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A note on global strong solutions of semilinear wave equations

Dedicated to Prof. Teruo Ikebe on his 60th birthday

By

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Introduction

Let L be a linear hyperbolic partial differential operator of second order with coefficients depending on $t \in [0, \infty)$ and $x = (x_1, \dots, x_n) \in \mathbf{R}^n$. Let Ω be a bounded or unbounded domain in $\mathbf{R}^n$ with a smooth boundary $\partial\Omega$, or the whole $\mathbf{R}^n$. We assume $3 \leqq n \leqq 6$. Let I be an interval in $[0, \infty)$ and let the time variable t run over I. We shall treat the following problem for a semilinear wave equation:

$$\text{[SLP]} \qquad \begin{cases} L[u] + F(u) = 0 & \text{in } I \times \Omega, \\ u(0, x) = f(x), \quad \partial_t u(x, 0) = g(x) & \text{in } \Omega, \\ B\,u(t, x) = 0 & \text{on } I \times \partial\Omega, \end{cases}$$

where F is a nonlinear term satisfying $|F(u)| \leqq \text{const.}\,(|u| + |u|^{\frac{n}{n-2}})$ as $|u| \to \infty$, and B is a suitable linear boundary operator. If $\Omega = \mathbf{R}^n$, of course we omit the boundary condition and consider the initial value problems. For mixed problems suitable boundary conditions are assigned. The explicit form of the linear hyperbolic operator L and the boundary operator B will not be given, because it is inessential to our argument.

Our aim is to show the existence of a global strong solution $u = u(t, x)$ of [SLP] for $I = [0, \infty)$. For that purpose we also need to solve [SLP] for some sufficiently short time intervals I. In order to solve [SLP] for some I we shall use some established results of the corresponding linear problem [LP],

$$\text{[LP]} \qquad \begin{cases} L[v] = G & \text{in } I \times \partial\Omega, \\ v(0, x) = f(x), \quad \partial_t v(0, x) = g(x) & \text{in } \Omega, \\ B\,v(t, x) = 0 & \text{on } I \times \partial\Omega, \end{cases}$$

where G is a given inhomogeneous term, and L, B and the data are the same as in [SLP].

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The literature concerning [SLP] or [LP] is now enormous (see e.g. [2], [3], [4], [5], [7], [8], [9], [10], [11] and [12]). Many authors (excepting Mizohata of [7]) convert [SLP] into the solution of integral equations by using the semigroup (or the evolution operators) generated by the linear operators L. Here we would like to adopt a different method with recourse to some established theories of linear hyperbolic differential equations, i.e. existence or uniqueness theorems, and energy estimates for L. This approach seems to be somewhat simpler than the integral equation method stated above. The author remarks that Mizohata also solves globally the initial value problem for $(\partial/\partial t)^2 u - \Delta u + F(u)$, not converting it into the problem of a nonlinear integral equation (see Chapter 7 in [7]).

We introduce some notations. Let S be a Banach space, I an interval in $[0, \infty)$ and m a nonnegative integer. By $C^m(I; S)$ we denote the set of all S-valued functions having all derivatives of order $\leqq m$ continuous on I. By $H^m(\Omega) = H^m$ we denote the usual Sobolev space defined on Ω, i.e. the set of all functions over Ω whose strong derivatives of order $\leqq m$ belong to $L^2(\Omega) = L^2$. H^0 equals L^2. By $\|u\|_m$ we denote the H^m norm of u, and we use $\|u\|$ as the L^2 norm of u, too. We set

$$X(I) = \{u \in \bigcap_{m=0}^{1} C^m(I; H^{2-m}) \ /Bu = 0 \text{ on } I \times \partial\Omega\}$$

and

$$Y(I) = X(I) \cap C^2(I; L^2).$$

We define the norm of u of $X(I)$ by

$$|||u|||_I = \sup_{t \in I} \{\|u(t)\|_2 + \|\partial_t u(t)\|_1\}.$$

If a solution of [SLP] belongs to $Y(I)$, we call it a strong solution. We define two kinds of energy for $u \in X(I)$ by

$$E_m(u(t)) = E_m(t) = \|u(t)\|_m + \|\partial_t u(t)\|_{m-1} \quad \text{for } m = 1, 2.$$

We remark that $|||u|||_I = \sup_{t \in I} E_2(u(t))$.

