfies

$$(1.2) \qquad [A, B] = (-1)^{ij}[B, A].$$

Moreover if C is a k-tensor, we have the generalized Jacobi identity

$$(1.3) \qquad (-1)^{ij}[[B, C], A] + (-1)^{jk}[[C, A], B] + (-1)^{ki}[[A, B], C] = 0.$$

A function $f \in C^{\infty}(\mathfrak{g}^*)$ is called a Casimir function if it satisfies $\{f, g\} = 0$ for all $g \in C^{\infty}(\mathfrak{g}^*)$. We denote by $\mathscr{C}$ a space of Casimir functions. Using the Schouten bracket, we can give another definition of Casimir functions and infinitesimal automorphisms. A function f is a Casimir function if $[P, f] = 0$, and a vector field X is an infinitesimal automorphism if $[P, X] = 0$. In the case of $\mathfrak{so}(3, R)^*$, by an easy consideration, we know that the space $\mathscr{C}$ is consisting of functions $\phi(x^2 + y^2 + z^2)$.

We denote by $\mathscr{L}$ the Lie algebra of infinitesimal automorphisms of $\mathfrak{so}(3, R)^*$. Let $\mathscr{I}$ be an ideal of $\mathscr{L}$ whose elements are tangent to coadjoint orbits. We shall prove $\mathscr{L} = \mathscr{I}$ in the following.

Lemma 1.1. *Let $X = f\partial_x + g\partial_y + h\partial_z$ be an element of $\mathscr{L}$. Then there exists $\phi \in \mathscr{C}$ such that $xf + yg + zh = \phi(x^2 + y^2 + z^2)$.*

Proof. By the genralized Jacobi identiy, we have

$$(1.4) \qquad [[X, P], \psi] + [[P, \psi], X] + [[\psi, X], P] = 0, \quad \text{for all } \psi \in \mathscr{C}.$$

Since $[X, P] = 0$ and $[P, \psi] = 0$, we get $X(\psi) \in \mathscr{C}$. Hence there exists $\tilde{\psi} \in \mathscr{C}$ such that $2\psi'(x^2 + y^2 + z^2)(xf + yg + zh) = \tilde{\psi}(x^2 + y^2 + z^2)$. Put $\phi = \tilde{\psi}/2\psi'$. Then ϕ is the desired Casimir function. q.e.d.

Theorem 1.2. *In the case of $\mathfrak{g}^* = \mathfrak{so}(3, R)^*$, it holds $\mathscr{L} = \mathscr{I}$.*

Proof. Let $X = f\partial_x + g\partial_y + h\partial_z$ be an element of $\mathscr{L}$. Then f, g and h satisfy the relations:

$$(1.5) \qquad \begin{cases} f = xg_y - yg_x + xh_z - zh_x, \\ g = yf_x - xf_y + yh_z - zh_y, \\ h = zf_x - xf_z + zg_y - yg_z. \end{cases}$$

Put $\operatorname{div}(X) = f_x + g_y + h_z$. Combining Lemma 1.1 with (1.5), we have

$$
(1.6)\qquad \begin{cases} x\operatorname{div}(X) = (xf + yg + zh)_x = 2x\,\phi'(x^2 + y^2 + z^2), \\ y\operatorname{div}(X) = (xf + yg + zh)_y = 2y\,\phi'(x^2 + y^2 + z^2), \\ z\operatorname{div}(X) = (xf + yg + zh)_z = 2z\,\phi'(x^2 + y^2 + z^2). \end{cases}
$$

Hence $\operatorname{div}(X) = f_x + g_y + h_z = 2\phi'(x^2 + y^2 + z^2)$ on $R^3 - \{0\}$. The continuity of $\operatorname{div}(X)$ and ϕ implies $\operatorname{div}(X) = 2\phi'(x^2 + y^2 + z^2)$ on R^3.

