DECOMPOSITIONS OF FINITELY ADDITIVE VECTOR MEASURES GENERATED BY BANDS OF FINITELY ADDITIVE SCALAR MEASURES

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Let V be an algebra of subsets of a space X and let $P(X)$ denote the sigma algebra of all subsets of the space X. For any Banach space Y, let $a(V, Y)$ and $ca(V, Y)$ denote respectively the space of finitely additive and countably additive, Y-valued functions on the algebra V. Elements of the space $a(V, Y)$ are referred to as vector charges and elements of the space $ca(V, Y)$ are referred to as vector measures.

For each vector charge $\mu \in a(V, Y)$, the semivariation

$$p(\cdot, \mu)\colon P(X) \to [0, \infty]$$

is defined on a set $E \in P(X)$ by the relation

$$p(E, \mu) = \sup\,(|\mu(A)|\colon A \in V, A \subseteq E).$$

The semivariation is increasing on the sigma algebra $P(X)$ and subadditive on the algebra V. Denote by $ab(V, Y)$ the space of charges $\mu \in a(V, Y)$ for which $p(X, \mu) < \infty$ and set $cab(V, Y) = ab(V, Y) \cap ca(V, Y)$. Elements of the spaces $ab(V, Y)$ and $cab(V, Y)$ are called respectively bounded charges and measures. A vector charge $\mu \in a(V, Y)$ is said to be strongly bounded (abbreviated s-bounded) on the algebra V if $\lim_n \mu(A_n) = 0$ for each disjoint sequence $A_n \in V, n \in N$. Denote by $R(V, Y)$ the space of s-bounded vector charges on the algebra V. The space $R(V, Y)$ was introduced by Rickart [17] who gave a general Lebesgue type decomposition of the space $R(V, Y)$ for V a σ-algebra. Rickart [17], also has shown the general inclusion $R(V, Y) \subset ab(V, Y)$. In [3] it is noted that these spaces coincide for the class of Banach spaces introduced by Gould [8] which can be seen to coincide with the class of Banach spaces not containing the space c_0 of sequences of scalars converging to zero.

The space $ab(V, Y)$ is a Banach space with the norm $\|\cdot\| = p(X, \cdot)$ and the subspaces $R(V, Y)$ and $cab(V, Y)$ are norm closed. In addition, the space $ab(V, R)$ is a Banach lattice with the total variation norm and the order induced by the cone

$$ab^{+}(V, R) = \{\mu \in ab(V, R)\colon \mu(A) \geq 0 \quad \text{for all sets } A \in V\}.$$

Peressini [16, pp. 41–42] gives a detailed discussion of the Banach lattice structure of the space $ab(V, R)$.

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Let R^+ denote the nonnegative reals and denote by $C(V)$ the space of all subadditive and increasing functions $p: V \to R^+$ which vanish at the empty set. Elements of the space $C(V)$ are referred to as contents on the algebra V. A charge $\mu \in ab(V, R)$ is said to be p-continuous for $p \in C(V)$ if it is a continuous function on the algebra V in the topology generated by the semimetric $\rho(A, B) = p(A \div B)$ for $A, B \in V$, where $\div$ denotes the symmetric difference operation in the algebra V. For a content $p \in C(V)$, denote by $ab(p, V, R)$ the space of all p-continuous elements in the space $ab(V, R)$. Two contents $p_1, p_2 \in C(V)$ are said to be equivalent if p_i is p_j continuous for $i, j = 1, 2$. For a charge $v \in ab^+(V, R)$, the space $ab(v, V, R)$ is topologically closed and forms a band. Bochner and Phillips [1] have shown that the space $ab(v, V, R)$ is the band generated by the charge $v \in ab^+(V, R)$.