Here let us state the assumptions we shall make on L, B and [LP], which do not involve the explicit form of L or B, however.

[A 1] The boundary operator B is given so that $X(I)$ is complete with its norm $|||\cdot|||_I$.

We remark that it is uncertain whether $X(I)$ is complete or not, as the explicit form of B is not given, and that [A 1] actually holds if B is the Dirchlet or the Neumann boundary operator.

[A 2] Let $I = [\tau, t]$ be any finite interval in $[0, \infty)$. If $u \in Y(I)$, then

$$E_2(u(t)) \leqq \exp(k(t-\tau))E_2(u(\tau)) + \int_\tau^t \exp(k(t-s))\|Lu(s)\|_1 ds,$$

where k is a positive constant depending only on L and B.

[A 3] Let $I=[\tau, t]$ be any finite interval in $[0, \infty)$. If $G \in C^0(I; H^1)$, $f \in H^2$ and $g \in H^1$, then [LP] has a unique solution v in $Y(I)$.

[A 2] and [A 3] have been studied extensively and are actually proved if L is regularly hyperbolic (see [4], [7], [8] and [11]).

§ 1. Existence of solutions

The nonlinear term F is assumed to satisfy

[A 4]

i) $F: \mathbf{C} \to \mathbf{C}$ is continuous and has a continuous Fréchet derivative as a map on $\mathbf{R}^2$ into itself, where $\mathbf{C}$ is the set of complex numbers.

ii) $F(0)=0$.

iii) There exists a nondecreasing continuous function $M: [0, \infty) \to [0, \infty)$ such that for every $u, v \in H^2$

$$\|F(u)-F(v)\|_1 \leqq M(\|u\|_2 + \|v\|_2)\|u-v\|_2. \tag{1.1}$$

We shall make several remarks on the Fréchet derivative of F (see [6] and [11] for detailed explanation). The Fréchet derivative $F'(z)$ is identified with the pair $(\partial F/\partial z, \partial F/\partial \bar{z})$. We define $F'(z)\cdot w$ for $w \in \mathbf{C}$ by

$$F'(z)\cdot w = (\partial F/\partial z)w + (\partial F/\partial \bar{z})\bar{w}$$

and the norm of $F'(z)$ by

$$|F'(z)| = |(\partial F/\partial z)(z)| + |(\partial F/\partial \bar{z})(z)|.$$

Then $|F'(z)\cdot w| \leqq |F'(z)|\,|w|$ holds. Under i) of [A 4], $|F'(z)|$ is continuous in z. Let $\partial_i = \partial/\partial x_i$. We have the following relations.

$$\partial_i F(u(t, x)) = F'(u(t, x))\cdot \partial_i u(t, x) \quad \text{for } i = 1, 2, \dots, n, \tag{1.2}$$

and

$$\begin{aligned} F(u)-F(v) &= \int_0^1 (d/ds)\{F(su+(1-s)v)\}\,ds \\ &= \int_0^1 F'(su+(1-s)v)\cdot(u-v)\,ds. \end{aligned} \tag{1.3}$$

[A 4] is verified for some explicit function, e.g. $F(u)=|u|^{\frac{2}{n-2}}u$ ($n=3$ or 4) in the next section.

We set $a = \|f\|_2 + \|g\|_1 + 1$ for $f \in H^2$ and $g \in H^1$, and for some finite interval $I \subset [0, \infty)$

$$Z_a(I) = \{u \in X(I);\ |||u|||_I \leqq a\}$$

Now we prove a local existence theorem for [SLP] by the contracting mapping principle. Here [A 3] plays an important role.

Theorem 1 (*Local Existence*). *Let t_0 be any nonnegative number, and $f \in H^2$ and $g \in H^1$. Let*

$$h = \text{Min.}\ \left[k^{-1}\log\frac{(k + M(a))\,a}{(a-1)\,k + aM(a)},\quad k^{-1}\log\left(1 + \frac{k}{2M(2a)}\right)\right].$$

Then [SLP] for $I = [t_0, t_0 + h]$ has a unique solution u in $Y(I)$ satisfying $|||u|||_I \leqq a$.