Let V be an open ball of a radius r centered at the origin. Let $\imath: \partial V \to V$ be the inclusion mapping. By the Stokes formula, we have

$$
(1.7)\qquad \int_V \operatorname{div}(X)dV = \int_{\partial V} \imath^*(i(X)dV).
$$

First compute the left hand side of (1.7). We use the polar coordinates of R^3.

$$
(1.8)\qquad \int_V \operatorname{div}(X)dV = \int_V (f_x + g_y + h_z)dV = 2\int_V \phi'(r^2)r^2 \sin\theta\, dr d\theta d\phi
$$

$$
= 8\pi \int_0^r \phi'(r^2)r^2 dr.
$$

On the other hand, we have

$$
(1.9)\qquad \int_{\partial V} \imath^*(i(X)dV) = \int_{\partial V} \phi(r^2) r \sin\theta\, d\theta\, d\phi = 4\pi\phi(r^2)r.
$$

Thus we get

$$
(1.10)\qquad 2\int_0^r \phi'(r^2)r^2 dr = \phi(r^2)r.
$$

Differentiating both sides of (1.10), we have $\phi(r^2) = 0$ for any $r > 0$. The continuity of ϕ implies $\phi(0) = 0$. Thus $\phi \equiv 0$ on R^3 and hence $xf + yg + zh = 0$. This means that a vector field X is tangent to each coadjoint orbit. (Note that $X = 0$ at the origin by (1.5).) q.e.d.

§ 2. Derivations of the polynomial Poisson algebra

Let $\{x, y, z\}$ be basis of $\mathfrak{so}(3, R)$ which satisfies (0.1). Let F_P be a space of homogeneous polynomials $f(x, y, z)$ with $\deg(f) = p + 1$, and put $F = \sum F_P$. Using the linear Poisson tensor P of (0.2), a Poisson bracket $\{\cdot, \cdot\}$ on F is defined. Since it holds $\{F_p, F_q\} \subset F_{p+q}$ $(p, q \geqq 0)$, F becomes a graded Lie algebra. We call F the polynomial Poisson algebra.

The following proposition can be proved by the same method as the case of $\mathfrak{sl}(2, R)$. So we omit the proof. (For the proof, see [3].)

Proposition 2.1. *Each space F_P $(p \geqq 2)$ is generated by F_1. Namely it holds $F_P = \{F_1, F_{P-1}\}$.*

Let $\mathfrak{g}^c$ be the complexification of $\mathfrak{g}=\mathfrak{so}(3, R)$. A subspace $\mathfrak{g}_0=\{x\}$ is a Cartan subalgebra of $\mathfrak{g}^c$ and a root decomposition of $\mathfrak{g}^c$ with respect to $\mathfrak{g}_0$ is given by

$$\mathfrak{g}^c=\mathfrak{g}_{-i}+\mathfrak{g}_0+\mathfrak{g}_i=\{y+iz\}+\{x\}+\{y-iz\}. \tag{2.1}$$

Let F_p^C be the complexification of F_p. A linear mapping $\mathrm{ad}(x)\colon F_p\to F_p$ is naturally extended to $\mathrm{ad}(x)\colon F_p^C\to F_p^C$. Then by direct calculations, we have

Proposition 2.2. *All eigen values of a linear mapping* $\mathrm{ad}(x)\colon F_p^C\to F_p^C$ *are* 0, $\pm i$, $\pm 2i,\dots, \pm(p+1)i$. *Let* $F_p(ki)$ *be an eigen space corresponding to an eigen value* ki. *Then we obtain* $F_p^C=\sum_{k=-(p+1)}^{p+1} F_p(ki)$, *where each* $F_p(ki)$ *is given by*:

(i) *If* p *is even, say* $p=2m$,

$$\begin{cases} F_p(0)=\langle x^{2m+1}, x^{2m-1}(y^2+z^2),\dots, x(y^2+z^2)^m\rangle, \\ F_p(i)=\langle x^{2m}(y-iz), x^{2m-2}(y-iz)(y^2+z^2),\dots,(y-iz)(y^2+z^2)^m\rangle, \\ F_p(-i)=\langle x^{2m}(y+iz), x^{2m-2}(y+iz)(y^2+z^2),\dots,(y+iz)(y^2+z^2)^m\rangle, \\ F_p(2i)=\langle x^{2m-1}(y-iz)^2, x^{2m-3}(y-iz)^2(y^2+z^2),\dots, x(y-iz)^2(y^2+z^2)^{m-1}\rangle \\ F_p(-2i)=\langle x^{2m-1}(y+iz)^2, x^{2m-3}(y+iz)^2(y^2+z^2),\dots, x(y+iz)^2(y^2+z^2)^{m-1}\rangle, \\ \qquad\vdots \\ F_p((2m+1)i)=\langle (y-iz)^{2m+1}\rangle, \\ F_p(-(2m+1)i)=\langle (y+iz)^{2m+1}\rangle. \end{cases}$$

