Ann. Funct. Anal. 7 (2016), no. 1, 206–216
http://dx.doi.org/10.1215/20088752-3462222
ISSN: 2008-8752 (electronic)
http://projecteuclid.org/afa

THE MATHEMATICAL WORK OF ANTHONY TO-MING LAU

M. S. MOSLEHIAN[1] and M. NEUFANG[2*]

Dedicated to Professor Anthony To-Ming Lau

Communicated by Q.-W. Wang

Abstract. In this paper, we review aspects of the biography and mathematical work of Professor Anthony To-Ming Lau.

1. Introduction

Professor Anthony (Tony) To-Ming Lau has made significant contributions to harmonic analysis and functional analysis. During the earlier stage of his career he worked mainly on the relation between amenability of semigroups and certain fixed-point properties. He then moved to the study of various operator and Banach algebras associated with locally compact groups, such as the group von Neumann algebra $VN(G)$, the Fourier algebra $A(G) = VN(G)_*$, and the Fourier–Stieltjes algebra $B(G)$. He has also studied topological center problems and Arens regularity of Banach algebras.

Some notable theorems and structures carry his name:

- **Lau algebra**: A Banach algebra A, which is the predual of a von Neumann algebra M such that the identity of M is a multiplicative linear functional on A, is called a *Lau algebra* (or F-*algebra*; cf. [39]).
- **Lau product**: Let $\mathcal{A}$ and $\mathcal{B}$ be Banach algebras, and let θ be in the spectrum of $\mathcal{B}$. Then the Cartesian product $\mathcal{A} \times \mathcal{B}$ equipped with the

Received Aug. 1, 2015; Accepted Nov. 25, 2015.
*Corresponding author.
2010 *Mathematics Subject Classification.* Primary 01A99; Secondary 01A60, 01A61, 43-03, 46-03, 47-03.
Keywords. bibliometrics.

algebra product

$$(a,b)(c,d) = \big(ac + \theta(d)a + \theta(b)c, bd\big),$$

and the norm $\|(a,b)\| = \|a\| + \|b\|$, is a Banach algebra, which is called the *θ-Lau product* of $\mathcal{A}$ and $\mathcal{B}$ and is denoted by $\mathcal{A} \times_\theta \mathcal{B}$. This product was first introduced by Tony [23] for F-algebras and generalized by Mehdi Sangani Monfared in [36] (see also [37]).

Tony has served as a member of the editorial boards of several journals, including *Canadian Journal of Mathematics*, *Canadian Mathematical Bulletin*, *Annals of Functional Analysis*, *Scientiae Mathematicae Japonicae*, and *Fixed Point Theory and Applications*. He has received several awards, including the following:

- Josephine Mitchell Graduate Mentorship Award 2012;
- 3M Teaching Award (June 2006)—national award, of which 10 are awarded each year for all universities in Canada and all disciplines;
- Distinguished University Professorship (from July 1, 2004);
- University Cup (September 2003);
- Award for Excellence in Mentoring, Faculty of Graduate Studies and Research (1999);
- Killam Annual Professorship (July 1, 1997–June 30, 1998);
- McCalla Professorship (September 1, 1991–May 31, 1992);
- Rutherford Award for Excellence in Undergraduate Teaching (May 1992);
- Faculty of Science Award for Excellent Teaching (May 1990).

2. Biography

Professor Anthony (Tony) To-Ming Lau (Figure 1) was born in Hong Kong, and he became interested in mathematics in elementary school. The best mathematics teacher he had in school was Mr. Jan-Hin Chan in grade 5 at the Kee Lap Elementary School in Wan Chai, Hong Kong. Mr. Chan motivated Tony's continuing interest in mathematics for the rest of his life. Tony completed his high school education at Raimondi College in Hong Kong before going to California in August 1961 for his undergraduate degree. He received his A.B. degree in mathematics on June 30, 1965, at the University of California, Berkeley.

At the University of British Columbia, he was very fortunate to be a Ph.D. student of Edmond E. Granirer, who gave him the opportunity to look for his own problems and to publish his own papers from his thesis. His earlier work was centered around amenability of semigroups and related fixed-point properties for nonlinear mappings (see [21]). Tony received his Ph.D. on May 1, 1969, and he has been a faculty member at the University of Alberta since 1970.

Later, under the influence of the work of Pierre Eymard and Ed Granirer, he began studying the group von Neumann algebra $VN(G)$, the Fourier algebra $A(G)$ and Fourier–Stieltjes algebra $B(G)$, Arens regularity and topological centers of the second dual of a Banach algebra, Beurling algebras, semigroups and their compactifications, and the structure of harmonic functionals. He greatly benefited from his one-year visit at the University of California, Berkeley, hosted by Marc Rieffel from 1976 to 1977, where he met Marc for the first time, as well

FIGURE 1. Anthony T.-M. Lau.

as Bill Bade and Theodore Palmer. They have greatly influenced the direction of Tony's research. During that year, Tony wrote his paper on F-algebras, and Marc introduced him to the theory of hypergroups. Tony was also much influenced by the works of Mahlon Day, Karl Hoffmann, and Ky Fan.