In this paper, the Lebesgue decomposition theorem for the space of Rickart vector charges on a σ-algebra, Rickart [17] is used to generate a general decomposition in the space of s-bounded vector charges for each band in the Banach lattice of scalar charges. The general decomposition theorem contains the Lebesgue type decompositions given by Darst [5], Rickart [17], Nakamura and Sunouski [12]. The decomposition theorem motivates a vector extension of the notion of pure finite additivity. It is shown that the spaces of countably additive and purely finitely additive vector charges are complementary when restricted to the space of s-bounded vector charges. Extensions of the type developed in this paper have been given by Traynor [18], [19], Huff [10], Ohba [14], [15] and Uhl [21].

1. Band decompositions of the space of strongly bounded vector charges

Let V be an algebra of subsets of a space X and let Y be a Banach space. Let $C_\infty(V)$ denote the space of all subadditive, nonnegative extended functions on the algebra V which vanish at the empty set. Elements of this space are referred to as extended contents on the algebra V. Rickart [17, Theorem 4.5, pp. 664–665] established the following general decomposition.

THEOREM 1. *Let W be a σ-algebra of subsets of a space X and let $p \in C_\infty(W)$ be a sigma subadditive, extended content. Then for each vector measure $\mu \in cab(W, Y)$ there exist unique vector measures $\mu_1, \mu_2 \in cab(W, Y)$ and a set $N \in W$ such that:*

(1) $\mu = \mu_1 + \mu_2$.
(2) *The vector measure μ_1 is p-continuous.*
(3) *For each set $A \in W$, $\mu_2(A \cap N) = \mu_2(A)$ and $p(N) = 0$.*

Application of the general Rickart decomposition requires the following proposition. It may be established by applying the general properties of the band projection operators given in Peressini [16, pp. 35–43] and Nakano [13, pp. 18–28].

PROPOSITION 1. *Let V be an algebra of subsets of a space X, let $B \subset ab(V, R)$ be a band and let $w \in ab^+(V, R)$ be arbitrary. Then*

$$P_B(ab^+(w, V, R)) = ab^+(P_B w, V, R)$$

where P_B denotes the projection operator generated by the band B (Peressini [16, pp. 35–43]).

Before applying Proposition 1 to Rickart's theorem, it is appropriate to review the construction and properties of the Brooks control measure for s-bounded vector measures. Let $\mu \in R(V, Y)$ be arbitrary and note that the s-boundedness insures that for each number $\varepsilon > 0$, there exists a finite set $y_1', y_2', \ldots, y_n' \in Y'$, all of norm one, and a number $\delta > 0$ such that $|y_k' \circ \mu|(A) < \delta$ for $k = 1, 2, \ldots, n$ and $A \in V$ yields $|\mu(A)| < \varepsilon$ (see [20] or [2]). Applying this property successively for the sequence $\varepsilon_n = 1/n$, $n \in N$, one obtains a family $y_{nj}' \in Y'$, $j = 1, 2, \ldots, k(n)$, $n \in N$ of functionals of norm one, and a sequence of numbers δ_n, $n \in N$ such that $|y_n' \circ \mu|(A) < \delta_n$ for $j = 1, 2, \ldots, k(n)$ and $A \in V$ yields $|\mu(A)| < 1/n$. Set

$$v(\cdot) = \sum_n \frac{1}{2^n} \sum_{j=1}^{k(n)} \frac{|y_{nj}' \circ \mu|(\cdot)}{k(n)}$$

and note that $v(\cdot) \in ab(V, R)$ and if $v(A) < \delta_n/2^n \cdot k(n)$ for some set $A \in V$, then $|y_{nj}' \circ \mu|(A) < \delta_n$ for $j = 1, 2, \ldots, k(n)$ and consequently $|\mu(A)| < 1/n$. Thus, the vector measure μ is v-continuous on the algebra V. Moreover, since the charge $v(\cdot)$ is monotone, we have that $p(\cdot, \mu)$ is v-continuous on the algebra V.

