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L_h^2-DOMAINS OF HOLOMORPHY IN THE CLASS OF UNBOUNDED HARTOGS DOMAINS

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ABSTRACT. A characterization of L_h^2-domains of holomorphy in the class of Hartogs domains in $\mathbb{C}^2$ is given.

There is a precise geometric characterization of bounded L_h^2-domains of holomorphy. Namely, we have the following theorem:

THEOREM 1 (see [PZ1]). *Let D be a bounded pseudoconvex domain in $\mathbb{C}^n$. Then D is an L_h^2-domain of holomorphy if and only if $U \setminus D$ is not pluripolar for any open set U with $U \setminus D \neq \emptyset$.*

As noted by M. A. S. Irgens, there is no hope that an analogous result holds in the unbounded case (see [Irg]); it is sufficient to consider the domain $\mathbb{C} \times \mathbb{D}$ which is not an L_h^2-domain of holomorphy (the space $L_h^2(\mathbb{C} \times \mathbb{D})$ is trivial), although the geometric condition from Theorem 1 is satisfied.

Therefore it is natural to try to find a characterization of unbounded L_h^2-domains of holomorphy. Recall that there is such a characterization in the case of planar domains.

THEOREM 2 (see, e.g., [Con], Chapter 21.9). *Let D be a domain in $\mathbb{C}$. Then D is an L_h^2-domain of holomorphy if and only if $U \setminus D$ is not polar for any open set U with $U \setminus D \neq \emptyset$. More precisely, for a point $a \in \partial D$ and an open neighborhood U of a there is an analytic continuation of any function $f \in L_h^2(D)$ onto U if and only if $U \setminus D$ is polar.*

Another class of domains in which a full description of L_h^2-domains of holomorphy is known is the class of Reinhardt domains (see [JP]).

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In this paper we present a characterization of L^2_h-domains of holomorphy in the class of unbounded Hartogs domains whose base is a planar domain. The results of the paper may also be seen as a continuation of results from [PZ2], where Bergman completeness in the class of unbounded Hartogs domains is studied.

For a domain $D \subset \mathbb{C}^n$ denote by $L^2_h(D)$ the class of square integrable holomorphic functions on D.

Recall that a domain $D \subset \mathbb{C}^n$ is called an *L^2_h-domain of holomorphy* if there are no domains $D_0, D_1 \subset \mathbb{C}^n$ with $\emptyset \neq D_0 \subset D_1 \cap D$, $D_1 \not\subset D$ such that for any $f \in L^2_h(D)$ there exists an $\tilde{f} \in \mathcal{O}(D_1)$ with $\tilde{f} = f$ on D_0.

For a subharmonic function $\rho : D \mapsto [-\infty, \infty)$, where D is a domain in $\mathbb{C}$, we define

$$G_{D,\rho} := \{(z_1, z_2) \in D \times \mathbb{C} : |z_2| < e^{-\rho(z_1)}\};$$

we call this domain a *Hartogs domain with base D*.

For $f \in L^2_h(G_{D,\rho})$ define

$$\rho_f(z_1) := \limsup_{j\to\infty} \frac{1}{j} \log |f_j(z_1)|, \ z_1 \in D,$$

where $f(z_1, z_2) = \sum_{j=0}^{\infty} f_j(z_1) z_2^j$, $(z_1, z_2) \in D$; the f_j's are the coefficients of the Hartogs expansion of f in $G_{D,\rho}$. Certainly, $\rho_f \leq \rho$ on D, so $\rho_f^* \leq \rho$ on D, where g^* denotes the upper regularization of the function g.

Then define $\tilde{\rho} := \sup_{f \in L^2_h(G_{D,\rho})} \rho_f^*$ on D. Clearly, $\tilde{\rho}^*$ is a subharmonic function on D, and $\tilde{\rho}^* \leq \rho$.

For a domain $D \subset \mathbb{C}$ define

$$\mathcal{S} := \mathcal{S}(D) := \{z \in \partial D : U \setminus D \text{ is polar for some open neighborhood } U \text{ of } z\}.$$

We can then reformulate Theorem 2 as follows: The domain $D \subset \mathbb{C}$ is an L^2_h-domain of holomorphy if and only if $\mathcal{S} = \emptyset$.