Proof. Let

$$h_1 = k^{-1}\log\left[\frac{\{k + M(a)\}\,a}{(a-1)\,k + aM(a)}\right].$$

Let $I_1 = [t_0, t_0 + h_1]$ and $u \in Z_a(I_1)$. We see from [A 4] that $F(u)$ is in $C^0(I_1; H^1)$. Hence by [A 3], [LP] with the inhomogeneous term $-F(u)$ has a unique solution v in $Y(I_1)$. We define the mapping S by $u \mapsto v = \mathrm{S}u$. Then by [A 2] $\mathrm{S}u$ fulfils

$$E_2(\mathrm{S}u(t)) \leqq \exp(k(t - t_0))(\|f\|_2 + \|g\|_1) + \int_{t_0}^{t}\exp(k(t-s))\,\|F(u(s))\|_1\,ds. \tag{1.4}$$

Since u is in $Z_a(I_1)$, we have, by ii) and iii) of [A 4]

$$\|F(u(t))\|_1 \leqq M(\|u\|_2)\,\|u\|_2 \leqq aM(a) \quad (t \in I_1).$$

Then, from (1.4) we have for $t \in I_1$

$$\begin{aligned} E_2(\mathrm{S}u(t)) &\leqq \exp(k(t - t_0))(a-1) + \int_{t_0}^{t}\exp(k(t-s))\,aM(a)\,ds \\ &\leqq \{a - 1 + (aM(a)/k)\}\exp(kh_1) - (aM(a)/k) \\ &\leqq a, \end{aligned}$$

whence $|||\mathrm{S}u|||_{I_1} \leqq a$. Thus S maps $Z_a(I_1)$ into itself.

Let us set

$$h_2 = k^{-1}\log\left[1 + \left(\frac{k}{2M(2a)}\right)\right].$$

Let $I_2 = [t_0, t_0 + h_2]$, and u_1 and u_2 be in $Z_a(I_2)$. Then $v = \mathrm{S}u_1 - \mathrm{S}u_2$ is the unique solution in $Y(I_2)$ of the linear problem

$$\begin{cases} L[v] = -(F(u_1) - F(u_2)) & \text{in } I_2 \times \Omega, \\ v(t_0) = \partial_t v(t_0) = 0 & \text{in } \Omega, \\ Bv(t, x) = 0 & \text{on } I_2 \times \partial\Omega. \end{cases}$$

Hence by [A 2], $\mathbf{S}u_1 - \mathbf{S}u_2$ satisfies for $t \in I_2$

$$E_2(\mathbf{S}u_1(t) - \mathbf{S}u_2(t)) \leqq \int_{t_0}^{t} \exp(k(t-s)) \| F(u_1(s)) - F(u_2(s)) \|_1 \, ds$$

Then for $t \in I_2$ we have

$$\begin{aligned} E_2(\mathbf{S}u_1(t) - \mathbf{S}u_2(t)) &\leqq M(2a) \int_{t_0}^{t} \exp(k(t-s)) \| u_1(s) - u_2(s) \|_1 \, ds \\ &\leqq \left(\frac{M(2a)(\exp(k(t-t_0)) - 1)}{k} \right) \||| u_1(s) - u_2(s) |\||_{I_2} \\ &\leqq (1/2) \||| u_1(s) - u_2(s) |\||_{I_2}. \end{aligned}$$

Thus we have $\||| \mathbf{S}u_1 - \mathbf{S}u_2 |\||_{I_2} \leqq (1/2) \||| u_1(s) - u_2(s) |\||_{I_2}$.

We define h by $h = \mathrm{Min.}\ (h_1, h_2)$. Let $I = [t_0, t_0 + h]$. Thus we have shown that S is a contracting mapping from $Z_a(I)$ into itself, and S maps $Z_a(I)$ into $Y(I)$. Noting that $X(I)$ is complete by [A 1], S has a unique fixed point u in $Z_a(I) \cap Y(I)$. Clearly this u is the unique solution of [SLP] for I. Thus we have completed the proof.

Next we shall show the existence of global solutions. To this end we need an apriori estimate which guarantees solutions to remain bounded in time for any finite time. We assume:

[A 5] Let T be any positive number, and $I = [0, T]$. Let u be a solution in $Y(I)$ of [SLP] with $f \in H^2$ and $g \in H^1$. Then there exists a nonnegative continuous function $K_2(t)$ such that for $0 \leqq t \leqq T$

$$E_2(u(t)) \leqq K_2(t).$$

[A 5] will be actually proved for some concrete nonlinear terms in the next section. For its proof we will use some estimate of $E_1(u(t))$, which is left not verified because it is well-known for general L and B (see [A 6] in §2).