THEOREM 2. *Let W be a sigma algebra of subsets of the space X and let $B \subset cab(W, R)$ be a band. Then for each vector measure $\mu \in cab(W, Y)$ there exists a unique decomposition $\mu = \mu_1 + \mu_2$, $u_i \in cab(W, Y)$, $i = 1, 2$, such that $y' \circ \mu_1 \in B$ and $y' \circ \mu_2 \in B^{\perp}$ for each functional $y' \in Y'$. Moreover, if a measure $w \in cab^+(W, R)$ is topologically equivalent to the semivariation $p(\cdot, \mu)$ on the sigma algebra W, then the vector measure μ_1 is $P_B w$-continuous and the vector measure μ_2 is $P_{B\perp} w$-continuous.*

Proof. Let $w \in cab^+(W, R)$ generate a uniformity equivalent to that of the semivariation $p(\cdot, \mu)$ (see [2, Theorems 1, 2, 3]) and let $v = P_B w$. By the general Rickart decomposition, there exist vector measures $\mu_1, \mu_2 \in cab(W, Y)$ and a set $N \in W$ such that $\mu = \mu_1 + \mu_2$, μ_1 is v-continuous, $\mu_2(\Delta) = \mu_2(\Delta \cap N)$ for each set $\Delta \in W$ and $v(N) = 0$.

Since the vector measure μ_1 is v-continuous, each signed measure $y' \circ \mu_1$ for $y' \in Y'$, belongs to the band generated by the measure v (denoted $[v]$). Moreover, $[v] \subset B$ so that $y' \circ \mu_1 \in B$ for each functional $y' \in Y'$. From the characterization of the complementary band $[v]^{\perp}$ as those measures singular with respect to v [1, pp. 319–320], and the relation $y' \circ \mu_2(\Delta) = y' \circ \mu_2(\Delta \cap N)$ for $y' \in Y'$ and $\Delta \in W$ we conclude that $y' \circ \mu_2 \in [v]^{\perp}$ for each functional $y' \in Y'$.

The existence of the decomposition is complete if it can be shown that $y' \circ \mu_2 \in B^\perp$ for each $y' \in Y'$. Indeed, let $u = P_{B\perp}w$ and from the Riesz theorem [16, pp. 39–40], $u + v = w \sim p(\cdot, \mu)$. Applying the Rickart decomposition to the charge $\mu_2 \in cab(W, Y)$ relative to the measure $u \in cab^+(W, R)$, there exist vector measures $\mu_{2,1}, \mu_{2,2} \in cab(W, Y)$ and a set $N' \in W$ with $\mu_2 = \mu_{2,1} + \mu_{2,2}$, the measure $\mu_{2,1}$ is u-continuous, $u(N') = 0$ and for each set $\Delta \in W$, $\mu_{2,2}(\Delta) = \mu_{2,2}(\Delta \cap N')$. Since the measure $\mu_{2,1}$ is u-continuous and $u(N') = 0$, for each set $\Delta \in W$ we have $\mu_{2,1}(\Delta \cap N') = \mu_{2,1}(\Delta \cap N \cap N') = 0$. From the representation $\mu_{2,2} = \mu_2 - \mu_{2,1}$ and the fact that μ_2 is supported by N and $\mu_{2,2}$ is supported by N', we obtain

$$\mu_{2,2}(\Delta) = \mu_{2,2}(\Delta \cap N \cap N')$$

for all $\Delta \in W$. Since the set $N \cap N'$ is a w-null set, the representation $\mu = \mu_1 + \mu_{2,1} + \mu_{2,2}$ yields $\mu_{2,2}(\Delta \cap N \cap N') = 0$ for all $\Delta \in W$. Consequently, $\mu_{2,2} \equiv 0$ and $\mu = \mu_1 + \mu_2$.

To see that the decomposition is unique, assume that there are two decompositions $\mu = \mu_1^i + \mu_2^i$ with $\mu_j^i \in cab(W, Y)$, for $i, j = 1, 2$, and $y' \circ \mu_1^i \in B$, $y' \circ \mu_2^i \in B^\perp$ for $i = 1, 2$, and all functionals $y' \in Y'$. Then for each functional $y' \in Y'$, $y' \circ (\mu_1^1 - \mu_1^2) = y' \circ (\mu_2^2 - \mu_2^1)$ and since the bands B and $B^\perp$ are complementary, we conclude $\mu_1^1 = \mu_1^2$ and $\mu_2^1 = \mu_2^2$.

