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CORRECTIONS TO HERMITIAN MORITA EQUIVALENCES BETWEEN MAXIMAL ORDERS IN CENTRAL SIMPLE ALGEBRAS

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ABSTRACT. Corrections to Dasgupta's "Hermitian Morita equivalences between maximal orders in central simple algebras" (*Illinois J. Math.* **53** (2009) 723–736) follow. $\operatorname{char} K = 2$ was not considered in the paper, nor was the fact that $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon$ could be equal to -1 at a couple of points in the proof of Proposition 3 of the paper. Changes in the proof of Proposition 3, in its statement and the corresponding changes in the statements of Theorems 1, 2 and 3 follow.

1. Correction to Proposition 3 of [1]

The corrected Proposition 3 should be thus:
"Let

$$\big(A, D, {}_AV_D, {}_D\big(V^*\big)_A, \mu, \tau\big)$$

be a Morita context where V is the set of $n \times 1$ matrices over D and V^* is the set of $1 \times n$ matrices over D and μ and τ are given by multiplication of matrices, that is, $\mu(v \otimes f) = vf$ and $\tau(f \otimes v) = fv$ where $v \in V$ and $f \in V^*$. Let y_1 satisfy $a^{\alpha} = y_1 a^{\beta} y_1^{-1}$. If $A = (A = M_n(D), \alpha, \epsilon)$ is Hermitian Morita equivalent to $D = (D, \beta, \delta)$ via

$$\big(A, D, \theta : {}_AV_D \longrightarrow {}_D\big(V^*\big)_A, \mu, \tau\big),$$

then $\theta : V \longrightarrow V^*$, the bijection, is defined by $\theta(v) = kv^{\beta}y_1^{-1}$, for some $k \in K$, if $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon = 1$, or if both $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon = -1$ and $\operatorname{char} K = 2$ are true. If $\theta : V \longrightarrow V^*$ is a bijection defined by $\theta(v) = kv^{\beta}y_1^{-1}$, for some $k \in K$,

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(I) then (A, α, ϵ) is HME to (D, β, δ), when $\delta^{-1} y_1^{\beta} y_1^{-1} \epsilon = 1$, or when $\delta^{-1} y_1^{\beta} \times y_1^{-1} \epsilon = -1$ if char $K = 2$, via

$$\big(A, D, \theta : {}_A V_D \longrightarrow {}_D \big(V^*\big)_A, \mu, \tau\big),$$

(II) then (A, α, ϵ) is HME to $D_1 = (D, \beta, -\delta)$, when $\delta^{-1} y_1^{\beta} y_1^{-1} \epsilon = -1$, or when $\delta^{-1} y_1^{\beta} y_1^{-1} \epsilon = 1$ if char $K = 2$, via

$$\big(A, D_1, \theta : {}_A V_{D_1} \longrightarrow {}_{D_1} \big(V^*\big)_A, \mu, \tau\big).\text{”}$$

The changes which need to be made to the proof of Proposition 3 of the paper follow.

The sentence “It is easily checked that these two equations form a consistent set of equations having the solution $\theta(v) = x^{-1} v^{\beta} y_1^{-1}$.” should be replaced by these following sentences.

“Note that since $\beta|_K = \mathrm{id}_K$, for $k \in K$, $k^{\beta} = k$. Further $-k^{\beta} = k$ if and only if char $K = 2$.

The second equation, $\theta(v) = x^{-1} v^{\beta} y_1^{-1}$, gives $x^{-1\beta} = x^{-1}$ since $x \in K$ by item (e) of the Hermitian equivalence data.

The first equation is $v = y_1 \theta(\epsilon^{-1} v \delta)^{\beta} x$.

So $v^{\beta} y_1^{-1\beta} = x^{\beta} \delta \theta(\epsilon^{-1} v \delta) \delta^{-1}$.

Then $x^{-1\beta} v^{\beta} y_1^{-1\beta} \delta = \delta \delta^{-1} \theta(v) \epsilon$.

Hence, $x^{-1\beta} v^{\beta} y_1^{-1\beta} \delta \epsilon^{-1} = \theta(v)$, that is,

$$\theta(v) = x^{-1\beta} v^{\beta} y_1^{-1} \quad \text{if } \delta^{-1} y_1^{\beta} y_1^{-1} \epsilon = 1 \tag{1.1}$$

and

$$\theta(v) = -x^{-1\beta} v^{\beta} y_1^{-1} \quad \text{if } \delta^{-1} y_1^{\beta} y_1^{-1} \epsilon = -1. \tag{1.2}$$

Equation (1.1) forms a consistent set of equations with $\theta(v) = x^{-1} v^{\beta} y_1^{-1}$ since $x^{-1} = x^{-1\beta}$, Equation (1.2) forms a consistent set of equations with $\theta(v) = x^{-1} v^{\beta} y_1^{-1}$ if $x^{-1} = -x^{-1\beta} = -x^{-1}$, that is if char $K = 2$.

Both sets of equations have the solution $\theta(v) = x^{-1} v^{\beta} y_1^{-1}$.”