(ii) *If* p *is odd, say* $p=2m-1$,

$$\begin{cases} F_p(0)=\langle x^{2m}, x^{2m-2}(y^2+z^2),\dots,(y^2+z^2)^m\rangle, \\ F_p(i)=\langle x^{2m-1}(y-iz), x^{2m-3}(y-iz)(y^2+Z^2),\dots, x(y-iz)(y^2+z^2)^{m-1}\rangle, \\ F_p(-i)=\langle x^{2m-1}(y+iz), x^{2m-3}(y+iz)(y^2+z^2),\dots, x(y+iz)(y^2+z^2)^{m-1}\rangle, \\ F_p(2i)=\langle x^{2m-2}(y-iz)^2, x^{2m-4}(y-iz)^2(y^2+z^2),\dots,(y-iz)^2(y^2+z^2)^{m-1}\rangle, \\ F_p(-2i)=\langle x^{2m-2}(y+iz)^2, x^{2m-4}(y+iz)^2(y^2+z^2),\dots,(y+iz)^2(y^2+z^2)^{m-1}\rangle, \\ \qquad\vdots \\ F_p(2mi)=\langle (y-iz)^{2m}\rangle, \\ F_p(-2mi)=\langle (y+iz)^{2m}\rangle. \end{cases}$$

A linear mapping $c\colon F\to F$ is called a derivation if it satisfies

$$c\{f, g\}=\{c(f), g\}+\{f, c(g)\} \quad \text{for all } f, g\in F. \tag{2.2}$$

In the rest of this section, we shall calculate all derivations of F. First define the degree of c. If a derivation c satisfies $c(F_p)\subset F_{p+r}$ for all p, we say that the degree of c is r, and write as $\deg c=r$. For any derivation c, we denote by $c_p^{(k)}$ the

$\mathrm{Hom}(F_p, F_{p+k})$-component of c. Define a new derivation $c^{(k)}$ by $c^{(k)}|F_p = c_p^{(k)}$. Then $c^{(k)}$ is a derivation of degree k, and c is written as $c = \sum c^{(k)}$.

By the same method as [3], we can prove

Proposition 2.3. *For a derivation* $c\colon F \to F$, *if* $\deg c \leqq -1$, *then* $c = 0$.

Consider the adjoint action of $F_0 = \mathfrak{so}(3, R)$ over F_p. Since F_0 is a simple Lie algebra, it holds $H^1(F_0, F_p) = 0$. Hence there exists an $f \in F_p$ such that $c|F_0 = \mathrm{ad}(f)$. Thus $(c - \mathrm{ad}(f))(F_0) = 0$. By this reason, hereafter, we always assume that a derivation c with non-negative degree satisfies $c(F_0) = 0$.

To determine a derivation c of non-negative degree p, we consider the subspace $c(F_1)$ in F_{p+1}. And we write it down according to the direct sum decomposition of F_{p+1}^C.

Proposition 2.4. *Let* $\deg c = p \geqq 0$. *Since* $c(F_1) \subset F_{p+1}^C$, *according to the direct sum decomposition of* F_{p+1}^C, *we can put*: $c(x^2) = \sum_{k=-p+2}^{p+2} a_{ki}$, $c(y^2) = \sum_{k=-p-2}^{p+2} b_{ki}$, $c(z^2) = \sum_{k=-p-2}^{p+2} c_{ki}$, $c(yz) = \sum_{k=-p-2}^{p+2} r_{ki}$. *Then* $c(x^2) = a_0$, $c(y^2) = ir_{-2i} + b_0 - ir_{2i}$, $c(z^2) = -ir_{-2i} + b_0 + ir_{2i}$ *and* $c(yz) = r_{-2i} + r_{2i}$. *Moreover* $a_0 + 2b_0 \in \mathscr{C}$.