The last part of the theorem follows from the fact that the projection operators P_B and $P_{B\perp}$ preserve absolute continuity.

Using the Stone representation of an algebra of sets as the algebra of open/closed subsets of a totally disconnected compact Hausdorff space, (see Dunford and Schwartz [7, pp. 38–44]) and the observation (which follows from the existence of the Brooks control measure) that a vector charge is s-bounded on an algebra V if and only if its image in the Stone space has a countably additive extension to the generated σ-algebra, it is possible to reformulate Theorem 2 for algebras of sets.

THEOREM 3. *Let V be an algebra of subsets of a space X and let $B \subset ab(V, R)$ be a band. Then for each vector charge $\mu \in R(V, Y)$, there exists a unique decomposition $\mu = \mu_1 + \mu_2$, with $\mu_i \in R(V, Y)$, for $i = 1, 2$, and for each functional $y' \in Y', y' \circ \mu_1 \in B$ and $y' \circ \mu_2 \in B^\perp$. Moreover, if $w \in ab^+(V, R)$ is topologically equivalent to the semivariation $p(\cdot, \mu)$ on the algebra V, then the charge μ_1 is P_Bw-continuous and the charge μ_2 is $P_{B\perp}w$-continuous on the algebra V.*

Proof. Let $\bar{V}$ denote the Stone representation of the algebra V as the algebra of open/closed subsets of a totally disconnected, compact Hausdorff space $\bar{X}$ (see Dunford-Schwartz [7, pp. 38–44]) and let $\bar{\bar{V}}$ denote the sigma algebra generated by the algebra $\bar{V}$. As noted above, the charge $\mu \in R(V, Y)$ generates a vector measure $\bar{\mu} \in cab(\bar{\bar{V}}, Y)$. The assertion of the theorem follows from Theorem 2 applied to the charge $\bar{\mu} \in cab(\bar{\bar{V}}, Y)$ after noting that the image $\bar{\bar{B}} \subset cab(\bar{\bar{V}}, R)$ of the band B is a band. This follows from the fact that the

Hahn extension is a vector lattice, topological isomorphism between the spaces $cab(\overline{V}, R)$ and $cab(\overline{\overline{V}}, R)$.

For the band of scalar charges continuous with respect to a nonnegative charge, Theorem 3 is the standard Lebesgue decomposition. For the σ-algebra generated by the closed subsets of a locally compact, Hausdorff space, Theorem 3 gives the decomposition into regular and antiregular charges as discussed in Ohba [14], [15]. The decomposition corresponding to the bands of countably additive and purely finitely additive scalar charges are discussed in Section 2.

For each algebra of sets V, each Banach space Y, and each band $B \subset ab(V, R)$, one may introduce a projection operator on the space $R(V, Y)$ as follows. For each charge $\mu \in R(Y, Y)$, set $\mu^* = V_B\mu$ if and only if $y' \circ \mu^* = P_B(y' \circ \mu)$ for each functional $y' \in Y'$. The corresponding operator for the complementary band $B^\perp$ is defined analogously. Theorem 3 insures that the operator V_B is well defined and has the properties: $V_B^2 = V_B$, $p(\cdot, V_B\mu) \leq p(\cdot, \mu)$, $R(V, Y) = B(Y) \oplus B^\perp(Y)$, where $B(Y) = V_B(R(V, Y))$ and $B^\perp(Y) = V_{B\perp}(R(V, Y))$.

2. Purely finitely additive vector charges

Let V be an algebra of subsets of the space X and let Y be a Banach space. A vector charge $\mu \in ab(V, Y)$ is said to be purely finitely additive if the only $p(\cdot, \mu)$-continuous, countably additive scalar charge is the identically zero charge. That is, a charge $\mu \in ab(V, Y)$ is purely finitely additive if and only if

$$cab(p(\cdot, \mu), V, R) = (0).$$

Denote by $pfa(V, Y)$ the space of purely finitely additive, bounded vector charges on the algebra V. It is clear that the definition of pure finite additivity extends the scalar definition and as in the scalar case, $cab(V, Y) \cap pfa(V, Y) = (0)$. Moreover, in the space of s-bounded vector charges, the condition of countable additivity (pure finite additivity) is equivalent to weak countable additivity (weak pure finite additivity).

