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MATHEMATICAL BIOGRAPHY OF PHIL GRIFFITH

By the late 1950s and throughout the 1960s the way in which problems in algebra were formulated and solved was greatly influenced by methods of homological algebra. Cartan and Eilenberg's seminal text, Homological Algebra, was published in 1956. No doubt for that reason Phil Griffith's early forays into mathematical research centered on problems that had a hint of homological flavor and for the most part could be translated into that language. His PhD advisor, Paul Hill, was a master in transfinite methods that he mostly applied to questions arising in the theory of abelian groups. The combination of view points proved to be lucky for the research fortunes of Griffith during the summer of 1967 in which he solved the Baer splitting problem for abelian groups (an abelian group G was called a Baer group provided any extension of a torsion group by G must be split exact; the problem had remained open for 30 years). Other results in the context of abelian groups were obtained by Griffith during this early period, e.g., he was first to construct, for prescribed $n > 0$, non-free abelian groups in which each subgroup of cardinality less than "aleph n" is free (a problem from Fuch's volumes on abelian groups). Perhaps it is fair to say that Griffith's main research success came from attempts to solve problems rather than to formulate and develop complex theoretical foundation. In 1970 he wrote a monograph, Infinite Abelian Group Theory, that was published by University of Chicago Press.

In 1968–1970 Griffith held a postdoctoral position at University of Chicago. Under the tutelage of Irving Kaplansky he developed a keen interest in the work of M. Auslander and H. Bass. While at Chicago, Griffith was fortunate to be in the company of D. Eisenbud, E.G. Evans and J.C. Robson. The research accomplished by Griffith during his Chicago experience was mainly focused on questions surrounding finite dimensional algebras and Artin rings. He collaborated with Eisenbud and Robson on these topics. However his interest in commutative algebra had begun in earnest, and he would later begin a ten year collaboration with E.G. Evans in that subject.

In 1970 Griffith became a member of the University of Illinois faculty, and in 1971 he was awarded an Alfred P. Sloan Foundation fellowship. In 1972 he was a visiting faculty member at Aarhus Universitet (Denmark). It was here while listening to lectures of many algebraic geometers (especially the French algebraic geometers) that Griffith's research became directed towards the newly developed area one might call "homological commutative algebra".

He collaborated with R. Fossum and H.-B. Foxby, the outgrowth of which was a joint article published in IHES on the structure of minimal injective resolutions (also with I. Reiten). The "homological era" in commutative algebra really began to flourish during this period as a result of the immense influence and success of sheaf theoretic tools developed by Serre and Grothendieck in tackling problems in algebraic geometry. In 1968 C. Peskine and L. Szpiro wrote their famous ground-breaking joint work published in IHES based on the methods of Serre and Grothendieck together with the "method of the Frobenius" inspired by E. Kunz. In this article they solved several of the "homological conjectures" in important cases, and they established relationships between many of these conjectures. By 1973 a second major breakthrough was to take place; namely M. Hochster would describe a procedure to construct a class of infinitely generated modules known as "maximal Cohen-Macaulay modules". These modules would have many applications by way of a general notion of "intersection theory" in settling a great number of the homological conjectures, and would later play a major role in the Evans-Griffith collaboration on the theory of syzygies. Hochster's construction (valid for equicharacteristic local rings) was based in part on his uncanny ability to formulate and solve "large" systems of equations for theoretical purposes.

Around 1975 Griffith teamed up with E.G. Evans to work on a "syzygy problem" about which Evans had been ruminating. The problem had been first stated in an unpublished PhD thesis by P. Hackman (Stockholm, Sweden) as a problem about "exterior powers and homology". The problem comes down to showing that a kth syzygy of finite projective dimension has rank at least k—or else it is free. In 1980 Evans and Griffith solved the problem for equicharacteristic local rings (later they discovered a proof in the mixed characteristic case for standard graded rings). A key point in their solution was to establish that order ideals of minimal generators for such non-free syzygy modules have height at least k. In turn the order ideal result rested upon proving an important fact about the length of certain finite free complexes that has come to be known as the "Improved New Intersection Theorem" (explicitly stated and named by Hochster). Although Hochster and Dutta (and later Hochster and Huneke) gave alternate proofs of the syzygy theorem, Evans and Griffith used Hochster's maximal Cohen-Macaulay modules as a basic tool for their arguments. Their original proof appeared in the Annals of Mathematics in 1981. In 1985 they co-authored a monograph, Syzygies, published by the London Mathematical Society. In all, Evans and Griffith collaborated on 15 articles related to the syzygy theorem. One of the pleasant byproducts of this theorem was the number of diverse applications that resulted, ranging from non-vanishing of cohomology of vector bundles to Cohen-Macaulay properties of three-generated ideals.

In 1983–1984 Fossum and Griffith were co-organizers for an NSF sponsored "Special Year" in commutative algebra. It should be mentioned that

Griffith benefited from year long visits at Illinois by renowned commutative algebraists, M. Auslander, L. Avramov, W. Bruns, H.-B. Foxby, C. Huneke, R.Y. Sharp and A. Singh. His weekly tea/coffee meetings with S. Dutta also have served as a time for amusement and inspiration.

Griffith, a native of Danville, Illinois, graduated from Northern Michigan University with a BS degree in 1963. After two years and receiving an MS degree at University of Missouri, he graduated from University of Houston with a PhD degree in 1968. He was married to his wife Judith in July 1960. They have two daughters, Keri and Lesli, and three grandchildren.

During his thirty-five years as a regular faculty member Griffith served on virtually every committee in the department. He served as research advisor for ten PhD students and as Director of Graduate Studies 2000-05.