Most of Tony's research papers published in the first 15 years of his career were written by himself, and he started to collaborate with other researchers more frequently around 1985. Tony also enjoys teaching, both at the graduate and undergraduate levels, and he often teaches at least one course for first-year students. For graduate students, he enjoys meeting with them individually, and he trains them to conduct independent research. He has supervised 21 Ph.D. students: Keith Taylor (1975), Maria Klawe (1977), Zhuocheng Yang (1987), Brian Forrest (1987), Mahatheva Skantharajah (1989), Tianxuan Miao (1990), Zhiguo Hu (1993), Qin Xu (1993), Ali Kamyabi-Gol (1997), Hossein Esslamzadeh (1998), Mehdi Sangani-Monfared (2002), Ross Stoke (2003), Monica Ilie (2003), Shawn Desaulniers (2008), Michael Yin Hei Cheng (2010), Danny Pak Keung Chan (2010), Nicolas Bouffoard (2011), Ben Wilson (2011), Sabestian Guex (2013), Matthew Mazowita (2014), and Nazanin Tahmasebi (2015), and he cosupervised Richard Mikael Slevinsky (2014). Tony has hosted many visiting graduate students and postdoctoral fellows from around the world, including Iran, Thailand, and India. He has also participated in many mathematics conferences (Figures 2 and 3).

Tony served as President of the Canadian Mathematical Society (CMS) from July 2008 to June 2010. With the assistance of Graham Wright (CMS Executive Officer), he was in particular able to raise significantly the number of memberships for the society.

At present, he is a professor at the Department of Mathematical and Statistical Sciences at the University of Alberta, Canada.

FIGURE 2. Lau and Moslehian—Korea, 2012.

FIGURE 3. Lau (fourth row, second from the left) and Neufang (top row, second from the right)—Canada, 2007.

3. Recent scientific work

The following are among Tony's main recent contributions to mathematics.

3.1. **Fourier–Stieltjes algebra of a topological group.** In the paper [26], the authors initiate the study of the Fourier–Stieltjes algebra $B(G)$ as the linear span of the continuous positive definite complex-valued functions on a topological group G, which is not assumed to be locally compact. The main challenge here is, of course, the lack of Haar measure beyond local compactness. A particularly interesting class of non-locally compact topological groups are the extremely amenable ones—that is, there is a multiplicative left-invariant mean on $LUC(G)$, the space of bounded left-uniformly continuous functions on G—a phenomenon that occurs (nontrivially) only beyond local compactness; indeed, as shown by Granirer and Tony [14], a locally compact group G is extremely amenable if and only if G is trivial.

3.2. **Measures on the Stone–Čech compactification of a semigroup.** In the memoir [6], the authors study the semigroup algebra $l^1(S)$ of a semigroup S and its second dual when viewed as the Banach algebra of measure $M(\beta(S))$ on the Stone–Čech compactification $\beta(S)$ of S, where $\beta(S)$ and $M(\beta(S)$ are taken with the first Arens multiplication. They prove that S is finite when $M(\beta(S))$ is amenable and study the case when $M(\beta(S))$ is weakly amenable. Three open questions are answered, including a conjecture of J. Duncan and A. Paterson formulated in 1990 on amenability of the semigroup algebra $l^1(S)$, and questions left open by Ghahramani, Loy, and Willis in their paper [12] from 1996. The authors determine exactly when the Banach algebra $l^1(S)$ of a semigroup S is amenable. In addition, it is shown that, for a locally compact group G, the Banach algebra $L^1(G)^{**}$ is weakly amenable if and only if G is finite. Moreover, the new concept of a "dct" set—that is, a set which is determining for the topological center—is introduced. The authors show that, for a large class of semigroups, a two-point dct exists, which is new even for the case when S is a group. This memoir has led to the study of dct sets for various other bidual Banach algebras associated to groups or semigroups by several authors.

3.3. **Second duals of Beurling algebras.** In the research monograph [7], the authors studied the interplay between the group structure of a locally compact group G and Banach algebras associated to G. More specifically, they study their radicals and when the Banach algebras have a strong Wedderburn decomposition. They also study the relationship between the topological centers defined by the two Arens multiplications, regularity and irregularity of Beurling algebras, particularly when G is the free group on two generators equipped with a weight function. Such investigations have their roots in the earlier works of Craw–Young [5], Işik–Pym–Ulger [16], Lau [24], Lau–Losert [25], Baker–Lau–Pym [2], and Bade–Dales–Lykova [1].