The equivalence of weak and strong countable additivity is an immediate consequence of the Orlicz-Pettis theorem. To see that weak pure finite additivity implies pure finite additivity, consider a vector charge $\mu \in R(V, Y)$ for which $y' \circ \mu \in pfa(V, R)$ for each functional $y' \in Y'$. From the strong boundedness of the charge μ, the set of scalar charges

$$\{y' \circ \mu(\cdot) \colon y' \in Y', |y'| = 1\}$$

is equivalent to a scalar charge $v \in ab^+(V, R)$ (the Brooks' control measure). Moreover, from the construction there exists a countable family $y_n' \in Y'$, $|y_n'| = 1$, for $n \in N$, and a summable sequence $c_n \in R$, $n \in N$ of positive numbers such that $v(\cdot) = \sum_n c_n|y_n' \circ \mu|(\cdot)$. Since the sequence of partial sums is an increasing sequence of charges (in the vector lattice order in the space $ab(V, R)$) we have $v(\cdot) = (\sup_n w_n)(\cdot)$ where

$$w_n(\cdot) = \sum_{k=1}^{n} c_n|y_n' \circ \mu|(\cdot).$$

Since the space $pfa(V, R)$ is a band in the vector lattice $ab(V, R)$, and $w_n \in pfa(V, R)$ for all $n \in N$, we conclude $v(\cdot) \in pfa(V, R)$. Since the charge $v \in ab^+(V, R)$ is topologically equivalent to the semivariation $p(\cdot, \mu)$, we have $\mu \in pfa(V, Y)$.

The properties of the equivalent content noted above may be formalized in the following proposition.

PROPOSITION 2. *Let V be an algebra of subsets of the space X. The following are equivalent for a charge $\mu \in R(V, Y)$.*

(1) *The vector charge μ is purely finitely additive.*
(2) *If a scalar charge $w \in ab(V, R)$ is $p(\cdot, \mu)$-continuous, then $w \in pfa(V, R)$.*

Theorem 3 may be used to show that in the space of s-bounded vector charges, the spaces of countably additive and purely finitely additive vector charges are complementary.

THEOREM 4. *Let V be an algebra of subsets of a space X and let Y be a Banach space. Then*

$$R(V, Y) = caR(V, Y) \cap pfaR(V, Y)$$

where

$$caR(V, Y) = R(V, Y) \cap ca(V, Y), \quad \textit{and} \quad pfaR(V, Y) = R(V, Y) \cap pfa(V, Y).$$

Proof. Theorem 3, applied to the band $B = cab(V, R)$, asserts that each vector charge $\mu \in R(V, Y)$ has a unique decomposition, $\mu = \mu_1 + \mu_2$, with $\mu_1, \mu_2 \in R(V, Y)$ and for each functional $y' \in Y'$, $|y'| = 1$, $y' \circ \mu_1 \in cab(V, R)$, $y' \circ \mu_2 \in pfa(V, R)$. The observations made prior to stating the theorem yield $\mu_1 \in cab(V, Y)$ and $\mu_2 \in pfa(V, Y)$. Since these spaces have only the null element in common, the proof is complete.

Remark. Diestel [6] has given a similar theorem for vector measures taking values in a vector lattice.

Heider [9] and Lloyd [10] have developed characterizations of the bands $cab(V, R)$ and $pfa(V, R)$ in terms of the corresponding Stone representations. The existence of the finitely additive control measure for elements of the space of s-bounded vector charges permit the scalar characterizations to be used to establish the corresponding vector formulations.