We denote by $\mathbb{D}$ the unit disc in $\mathbb{C}$.

Our main aim is to prove the following theorem:

THEOREM 3. *Assume that ρ is bounded from below.*

(a) *If $D \neq \mathbb{C}$, then $G_{D,\rho}$ is an L^2_h-domain of holomorphy if and only if $\limsup_{D \ni z \to z_0} \rho(z) = \infty$ for any $z_0 \in \mathcal{S}$.*
(b) *If $D = \mathbb{C}$, then $G_{\mathbb{C},\rho}$ is an L^2_h-domain of holomorphy if and only if ρ is not constant.*

Let us begin with some lemmas.

LEMMA 4. *Assume that ρ is bounded from below and let $z_1^0 \in D$ be such that there is no open connected neighborhood $U \subset D$ of z_1^0 such that $\rho_{|U}$ is constant. Then $\rho(z_1^0) = \tilde{\rho}^*(z_1^0)$.*

Proof. Assume that $\rho \geq M$ on D. Suppose that $\tilde{\rho}^*(z_1^0) < M_1 < \rho(z_1^0)$. Then there is an open disc $U \subset D$ centered at z_1^0 such that $\rho_f(z_1) \leq \rho_f^*(z_1) \leq \tilde{\rho}(z_1) < M_1$ for any $z_1 \in U$ and $f \in L_h^2(G_{D,\rho})$. Therefore, for any $f \in L_h^2(G_{D,\rho})$ the function $\mathcal{F}_f$ defined as

$$\mathcal{F}_f(z_1, z_2) := \sum_{j=0}^{\infty} f_j(z_1) z_2^j, \ (z_1, z_2) \in U \times e^{-M_1}\mathbb{D}$$

is a well defined holomorphic function—it follows from the Hartogs Lemma that the Hartogs series defining $\mathcal{F}_f$ is locally uniformly convergent in $U \times e^{-M_1}\mathbb{D}$. Certainly, $\mathcal{F}_f = f$ on $G_{D,\rho} \cap (U \times e^{-M_1}\mathbb{D})$. Therefore $\mathcal{F}_f$ is a holomorphic continuation of f onto $U \times e^{-M_1}\mathbb{D}$.

On the other hand, let $z_2^0 \in \mathbb{C}$ be such that $|z_2^0| = e^{-\rho(z_1^0)}$. Note that $(z_1^0, z_2^0) \in (\partial G_{D,\rho} \cap (U \times e^{-M_1}\mathbb{D}))$. For $z_2 \in e^{-M_1}\mathbb{D}$ define

$$U(z_2) := \{z_1 \in U : (z_1, z_2) \in G_{D,\rho}\}.$$

We claim that there is a $\tilde{z}_2 \in e^{-M_1}\mathbb{D}$ such that $U(\tilde{z}_2) \neq \emptyset$ and $U \setminus U(\tilde{z}_2)$ is not polar. Actually, since $\rho_{|U}$ is not constant, we easily get the existence of a $\tilde{z}_2 \in e^{-M_1}\mathbb{D}$ such that $U(\tilde{z}_2) \neq \emptyset$ and $U(\tilde{z}_2) \neq U$. Note that $U(\tilde{z}_2) = \{z_1 \in U : \rho(z_1) < -\log|\tilde{z}_2|\}$. Suppose that $U \setminus U(\tilde{z}_2)$ is polar. Then $\rho(z_1) \geq -\log|\tilde{z}_2|$ for any $z_1 \in U(\tilde{z}_2)$. Since $U \setminus U(\tilde{z}_2)$ is polar, $\rho(z_1) \geq -\log|\tilde{z}_2|$ for any $z_1 \in U$, so $U(\tilde{z}_1) = \emptyset$, which is a contradiction. Therefore $U \setminus U(\tilde{z}_2) \neq U$ is not polar. In particular, there is a function $f \in L_h^2(V_1 \times \{\tilde{z}_2\})$, where $V_1 := \{z_1 \in D : (z_1, \tilde{z}_2) \in G_{D,\rho}\}$, which does not have a holomorphic continuation on $U \times \{\tilde{z}_2\}$ (see Theorem 2).

