APPENDIX TO MY PAPER "ON UNIQUE FACTORIZATION IN ALGEBRAIC FUNCTION FIELDS"

BY

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1. Introduction. The proof in §4 of the paper of the title [1] is much too sketchy and open to some quite alarming misinterpretations. For example, Dr. E. Kunz has pointed out to me that it would appear to "prove" that $\mathbf{Q}^2$ viewed as a vector space over $\mathbf{Q}$ is a finite union of lines through the origin. In fact, in order to justify the assertion (12) in [1] one must use the relation between the linear topology and the topology defined by the valuation $\| \alpha \|_{\mathfrak{P}_j}$. In this appendix, we supply the details of the argument.

2. Proof of the finite intersection property. We begin by amplifying the remark on the ordering of the linear spaces $L = \mathfrak{L}(\mathfrak{b}, S) + R$. Observe that one need consider only a finite number of possibilities for deg $(\mathfrak{b})$. Now order the L by considering $\sum_{j=1}^{s} (\nu_{\mathfrak{P}_j}(\mathfrak{b}))^2$. From the resulting sequence $\Lambda = (L_i)$, we select subsequences $\Lambda^{(j)} = (L_i^{(j)})$, $1 \leq j \leq s$, which together give Λ and such that in $\Lambda^{(j)}$, $\nu_{\mathfrak{P}_j}(\mathfrak{b}_i)$ is strictly increasing.

Now in (12) of [1] the argument ought to run as follows. If (11) holds for every n, then either (12) holds (in which case we have finished) or for every sequence (λ_i) of coset representatives the finite intersection property fails for some n. It is this latter possibility which was dismissed without comment in [1]. Suppose that we are in this case and assume for the moment that one can construct a sequence (λ_i) of coset representatives which is a finite union of convergent subsequences (in the sense of the topology defined by $\| \quad \|_{\mathfrak{P}_j}$). Let $\hat{\lambda} \in \hat{K}$ denote the limit of one of them. Then for $\lambda \in K$ and arbitrarily close to $\hat{\lambda}$ at $\mathfrak{P}_1, \cdots, \mathfrak{P}_s$ (approximation theorem, see Chevalley, [2, Chapter I, §6]) either $\lambda \notin \Lambda$ (in which case we already have a contradiction to (3) of [1]) or $\lambda \in$ some L. But then, since $\hat{E}$ is an ultrametric space, all the λ_i sufficiently close to λ are in L. By considering the other subsequences, we obtain a finite covering as in (8) of [1].

It remains to justify our assumption on the existence of such subsequences in the case in question. Suppose for simplicity of notation, that $\lambda_1^{(1)} \notin L_1^{(1)}$ but $\lambda_1^{(1)} \in L_2^{(1)}$. (If $\lambda_1^{(1)}$ is not in any member of $\Lambda^{(1)}$, then we can ignore $\Lambda^{(1)}$.) We construct $\lambda_2^{(1)} \notin L_1^{(1)} \cup L_2^{(1)}$ and such that

$$\nu_{\mathfrak{P}_1}(\lambda_2^{(1)}) > \nu_{\mathfrak{P}_1}(\lambda_1^{(1)}) \quad \text{and} \quad \nu_{\mathfrak{P}_j}(\lambda_2^{(1)} - \lambda_1^{(1)})$$

is arbitrarily large for $2 \leq j \leq s$. From now on, we omit the superfix 1.

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Since $\lambda_1 \in L_2$, we know that there exists $\xi_1 \in R$ such that

$$\nu_{\mathfrak{P}_1}(\lambda_1 - \xi_1) \geqq \nu_{\mathfrak{P}_1}(\mathfrak{b}_2) > \nu_{\mathfrak{P}_1}(\mathfrak{b}_1)$$

and

$$\nu_{\mathfrak{P}_j}(\lambda_1 - \xi_1) \geqq \nu_{\mathfrak{P}_j}(\mathfrak{b}_2) \qquad (2 \leq j \leq s).$$

Choose $\hat{\lambda}_1 \in E$ such that $\nu_{\mathfrak{P}_1}(\lambda_1 - \hat{\lambda}_1) = \nu_{\mathfrak{P}_1}(\mathfrak{b}_2) - 1$ and $\nu_{\mathfrak{P}_j}(\lambda_1 - \hat{\lambda}_1)$ $(2 \leq j \leq s)$ is arbitrarily large. Then choose $\lambda_2 \in K$ such that $\nu_{\mathfrak{P}_j}(\lambda_2 - \hat{\lambda}_1)$ is arbitrarily large for $1 \leq j \leq s$ (approximation theorem again). It is easy to verify that $\lambda_2 \notin L_1 \cup L_2$ and that the subsequence constructed by repeating the argument converges. One gets the other subsequences by looking at the places $\mathfrak{P}_2, \cdots, \mathfrak{P}_s$.

This completes our account of the elided details of §4 and of the proof of the theorem of [1].

References

1. J. V. Armitage, *On unique factorization in algebraic function fields*, Illinois J. Math., vol. 11 (1967), pp. 280–283.

2. C. Chevalley, *Algebraic functions of one variable*, Amer. Math. Soc. Mathematical Surveys, no. 6, 1951.

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