Proof. By the equation $0 = c\{x, x^2\} = \{x, c(x^2)\} = \sum kia_{ki}$, we have $a_{ki} = 0$ if $k \neq 0$. Thus $c(x^2) = a_0$. Another equation $c\{x, y^2\} = \{x, c(y^2)\} = 2c(yz)$ implies that $\{x, \sum b_{ki}\} = \sum kib_{ki} = 2\sum r_{ki}$. Thus we get $r_0 = 0$ and $b_{ki} = (-2i/k)r_{ki}$ if $k \neq 0$. Similarly $c\{x, z^2\} = \{x, c(z^2)\} = -2c(yz)$ implies $c_{ki} = (2i/k)r_{ki}$ if $k \neq 0$. On the other hand, it holds that $c\{x, yz\} = \{x, c(yz)\} = c(z^2 - y^2) = c(z^2) - c(y^2)$. This equation implies $kir_{ki} = (4i/k)r_{ki} + c_0 - b_0$. Hence we have $b_0 = c_0$ and $r_{ki} = 0$ if $k \neq \pm 2$. Thus $b_{2i} = -ir_{2i}$, $b_{-2i} = ir_{-2i}$, $c_{2i} = ir_{2i}$ and $c_{-2i} = -ir_{-2i}$. A derivation c leaves the space $\mathscr{C}$ invariant. Hence $c(x^2 + y^2 + z^2) = a_0 + 2b_0 \in \mathscr{C}$. q.e.d.

Using the above proposition, we shall prove

Propositon 2.5. (i) *If* $\deg c = 2m - 1$ $(m \geqq 1)$, *then c is an inner derivation.*

(ii) *If* $\deg c = 2m$ $(m \geqq 0)$, *then c is an outer derivation. More precisely, c is essentially defined as follows*:

For all $p \geqq 0$, $c(u_p) = pu_p(x^2 + y^2 + z^2)^m$ for all $u_p \in F_p$.

Proof. The proof proceeds in the same way as the case of $\mathfrak{sl}(2, R)$. (i) Note that $c(F_1) \subset F_{2m}$. Since there are no Casimir functions in F_{2m}, we can put $b_0 = -a_0/2$. According to the direct sum decomposition of F_{2m}^C, we can write a_0, r_{2i} and r_{-2i} as follows:

$$(2.3)\quad \begin{cases} a_0 = a_1 x^{2m+1} + a_2 x^{2m-1}(y^2 + z^2) + \cdots + a_{m+1} x (y^2 + z^2), \\ r_{2i} = c_1 x^{2m-1}(y - iz)^2 + c_2 x^{2m-3}(y - iz)^2 (y^2 + z^2) + \cdots \\ \qquad\qquad + c_m x (y - iz)^2 (y^2 + z^2)^{m-1}, \end{cases}$$

$$\left|\begin{array}{l} r_{-2i} = d_1 x^{2m-1}(y+iz)^2 + d_2 x^{2m-3}(y+iz)^2(y^2+z^2) + \cdots \\ \qquad\qquad\qquad\qquad\qquad\qquad \times\, d_m x(y+iz)^2(y^2+z^2)^{m-1}. \end{array}\right.$$

Substituting (2.3) into $\{y, c(y^2)\} = c\{y, y^2\} = 0$, and equating coefficients of z^k $(k = 0, 1, 2, \ldots, 2m+1)$ to zero, we have $a_i = c_k = d_k = 0$ $(1 \leqq j \leqq m+1,\ 1 \leqq k \leqq m)$. Thus $c(x^2) = c(y^2) = c(z^2) = 0$. Since $c(F_0) = 0$, we also have $c(xy) = c(xz) = c(yz) = 0$. Hence $c(F_1) = 0$. By Proposition 2.1, $c(F_p) = 0$ $(p \geqq 1)$, and hence $c = 0$ on F.

(ii) Since $c(F_1) \subset F_{2m+1}$, we can write down a_0, r_{2i} and r_{-2i} according to the direct sum decomposition of F^C_{2m+1} as follows:

$$(2.4)\quad \begin{cases} a_0 = a_1 x^{2m+2} + a_2 x^{2m}(y^2+z^2) + \cdots + a_{m+2}(y^2+z^2)^{m+1}, \\ r_{2i} = c_1 x^{2m}(y-iz)^2 + c_2 x^{2m-2}(y-iz)^2(y^2+z^2) + \cdots \\ \qquad\qquad\qquad\qquad + c_{m+1}(y-iz)^2(y^2+z^2)^m, \\ r_{-2i} = d_1 x^{2m}(y+iz)^2 + d_2 x^{2m-2}(y+iz)^2(y^2+z^2) + \cdots \\ \qquad\qquad\qquad\qquad + d_{m+1}(y+iz)^2(y^2+z^2)^m. \end{cases}$$