It follows from a result of T. Ohsawa (see [Ohs]), applied to $\Psi(\cdot) := 2g_{e^{-M}\mathbb{D}}(\tilde{z}_2, \cdot)$ (where $g_D(p, \cdot)$ denotes the Green function of the domain D with logarithmic pole at p), that there is a function $F \in L_h^2(G_{D,\rho})$ such that $F_{|V_1 \times \{\tilde{z}_2\}} \equiv f$. (Here and in the sequel, when referring to the result of Ohsawa, we use the formulation in [CKO, p. 706].) But it follows from the earlier property that F extends to a holomorphic function on $U \times e^{-M_1}\mathbb{D}$. Therefore f extends holomorphically onto $U \times \{\tilde{z}_2\}$, which is a contradiction. □

Remark. Let us make a remark on the proof of the above lemma. We provided the proof with the help of a new extension result of Ohsawa. The result in [Ohs] applies to the unbounded case (unlike the one in the standard version of the extension theorem in [OT]), but there are some limits. Namely, the possibility of the extension of an L_h^2-function from the hyperplane depends on the existence of a suitable plurisubharmonic function Ψ—in our proof this is the Green function of the projection of the domain $G_{D,\rho}$ onto the second variable. At this place it is important that the projection is bounded, in other words, that the function ρ is bounded from below.

LEMMA 5. *Let ρ be bounded from below and not constant and let $z_1^0 \in D$ be such that there is an open connected neighborhood $U \subset D$ of z_1^0 such that $\rho_{|U}$ is constant. Then $\tilde{\rho}^* \equiv \rho$ on U.*

Proof. Let $\tilde{V}_1$ denote the set of all $z_1 \in D$ such that there is an open neighborhood V of z_1 such that $\rho_{|V}$ is constant. Certainly $\tilde{V}_1$ is open and $\tilde{V}_1 \supset U$. Denote by V_1 the connected component of $\tilde{V}_1$ such that $V_1 \supset U$. Then $\rho_{|V_1} \equiv C$ for some $C \in \mathbb{R}$.

We claim that

$$(*) \qquad \tilde{\rho}^* \text{ is constant on } V_1.$$

Without loss of generality assume that $\tilde{\rho}^* \not\equiv -\infty$. Then $(\tilde{\rho}^*)^{-1}(-\infty)$ is polar. We observe that in order to prove $(*)$ it is sufficient to show the following property:

$(**)$ For any $z_1 \in V_1$ such that $\tilde{\rho}^*(z_1) > -\infty$ there is an open neighborhood $U(z_1) \subset V_1$ of z_1 such that $\tilde{\rho}^*$ is constant on $U(z_1)$.

First we prove the implication "$(**) \Rightarrow (*)$". Assume that $(**)$ holds. Then the set $\{z_1 \in V_1 : \tilde{\rho}^*(z_1) > -\infty\}$ is open, so $(\tilde{\rho}^*)^{-1}(-\infty) \cap V_1$ is closed in V_1 and polar. Therefore the set $\{z_1 \in V_1 : \tilde{\rho}^*(z_1) > -\infty\}$ is connected. But $\tilde{\rho}^*$ is locally constant there, so it is constant on $V_1 \setminus (\tilde{\rho}^*)^{-1}(-\infty)$. The subharmonicity of $\tilde{\rho}^*$ implies that $\tilde{\rho}^*$ is constant on V_1.

Now we show the property $(**)$.