Theorem 2 (*Global Existence*). *Assume [A 5]. Let T be any positive number, $I = [0, T]$, and $f \in H^2$ and $g \in H^1$. Then [SLP] for I has a unique global solution belonging to $Y(I)$.*

Proof. Let $A_0 = \sup_{t \in I} K_2(t)$, $A = A_0 + 1$ and

$$H = \mathrm{Min.} \left[k^{-1} \log \frac{(k + M(A))\,A}{(A-1)\,k + AM(A)},\ k^{-1} \log \left(1 + \frac{k}{2M(2A)} \right) \right].$$

Noting $\|f\|_2 + \|g\|_1 \leqq A_0$, by Theorem 1 we can construct a unique solution u_1 of [SLP] for $[0, H]$ in $Y([0, H])$ satisfying $\|u_1\|_{[0,H]} \leqq A_0$. For $t \geqq H$ we consider the mixed problem of $L[u] + F(u) = 0$, with $u(H) = u_1(H)$, $\partial_t u(H) = \partial_t u_1(H)$ as the initial data and with the same boundary condition as in [SLP]. Noting $\|u_1(H)\|_2 + \|\partial_t u_1(H)\|_1 \leqq A_0$, again we can construct by Theorem 1 a unique solution u_2 of this problem for $[H, 2H]$. We define u by $u = u_1$ in $[0, H]$ and by $u = u_2$ in $[H, 2H]$. In the same way we can show that for any interval $I_0 = [t, t + H]$ contained in $[0, 2H]$, a unique solution of [SLP] with the initial data $u(t)$ and $\partial_t u(t)$ exists, and that its solution equals u in I_0. Then by the uniqueness of solutions, u is the unique solution of [SLP] for $[0, 2H]$ belonging to $Y[0, 2H])$. Repeating the same procedure a finite numbers of times, we can construct the unique solution of [SLP] for I, which completes the proof.

§ 2. Some estimates

We shall apply the results of § 1 to [SLP] with some explicit nonlinear terms. For that purpose we shall prove [A 4] and [A 5].

By $|\cdot|_p$ we denote the norm of $L^p(\Omega)$ with $1 \leqq p \leqq \infty$, but use $\|\cdot\|$ for the $L^2(\Omega)$ norm as before. We often denote various positive constants by the same symbol C.

We assume:

[A 6] Let T be any positive number, and $I = [0, T]$. Let u be a solution in $X(I)$ of [SLP] with $f \in H^2$ and $g \in H^1$. Then there exists a nonnegative continuous function $K_1(t)$ such that for $0 \leqq t \leqq T$

$$E_1(u(t)) \leqq K_1(t).$$

[A 6] is known to be satisfied for [SLP] with sufficiently general L. The proof of [A 6] is comparatively easy (see our references [2], [3], [7], [8], [9], [10], [11] and [12]). [A 6] is applied only to the proof of [A 5] (see Lemma 3 in this section).

The nonlinear term F is assumed to satisfy the following conditions.

[C] i) $F: \mathbf{C} \to \mathbf{C}$ is continuous and has a continuous Fréchet derivative as a map on $\mathbf{R}^2$ into itself.

ii) $F(0) = 0$.

iii) There exists a real valued nonnegative function $G: \mathbf{C} \to \mathbf{R}_+$ satisfying $F(z) = \partial G(z)/\partial \bar{z}$ and $G(0) = 0$.

iv) $|F(z)| \leqq C(|z| + |z|^{\frac{n}{n-2}})$ for $z \in \mathbf{C}$.

v) $|F'(z)| \leqq C(1 + |z|^{\frac{2}{n-2}})$ for $z \in \mathbf{C}$.

vi) $|F'(z_1) - F'(z_2)| \leqq C|z_1 - z_2|$ for z_1 and $z_2 \in \mathbf{C}$.

We remark that iii) will not be applied ostensibly to coming proofs, but if we actually try to prove [A 6] for concrete L and B, we shall use iii).