Let V be an algebra of subsets of a space X and $\bar{V}$ denote the Stone representation of the algebra V as the algebra of open/closed subsets of a totally disconnected, compact Hausdorff space $\bar{X}$. As noted above, each vector charge $\mu \in R(V, Y)$ generates a vector charge $\bar{\mu} \in cab(\bar{\bar{V}}, Y)$, where $\bar{\bar{V}}$ denotes the σ-algebra generated by the algebra $\bar{V}$. It is not difficult to see that the σ-algebra $\bar{\bar{V}}$ is precisely the σ-algebra of Baire sets in the space $\bar{X}$. Heider [9] has shown that a scalar charge $\lambda \in ab(V, R)$ is countably additive if and only if the corresponding Baire measure $\bar{\lambda} \in cab(\bar{\bar{V}}, R)$ vanishes on the class of Baire sets of the first category, and the charge $\lambda \in ab(V, R)$ is purely finitely additive if and only if the Baire measure $\bar{\bar{\lambda}} \in cab(\bar{\bar{V}}, R)$ is supported by a Baire set of the first category. Thus, a vector charge $\mu \in R(V, Y)$ is countably additive if and only if its image in the Stone space $\bar{\bar{\mu}} \in cab(\bar{\bar{V}}, Y)$ vanishes on Baire subsets of the first category. To see that an analogous characterization holds for the subspace $pfa(V, Y)$, consider a vector charge $\mu \in pfaR(V, Y)$ and let $v \in ab^+(V, R)$ be chosen as above to be equivalent to the semivariation $p(\cdot, \mu)$. Then $v \in pfa^+(V, R)$ and thus there exists a Baire set of the first category which supports the finite Baire measure $\bar{\bar{v}} \in cab^+(\bar{\bar{V}}, R)$. Since the vector measure $\bar{\bar{\mu}} \in cab(\bar{\bar{V}}, Y)$ is $\bar{\bar{v}}$-continuous, it too is supported by a Baire set of the first category. Conversely, assume that a charge $\mu \in R(V, Y)$ has a property that its Stone extension $\bar{\bar{\mu}} \in cab(\bar{\bar{V}}, Y)$ is supported by a Baire set of the first category. Then the semivariation $p_\sigma(\cdot, \bar{\bar{\mu}})$, defined by sets in the σ-algebra $\bar{\bar{V}}$, is zero outside a Baire set of the first category. Moreover, the semivariations $p(\cdot, \bar{\mu})$ and $p_\sigma(\cdot, \bar{\bar{\mu}})$ generate equivalent semimetrics on the algebra $\bar{V}$ and the algebra $\bar{V}$ is dense in the σ-algebra $\bar{\bar{V}}$ in the semimetric generated by the semivariation $p_\sigma(\cdot, \bar{\bar{\mu}})$. Now if $w \in cab(V, R)$ is $p(\cdot, \mu)$-continuous, the Stone image $\bar{w} \in cab(\bar{V}, R)$ is $p_\sigma(\cdot, \bar{\bar{\mu}})$-continuous on the algebra $\bar{V}$. Thus, the extension $\bar{\bar{w}} \in cab(\bar{\bar{V}}, R)$ is $p_\sigma(\cdot, \bar{\bar{\mu}})$-continuous on the σ-algebra $\bar{\bar{V}}$. Thus, the Baire measure $\bar{\bar{w}}$ vanishes outside a Baire set of the first category. Since the zero measure is the only countably additive measure with this property, we conclude $\bar{\bar{w}} = 0$. Thus, the charge $\mu \in R(V, Y)$ is purely finitely additive.

The above arguments may be summarized.

THEOREM 5. *Let V be an algebra of subsets of a space X and let $\bar{\bar{V}}$ denote the Baire σ-algebra in the Stone representation space of the algebra V.*

(1) *A vector charge $\mu \in R(V, Y)$ is countably additive if and only if its Baire extension $\bar{\bar{\mu}} \in cab(\bar{\bar{V}}, Y)$ vanishes on all Baire sets of the first category.*

(2) *A vector charge $\mu \in R(V, Y)$ is purely finitely additive if and only if its Baire extension $\bar{\bar{\mu}} \in cab(\bar{\bar{V}}, Y)$ is supported by a Baire set of the first category.*

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