Suppose that there is a $\tilde{z}_1 \in V_1$ such that $(\tilde{\rho})^*(\tilde{z}_1) > -\infty$ and $\tilde{\rho}^*$ is not constant on any neighborhood of $\tilde{z}_1$. Without loss of generality we may assume that $\tilde{\rho}^*(\tilde{z}_1) < \rho(\tilde{z}_1)$. Let $-\infty < M < \tilde{\rho}^*(\tilde{z}_1)$ and $M \leq \rho$ on D. The function $\psi := \max\{M, \frac{\rho+\tilde{\rho}^*}{2}\}$ defined on D is subharmonic, bounded from below, satisfies $\psi \leq \rho$, $\tilde{\rho}^*(\tilde{z}_1) < \psi(\tilde{z}_1) < \rho(\tilde{z}_1)$, and is not constant on any neighborhood of $\tilde{z}_1$.

Let $\tilde{\psi}$ denote the function defined for ψ in the same way as the function $\tilde{\rho}$ was defined for ρ. Note that $\tilde{\psi}^* \leq \tilde{\rho}^*$ on D, so $\tilde{\psi}^*(\tilde{z}_1) \leq \tilde{\rho}^*(\tilde{z}_1) < \psi(\tilde{z}_1)$. However, it follows from Lemma 4 applied to ψ that $\psi(\tilde{z}_1) = \tilde{\psi}^*(\tilde{z}_1)$, which is a contradiction.

Consequently, $(*)$ is satisfied, so $\tilde{\rho}^* \equiv \tilde{C} \in [-\infty, \infty)$ on V_1.

We want to show that $\tilde{\rho}^* \equiv \tilde{C} = C \equiv \rho$ on V_1. If $D \setminus V_1$ is polar, then ρ is constant on D, which is a contradiction. Therefore $D \setminus V_1$ is not polar, so $\partial V_1 \cap D$ is not polar either. Hence there is a point $\tilde{z}_1 \in \partial V_1 \cap D$ such that V_1 is not thin at $\tilde{z}_1$, so $\tilde{\rho}^*(\tilde{z}_1) = \tilde{C}$. Moreover, ρ is not constant on any neighborhood of $\tilde{z}_1$, so in view of Lemma 4, $\tilde{\rho}^*(\tilde{z}_1) = \rho(\tilde{z}_1) = C$, so $C = \tilde{C}$. □

As a consequence of Lemmas 4 and 5 we get the following result.

Corollary 6. *Let ρ be bounded from below and not constant. Then $\rho = \tilde{\rho}^*$ on D.*

Let us define $\hat{D}$ to be the set of points from D and those points $\hat{z}_1 \in \mathcal{S}$ for which $\limsup_{D\ni z_1\to\hat{z}_1} \rho(z_1) < \infty$. Note that $\hat{D}$ is a domain with $D \subset \hat{D} \subset D \cup \mathcal{S}$. We may also define the function

$$\hat{\rho}(\tilde{z}_1) := \begin{cases} \rho(z_1), & \text{if } z_1 \in D, \\ \limsup_{D\ni z_1\to\hat{z}_1} \rho(z_1), & \text{if } z_1 \in \tilde{D} \setminus D. \end{cases}$$

Note that $\hat{\rho}$ is subharmonic on $\hat{D}$.

Proof of Theorem 3. Let $\rho \geq M$ on D.

If ρ is constant, $\mathcal{S} = \emptyset$ and $\mathbb{C} \setminus D$ is not polar, then the domain D is an L_h^2-domain of holomorphy (see Theorem 2) and $G_{D,\rho} = D \times e^{-M_1}\mathbb{D}$ for some $M_1 \in \mathbb{R}$ ($M_1 \equiv \rho$). Consequently, $G_{D,\rho}$ is an L_h^2-domain of holomorphy.

First we show the sufficiency of the condition.

Assume that $\limsup_{D\ni z\to z_0} \rho(z) = \infty$ for any $z_0 \in \mathcal{S}$ and that ρ is not constant. (In the case $\mathcal{S} \neq \emptyset$ the second condition follows directly from the first one.) Suppose that $G_{D,\rho}$ is not an L_h^2-domain of holomorphy. Then there are discs P_j, Q_j, $j = 1, 2$, such that $P_j \subset\subset Q_j$, $j = 1, 2$, $P := P_1 \times P_2 \subset G_{D,\rho}$, $\partial G_{D,\rho} \cap \partial P \neq \emptyset$, and for any $f \in L_h^2(G_{D,\rho})$ there is a $g \in \mathcal{O}(Q_1 \times Q_2)$ such that $f = g$ on $P_1 \times P_2$.