To prove [A 4] and [A 5] from [C] we prepare the following well-known Sobolev imbedding theorem:

Lemma 1. *Let* $2 \leqq p < \infty$ *and* $(1/p) \geqq (1/2) - (m/n)$. *Then for* $u \in H^m$

$$|u|_p \leqq C \|u\|_m.$$

For the proof of this lemma see Lemma 5 of [1].

Lemma 2. *Let* $3 \leqq n \leqq 6$ *and F fulfil* [*C*]. *Then F satisfies* [*A* 4].

Proof. Let u and v belong to H^2. By (1.3) we have

$$F(u) - F(v) = \int_0^1 F'(su + (1-s)v) \cdot (u - v) ds.$$

Then noting an inequality $(a+b)^p \leqq 2^{p-1}(a^p + b^p)$ for $a, b, p > 0$, we have from v) of [C]

$$\begin{aligned} \|F(u) - F(v)\| &\leqq \left\| \int_0^1 |F'(su + (1-s)v)| \, |(u-v)| ds \right\| \\ &\leqq C \|(1 + |u|^{\frac{2}{n-2}} + |v|^{\frac{2}{n-2}})(u - v)\| \\ &\leqq C(\|u - v\| + \|(u-v)|u|^{\frac{2}{n-2}}\| + \|(u-v)|v|^{\frac{2}{n-2}}\|). \end{aligned}$$

Applying the Hölder inequality and Lemma 1,

$$\|(u-v)|u|^{\frac{2}{n-2}}\| \leqq |u - v|_p (|u|_p)^{\frac{2}{n-2}} \leqq C \|u - v\|_1 (\|u\|_1)^{\frac{2}{n-2}},$$

where $p^{-1} = 2^{-1} - n^{-1}$. Thus we have

$$\|F(u) - F(v)\| \leqq C[\|u - v\| + \|u - v\|_1 \{(\|u\|_1)^{\frac{2}{n-2}} + (\|v\|_1)^{\frac{2}{n-2}}\}]. \tag{2.1}$$

Next we shall estimate $\|\partial_i F(u) - \partial_i F(v)\|$. By (1.2) we have

$$\|\partial_i F(u) - \partial_i F(v)\| \leqq \|F'(u) \cdot (\partial_i v - \partial_i u)\| + \|(F'(u) - F'(v)) \cdot \partial_i v\|. \tag{2.2}$$

By an estimation similar to the above, we have for $i = 1, 2, \dots, n$

$$\begin{aligned} \|F'(u) \cdot (\partial_i u - \partial_i v)\| &\leqq C\{\|\partial_i u - \partial_i v\| + \|(\partial_i u - \partial_i v)|u|^{\frac{2}{n-2}}\|\} \\ &\leqq C\{\|u - v\|_1 + \|u - v\|_2 (\|u\|_1)^{\frac{2}{n-2}}\}. \end{aligned} \tag{2.3}$$

From vi) of [C] we have

$$\|(F'(u) - F'(v)) \cdot \partial_i v\| \leqq C \|(u - v)\partial_i v\|.$$

Noting $3 \leqq n \leqq 6$, we can choose p and $q \geqq 2$ such that $p^{-1} + q^{-1} = 2^{-1}$, $p^{-1} \geqq 2^{-1} - (2/n)$ and $q^{-1} \geqq 2^{-1} - n^{-1}$. Hence from the Hölder inequality and Lemma 1 we have for $i = 1, \dots, n$

$$\|(F'(u)-F'(v))\cdot\partial_i v\| \leqq C|u-v|_p|\partial_i v|_q \leqq C\|u-v\|_2\|v\|_2 \tag{2.4}$$

Thus by (2.1), (2.2), (2.3) and (2.4) we have

$$\begin{aligned}\|F(u)-F(v)\|_1 &\leqq C\{\|F(u)-F(v)\| + \sum_{i=1}^{n}\|F'(u)\cdot(\partial_i u-\partial_i v)\| \\ &\qquad + \sum_{i=1}^{n}\|(F'(u)-F'(v))\cdot\partial_i v\|\} \\ &\leqq C[\|u-v\| + \|u-v\|_1\{(\|u\|_1)^{\frac{2}{n-2}} + (\|v\|_1)^{\frac{2}{n-2}}\} \\ &\qquad + \|u-v\|_1 + \|u-v\|_2(\|u\|_1)^{\frac{2}{n-2}} + \|u-v\|_2\|v\|_2] \\ &\leqq C\{2+2(\|u\|_2+\|v\|_2)^{\frac{2}{n-2}} + (\|u\|_2+\|v\|_2)\}\|u-v\|_2.\end{aligned}$$