In the converse part of the theorem, part (f) line 4 should be “$= v k v'^{\beta} y_1^{-1}$ when $y_1 y_1^{-1\beta} \delta \epsilon^{-1} = 1$ since $\beta|_K = \mathrm{id}_K$ and when $y_1 y_1^{-1\beta} \delta \epsilon^{-1} = -1$ since char $K = 2$.” and part (g) line 3 should be “$= (\delta^{-1} v'^{\beta} (y_1^{-1})^{\beta} \delta v)^{\beta} k$ when $\delta^{-1} y_1^{\beta} y_1^{-1} \epsilon = 1$ since $\beta|_K = \mathrm{id}_K$ and when $\delta^{-1} y_1^{\beta} y_1^{-1} \epsilon = -1$ since char $K = 2$.”

Further in (II) add this sentence at the end, “If $y_1 y_1^{-1\beta} \delta \epsilon^{-1} = 1$, result follows since char $K = 2$.”

That concludes the changes to the proof.

2. Corrected Theorems 1, 2, and 3 statements

The corresponding changes to statements of Theorems 1, 2 and 3 follow.

In particular, Theorem 1 should be thus:

"If $\Lambda = (\Lambda, \alpha, \epsilon)$ is Hermitian Morita equivalent to $\Delta = (\Delta, \beta, \delta)$ via the Hermitian equivalence data,

$$\big(\Lambda, \Delta, \theta : {}_{\Lambda}M_{\Delta} \longrightarrow {}_{\Delta}M^{*}_{\Lambda}, \mu, \tau\big),$$

then $\theta : {}_{\Lambda}M_{\Delta} \longrightarrow {}_{\Delta}M^{*}_{\Lambda}$, the bijection, is given by $\theta(m) = km^{\beta}y_1^{-1}$ for some $k \in K$ if $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon = 1$, or if both $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon = -1$ and $\operatorname{char} K = 2$ are true. If $\theta : {}_{\Lambda}M_{\Delta} \longrightarrow {}_{\Delta}M^{*}_{\Lambda}$ is a bijection defined by $\theta(m) = km^{\beta}y_1^{-1}$ for some $k \in K$, then

(a) $(\Lambda, \alpha, \epsilon)$ is HME to (Δ, β, δ) when $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon = 1$, or when $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon = -1$ if $\operatorname{char} K = 2$, via

$$\big(\Lambda, \Delta, \theta : {}_{\Lambda}M_{\Delta} \longrightarrow {}_{\Delta}\big(M^{*}\big)_{\Lambda}, \mu, \tau\big).$$

(b) $(\Lambda, \alpha, \epsilon)$ is HME to $\Delta_1 = (\Delta, \beta, -\delta)$ when $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon = -1$, or when $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon = 1$ if $\operatorname{char} K = 2$, via

$$\big(\Lambda, \Delta_1, \theta : {}_{\Lambda}M_{\Delta_1} \longrightarrow {}_{\Delta_1}\big(M^{*}\big)_{\Lambda}, \mu, \tau\big)."$$

Theorem 2 should be thus:

"If $\Lambda = (\Lambda, \alpha, \epsilon)$ is Hermitian Morita equivalent $\Delta = (\Delta, \beta, \delta)$ via the Hermitian equivalence data,

$$\big(\Lambda, \Delta, \theta : {}_{\Lambda}M_{\Delta} \longrightarrow {}_{\Delta}M^{*}_{\Lambda}, \mu, \tau\big),$$

then for some $k \in K$, $kP_1 = \Delta$ if $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon = 1$, or if both $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon = -1$ and $\operatorname{char} K = 2$ are true. If $kP_1 = \Delta$ for some $k \in K$, then

(a) $(\Lambda, \alpha, \epsilon)$ is HME to (Δ, β, δ) when $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon = 1$, or when $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon = -1$ if $\operatorname{char} K = 2$, and

(b) $(\Lambda, \alpha, \epsilon)$ is HME to $(\Delta, \beta, -\delta)$ when $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon = -1$, or when $\delta^{-1}y_1^{\beta} \times y_1^{-1}\epsilon = 1$ if $\operatorname{char} K = 2$."

Theorem 3 should read "Let $a^{\alpha} = y_1 a^{\beta} y_1^{-1}$. Let $V = KM$. Assume that $\dim V_D \geq 2$. If y_1 can be chosen to be a unit of Λ, then

(a) $(\Lambda, \alpha, \epsilon)$ is HME to (Δ, β, δ) when $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon = 1$, or when $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon = -1$ if $\operatorname{char} K = 2$, and

(b) $(\Lambda, \alpha, \epsilon)$ is HME to $(\Delta, \beta, -\delta)$ when $\delta^{-1}y_1^{\beta}y_1^{-1}\epsilon = -1$, or when $\delta^{-1}y_1^{\beta} \times y_1^{-1}\epsilon = 1$ if $\operatorname{char} K = 2$."

Reference

[1] B. Dasgupta, *Hermitian Morita equivalences between maximal orders in central simple algebras*, Illinois J. Math. **53** (2009) 723–736. MR 2727351

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