Let us consider three cases.

Case I. $\partial P \cap \partial G_{D,\rho} \subset \mathcal{S} \times \mathbb{C}$. Then our assumption implies that there is a point $(z_1^0, z_2^0) \in \partial P \cap \partial G_{D,\rho}$ such that $z_1^0 \in \mathcal{S}$, $z_2^0 \neq 0$ and $z_2^0 \in P_2$. Consider the set

$$U := \{z_1 \in Q_1 \cap D : (z_1, z_2^0) \in G_{D,\rho}\}.$$

Note that $U \neq \emptyset$ (because $P_1 \subset U$). Note also that $Q_1 \setminus U$ is not polar. In fact, the assumption on the boundary behaviour of ρ implies that there is a point $\tilde{z}_1 \in Q_1 \cap D$ such that $(\tilde{z}_1, z_2^0) \notin G_{D,\rho}$, so $\rho(\tilde{z}_1) \geq -\log|z_2^0|$. The existence of only a polar set of such points would, however, lead to a contradiction with the mean value property of subharmonic functions. Therefore there is a function $f \in L_h^2(\tilde{U})$, where $\tilde{U} := \{z_1 \in D : (z_1, z_2^0) \in G_{D,\rho}\}$, which does not have a holomorphic continuation on Q_1 (see [Con]). There is a function $F \in L_h^2(G_{D,\rho})$ such that $F(z_1, z_2^0) = f(z_1)$, $z_1 \in \tilde{U}$ (apply [Ohs] with $\Psi(z_1, z_2) := 2g_{e^{-M}\mathbb{D}}(z_2^0, z_2)$). But such a function F has a holomorphic continuation on $Q_1 \times Q_2$, from which we conclude the existence of a holomorphic continuation of f on Q_1, which is a contradiction.

Case II. $\partial P \cap \partial G_{D,\rho} \cap ((\partial D \setminus \mathcal{S}) \times \mathbb{C}) \neq \emptyset$. The proof in this case is similar to that in Case I. There is a point $(z_1^0, z_2^0) \in \partial P \cap \partial G_{D,\rho}$ such that $z_1^0 \in \partial D \setminus \mathcal{S}$ and $z_2^0 \in P_2$ (but not necessarily $z_2^0 \neq 0$). Consider the set

$$U := \{z_1 \in Q_1 \cap D : (z_1, z_2^0) \in G_{D,\rho}\}.$$

Note that $U \neq \emptyset$ (because $P_1 \subset U$). Note also that $Q_1 \setminus U$ is not polar. In fact, this follows directly from the fact that $z_1^0 \in \partial D \setminus \mathcal{S}$, the definition of $\mathcal{S}$ and the inclusion $Q_1 \setminus D \subset Q_1 \setminus U$. Therefore there is a function $f \in L_h^2(\tilde{U})$, where $\tilde{U} := \{z_1 \in D : (z_1, z_2^0) \in G_{D,\rho}\}$, which does not have a holomorphic continuation on Q_1 (see Theorem 2). But then the function F defined by the formula $F(z_1, z_2) := f(z_1)$, $(z_1, z_2) \in G_{D,\rho}$ is from the class $L_h^2(G_{D,\rho})$. But such a function F has a holomorphic continuation on $Q_1 \times Q_2$, from which we conclude the existence of a holomorphic continuation of f on Q_1, which is a contradiction.

Case III. $\partial P \cap \partial G_{D,\rho} \cap (D \times \mathbb{C}) \neq \emptyset$. Set $M_1 := \sup\{|z_2| : z_2 \in P_2\}$, $M_2 := \sup\{|z_2| : z_2 \in Q_2\}$. Then $0 < M_1 < M_2$. From our assumption we conclude the existence of a point $\tilde{z}_1 \in \bar{P}_1 \cap D$ such that $\rho(\tilde{z}_1) \geq -\log M_1$. On the other hand, the extension property implies that for any $f \in L_h^2(G_{D,\rho})$ and for any $z_1 \in P_1$ the inequality $e^{-\rho_f(z_1)} \geq M_2$ holds, so $\tilde{\rho}^*(z_1) \leq -\log M_2$, $z_1 \in P_1$. This implies the inequality $\tilde{\rho}^*(\tilde{z}_1) \leq -\log M_2 < -\log M_1$, which contradicts the equality $\rho(\tilde{z}_1) = \tilde{\rho}^*(\tilde{z}_1)$ that holds by Corollary 6.