We set $M(s)=C(2+2s^{\frac{2}{n-2}}+s)$. Then $M(s)$ is nonnegative, nondecreasing and continuous in $s \geqq 0$, and

$$\|F(u)-F(v)\|_1 \leqq M(\|u\|_2+\|v\|_2)\|u-v\|_2$$

holds, which was to be shown.

It remains to show that the energy E_2 for a solution of [SLP] is apriori bounded on any finite interval $I=[0, T]$.

Lemma 3. *Let F fulfil* [C]. *Assume* [A6]. *Then* [A5] *actually holds.*

Proof. By iv) of [C] and Lemma 1 we have

$$\begin{aligned}\|F(u)\| &\leqq C(\||u|^{\frac{n}{n-2}}\| + \|u\|) = C\{(|u|_{\frac{2n}{n-2}})^{\frac{n}{n-2}} + \|u\|\} \\ &\leqq C\{(\|u\|_1)^{\frac{n}{n-2}} + \|u\|\}.\end{aligned}$$

Applying the Hölder inequality, we have

$$\begin{aligned}\||u|^{\frac{2}{n-2}}\partial_i u\| &\leqq ||u|^{\frac{2}{n-2}}|_n|\partial_i u|_{\frac{2n}{n-2}} \\ &= (|u|_{\frac{2n}{n-2}})^{\frac{2}{n-2}}|\partial_i u|_{\frac{2n}{n-2}}.\end{aligned}$$

Then by (1.2), v) of [C] and Lemma 1 we have for $i=1,\dots,n$

$$\begin{aligned}\|\partial_i F(u))\| = \|F'(u)\cdot\partial_i u\| &\leqq C(\|\partial_i u\| + \||u|^{\frac{2}{n-2}}\partial_i u\|) \\ &\leqq C\{\|\partial_i u\| + (|u|_{\frac{2n}{n-2}})^{\frac{2}{n-2}}|\partial_i u|_{2n/n-2}\} \\ &\leqq C\{\|\partial_i u\| + (\|u\|_1)^{\frac{2}{n-2}}\|\partial_i u\|_1\}\end{aligned}$$

$$\leqq C\{\|u\|_1 + (\|u\|_1)^{\frac{2}{n-2}}\|u\|_2\}.$$

Thus, noting $\|u\|_1 \leqq E_1(t) \leqq K_1(t)$ and $\|u\|_2 \leqq E_2(t)$, we have

$$\|F(u))\|_1 \leqq \|F(u))\| + \sum_{i=1}^{n} \|F'(u)\cdot\partial_i u\|$$

$$\leqq C\{2K_1(t)^{\frac{n}{n-2}} + K_1(t)^{\frac{2}{n-2}}E_2(t)\}.$$

So we have by [A 2]

$$E_2(t) \leqq \exp(kt)(\|f\|_2 + \|g\|_1)$$

$$+ C\int_0^t \exp(k(t-s))\{2K_1(s) + K_1(s)^{\frac{n}{n-2}} + K_1(s)^{\frac{2}{n-2}}E_2(t)\}\,ds.$$

Hence by the Grownwall inequality we can find a nonnegative continuous function $K_2(t)$ such that $E_2(t) \leqq K_2(t)$. Thus the proof is complete.

Combining with the last two lemmas and Theorem 1 and 2, we obtain:

Theorem 3. Let $f \in H^2$ and $g \in H^1$. Let F satisfy [C]. Assume [A 1], [A 2], [A 3] and [A 6]. Then [S L P] has the unique solution in $Y([0, \infty))$.

Apped in proof. [A 2] is well-known for the initial value problems, but it seems doubtful for the initial-boundary value problems. Instead of [A 2] we assume the energy inequality (2.21) in Proposition 2.6 of [4] and we assume $G \in C^1(I; L^2)$ in [A 3] instead of $G \in C^0(I; H^1)$. Then we can show the same results for the initial-boundary value problems by the slight modification of our argument. The detailed exposition will be published.

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