

## CORRIGENDUM TO “THE FOLIATED STRUCTURE OF CONTACT METRIC $(\kappa, \mu)$ -SPACES”

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ABSTRACT. We correct the statement of a theorem in Illinois J. Math. **53** (2009), 1157–1172 concerning the Betti numbers of a compact  $(\kappa, \mu)$ -spaces.

### 1. Betti numbers of $(\kappa, \mu)$ -spaces

In [3] the author studied the geometry of the two Legendre foliations  $\mathcal{D}(\lambda)$  and  $\mathcal{D}(-\lambda)$  canonically attached to any non-Sasakian  $(\kappa, \mu)$ -space. In particular he obtained a geometrical interpretation of the Boeckx’s classification of contact metric  $(\kappa, \mu)$ -manifolds based on the Boeckx invariant  $I_M = \frac{1-\mu}{\sqrt{1-\kappa}}$  [2]. Moreover, he proved the following theorem.

**THEOREM 1.1.** *Any contact metric  $(\kappa, \mu)$ -manifold  $(M, \varphi, \xi, \eta, g)$  such that  $|I_M| > 1$  admits a Sasakian metric compatible with the original contact form  $\eta$ .*

Then Theorem 1.1 allows us to obtain some topological obstructions for a compact  $(\kappa, \mu)$ -space. In fact Fujitani [4] proved that in any compact Sasakian manifold of dimension  $2n + 1$  the  $p$ th Betti number  $b_p$  is even for  $p$  odd and  $1 \leq p \leq n$ , and, by duality, for  $p$  even and  $n + 1 \leq p \leq 2n$  (see also [1]).

However, due to just an incorrect recalling of the above result of Fujitani, in [3] it was stated that in any compact  $(\kappa, \mu)$ -manifold of dimension  $2n + 1$  such that  $|I_M| > 1$  all the Betti numbers  $b_p$ , with  $1 \leq p \leq 2n$ , are even [3, Corollary 10].

Hence we correct the above misprint and we can state the following theorem.

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**THEOREM 1.2.** *Let  $(M, \varphi, \xi, \eta, g)$  be a compact contact metric  $(\kappa, \mu)$ -manifold of dimension  $2n + 1$  such that  $|I_M| > 1$ . Then the  $p$ th Betti number  $b_p$  of  $M$  is even if  $1 \leq p \leq n$  with  $p$  odd and if  $n + 1 \leq p \leq 2n$  with  $p$  even.*

Theorem 1.2 is the first topological obstruction known for  $(\kappa, \mu)$ -spaces.

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