Now we prove the necessity of the condition.

Recall that $\hat{D} \setminus D$ is a polar set. Therefore $(\hat{D} \setminus D) \cap G_{\hat{D},\hat{\rho}}$ is pluripolar. Since L_h^2-holomorphic functions extend through pluripolar sets, it is easy to see that $L_h^2(G_{D,\rho}) = L_h^2(\,G_{\hat{D},\hat{\rho}})_{|G_{D,\rho}}$, which gives us the necessity of the condition. □

The following description of L_h^2-holomorphic hulls of domains $G_{D,\rho}$ follows from Theorem 3 and its proof.

Corollary 7. *Assume that ρ is bounded from below. Assume that $\mathbb{C} \setminus D$ is not polar or ρ is not constant. Then the L_h^2-holomorphic hull of $G_{D,\rho}$ equals $G_{\hat{D},\hat{\rho}}$. If $\mathbb{C} \setminus D$ is polar and ρ is constant, then the L_h^2-holomorphic hull of $G_{D,\rho}$ equals $\mathbb{C}^2$.*

Remark. We are far from a full understanding of the structure of L_h^2-domains of holomorphy. For instance, the natural question whether we may remove the assumption of lower boundedness of the function ρ in Theorem 3 remains open. On the other hand, the methods used in the paper may be easily transferred to Hartogs domains with higher dimensional bases. However, because of the lack of a full description of L_h^2-domains of holomorphy in $\mathbb{C}^n$, $n \geq 2$, the results obtained in this case would be much more incomplete. We believe that to obtain a complete characterization of L_h^2-domains of holomorphy in the class of Hartogs domains in the two-dimensional case (as well as in higher dimensional cases of Hartogs domains or even in the general case of unbounded domains), a completely different approach is needed.

Some other problems remain also open. For instance: Is it true that if D is a pseudoconvex domain and locally an L_h^2-domain of holomorphy (which means that the geometric condition from Theorem 1 is satisfied) and $L^2_h(D) \neq \{0\}$, then D must be an L^2_h-domain of holomorphy? Another natural problem is to find a description of L^p_h-domains of holomorphy.

We conclude this paper by presenting a sufficient condition for a pseudoconvex domain D to have an infinite-dimensional Bergman space $L^2_h(D)$. This gives a partial answer to the following question: Is there a pseudoconvex domain having a finite-dimensional but non-trivial Bergman space? A non-pseudoconvex example of that type was given in [Wie].

Proposition 8. *Let $D \subset \mathbb{C}^n$, $D \neq \mathbb{C}^n$, be an L^2_h-domain of holomorphy and let $\{\varphi_j\}_{j\in J}$ be a complete orthonormal system in $L^2_h(D)$. Assume that there is an open set U such that $U \cap D \neq \emptyset$, $U \not\subset D$, and for any $j \in J$ the function φ_j has an analytic continuation onto U. Then* $\dim L^2_h(G_{D,\rho}) = \infty$. *In particular, any L^2_h-domain of holomorphy which is balanced, a Hartogs domain or a Laurent-Hartogs domain different from $\mathbb{C}^n$, has infinite-dimensional Bergman space.*

Proof. Suppose the contrary. Then J is finite and $J \neq \emptyset$. Since D is an L^2_h-domain of holomorphy, there is a function $f \in L^2_h(D)$ which does not have an analytic continuation onto U. But $f = \sum_{j\in J} \lambda_j \varphi_j$, where $\lambda_j \in \mathbb{C}$. Since J is finite, f has an analytic continuation onto U, which is a contradiction. □

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