In F_{2m+1}, there is one dimensional subspace of Casimir functions whose basis is $(x^2+y^2+z^2)^{m+1}$. Hence we need put $b_0 = (K(x^2+y^2+z^2)^{m+1} - a_0)/2$. Substituting (2.4) into $\{y, c(y^2)\} = 0$, and equating coefficients of z^k $(k = 1, 2, \ldots, 2m+2)$ to zero, we have

$$(2.5)\quad \begin{cases} c(x^2) = 4d_1 i x^2(x^2+y^2+z^2)^m + a_{m+2}(x^2+y^2+z^2)^{m+1}, \\ c(y^2) = 4d_1 i y^2(x^2+y^2+z^2)^m - (1/2)(a_{m+2} - K + 4d_1 i)(x^2+y^2+z^2)^{m+1}, \\ c(z^2) = 4d_1 i z^2(x^2+y^2+z^2)^m - (1/2)(a_{m+2} - K + 4d_1 i)(x^2+y^2+z^2)^{m+1}. \end{cases}$$

Using (2.5), we also have

$$(2.6)\quad \begin{cases} c(xy) = c\{y^2, z\}/2 = \{c(y^2), z\}/2 = 4d_1 i xy(x^2+y^2+z^2)^m, \\ c(xz) = c\{x^2, y\}/2 = \{c(x^2), y\}/2 = 4d_1 i xz(x^2+y^2+z^2)^m, \\ c(yz) = c\{x, y^2\}/2 = \{x, c(y^2)\}/2 = 4d_1 i yz(x^2+y^2+z^2)^m. \end{cases}$$

By equations (2.5) and (2.6), we know that a derivation c can be essentially written as $c(u_1) = u_1(x^2+y^2+z^2)^m$ for all $u_1 \in F_1$. (Recall that c is a "real" derivation.) Since F_1 generates F_p $(p \geqq 2)$, we also obtain that $c(u_p) = pu_p(x^2+y^2+z^2)^m$ for all $u_p \in F_p$. Finally we see that a derivation c is outer. In fact, if c is inner, there exists a function $f \in F_{2m}$ such that $c = \mathrm{ad}(f)$. Then $0 = c(x) = \{f, x\}$. Thus we have $c(x^2) = \{f, x^2\} = 2x\{f, x\} = 0$. On the other hand, $c(x^2) = x^2(x^2+y^2+z^2) \neq 0$. This is a contradiction. q.e.d.

We have thus determined the derivation algebra of F. We shall resume all results in

Theorem 2.6. *Let $c: F \to F$ be a derivation. Then* $c \equiv \sum_{m \geqq 0} \alpha_m c^{(2m)}$ (mod ad(F)), *where* $c^{(2m)}$ *is derivation defined by*

$$c^{(2m)}(u_p) = pu_p(x^2 + y^2 + z^2)^m \quad \textit{for all } u_p \in F_p, \tag{2.7}$$

and α_m *is some constant depending on* c. *In particular, all* $c^{(2m)}$ *are outer derivations, hence* $H^1(F, F)$ *is infinite dimensional.*

Let L be the formal Lie algebra of $\mathscr{L}$ at the origin. (For the precise definition of the formal Lie algebras, see [4].) Then the derivation c obtained in Theorem 2.6 induces a derivation $\bar{c}$ of L. The form of $\bar{c}$ is explicitly given by $\bar{c} = \mathrm{ad}((x^2 + y^2 + z^2)^m(x\partial_x + y\partial_y + z\partial_z))$.

We shall consider here C^∞-versions of the results obtained in the above theorem. For all non-negative integers m, let $X = (x^2 + y^2 + z^2)^m(x\partial_x + y\partial_y + z\partial_z)$ be a smooth vector field on $\mathfrak{so}(3, R)^* = R^3$. Then for all $Y = f\partial_x + g\partial_y + h\partial_z \in \mathscr{L}$, we have

$$\begin{aligned}\mathscr{L}([X, Y])P &= -\mathscr{L}(Y)L(X)P = \mathscr{L}(Y)\{(x^2 + y^2 + z^2)^m P\} \\ &= Y\{(x^2 + y^2 + z^2)^m\} P \\ &= 2m(x^2 + y^2 + z^2)^{m-1}(xf + yg + zh)P = 0.\end{aligned}$$

(Recall that $\mathscr{L} = \mathscr{I}$ by Theorem 1.2, hence $xf + yg + zh = 0$.) This implies $[X, Y] \in \mathscr{L}$ and thus ad(X) is a derivation of $\mathscr{L}$ for all $m \geqq 0$. This fact is quite different from the case of $\mathfrak{sl}(2, R)$. (See Proposition 3.1 in [3].)

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