3.4. **Multipliers of commutative Banach algebras, with applications to Fourier and Fourier–Stieltjes algebras.** Let A be a semisimple and regular commutative Banach algebra. In the paper [18], the authors establish a general

Foguel-type theorem for multipliers of A. This result is employed to obtain various properties of ideals associated with power-bounded multipliers. If A is Tauberian and has a bounded approximate identity, then the authors prove a generalization of the classical theorem of Choquet and Deny [3]. The main applications are concerned with Fourier and Fourier–Stieltjes algebras of locally compact groups. In particular, it is shown that if u is a power bounded element of $B(G)$, then the set of all x in G such that $u(x) = 1$ is in the closed coset ring of G, which was completely characterized by Forrest [9] (and previously by Gilbert [13] and Schreiber [40] in the abelian case). Further related investigations are carried out in the articles [19] and [20], continuing Tony's collaboration with E. Kaniuth and A. Ülger.

3.5. **Geometry and nonlinear fixed-point properties.** In the paper [8], the authors show that the Fourier–Stieltjes algebra $B(G)$ of a noncompact locally compact group G cannot have the weak*-weak* fixed-point property for nonexpansive mappings. This answers two open problems posed at a conference in Marseille–Luminy in 1989. They also show that a locally compact group is compact exactly when the asymptotic center of any nonempty weak*-closed bounded convex subset C in $B(G)$ with respect to a decreasing net of bounded subsets is a nonempty norm compact subset. In particular, when G is compact, $B(G)$ has the weak*-weak* fixed-point property for left-reversible semigroups. This is a significant generalization of a classical result of Lim [34] for the circle group. This work is related to and was motivated by Tony's earlier work with Mah [31] and Ülger [27].

3.6. **Topological centers of Banach algebras related to groups and semigroups.** In the memoir [11], the authors study in depth the second dual of the measure algebra $M(G)$ of a locally compact group with the Arens product. They study the hyper-Stonean envelope $\widetilde{G}$ of G, that is, the spectrum of the commutative C^* algebra $M(G)^*$ with the first Arens product. They show that $\widetilde{G}$ determines the locally compact group G. They show that $\widetilde{G}$ is a semigroup if and only if G is discrete. They also give a partial solution to the topological center problem regarding the second dual of the measure algebra $M(G)$ posed in 1995 by Gharamani and Tony [35]. This problem (known as the *Ghahramani–Lau conjecture*) has recently been completely solved (see [35], [38]).

3.7. **Geometry of continuous positive definite functions on groups, and group von Neumann algebras.** In the paper [17], the authors study Hahn–Banach and separation properties for positive definite functions on groups. Let G be a locally compact group, and let H be a closed subgroup of G. Consider the property that the continuous positive definite functions on G which are identically 1 on H separate points in $G \setminus H$ from points in H. Forrest [9] shows that if G is a [SIN]-group, then G has this separation property for every closed subgroup. In the paper [10], the authors show that the converse also holds when G is almost connected. For such groups, the separation property has been shown to be related to the extension property. This article shows that outside the class of [SIN]-groups, the two properties can be vastly different.

3.8. **Fixed-point and ergodic-type properties for semigroups of nonlinear mappings.** In recent years, there has been considerable interest in the study of when a closed convex subset K of a Banach space has the fixed-point property, that is, if T is a nonexpansive mapping from K into K, then K contains a fixed point for T. In the papers [28]–[30] and [32], the authors study fixed-point properties of semigroups of nonexpansive mappings on weakly compact convex subsets of a Banach space (or, more generally, a locally convex space). By considering the classes of bicyclic semigroups they answer two open questions: one posed earlier by Tony in 1976 and the other posed by T. Mitchell in 1984. They also provide a characterization for the existence of a left-invariant mean on the space of weakly almost-periodic functions on separable semitopological semigroups in terms of a fixed-point property for nonexpansive mappings related to another open problem raised by Tony in 1976.

3.9. **Fixed-point sets of positive definite functions and measure on groups: Harmonic functions.** Harmonic functions are functions in $L_\infty(G)$ fixed under the convolution action by a probability measure on G. In the monograph [4], the authors introduce new aspects to the theory of harmonic functions and related topics. The work brings together recent developments in abstract harmonic analysis, semigroups, and nonassociative functional analysis. More specifically, the authors study the algebraic and analytic structure of the space of bounded complex harmonic functions on a locally compact group G and its noncommutative analogue, the space of harmonic functionals on the Fourier algebra, which is the fixed-point set of a continuous positive definite function on G acting canonically on the group von Neumann algebra $VN(G)$. Moreover, they show that the fixed points for the action of an element of $B(G)$ of norm 1 form the range of a contractive projection on $VN(G)$ and therefore admit a Jordan algebra structure. This provides a natural setting to apply new methods and results from nonassociative analysis and Fourier algebras. They use these devices to study, in particular, the nonassociative geometric structure of the space of harmonic functionals.

3.10. **Finite-dimensional invariant subspace property of Ky Fan and the Hahn–Banach extension property for amenable Banach algebras.** Motivated by a result of Ky Fan in 1965, the authors of [33] establish a characterization of a left-amenable F-algebra (which includes the group algebra and the Fourier algebra of a locally compact group, the predual $L_1(\mathcal{G})$ of a locally compact quantum group $\mathcal{G}$, or, more generally, the predual algebra of a Hopf–von Neumann algebra) in terms of a finite-dimensional invariant subspace property. This is done by first exhibiting a fixed-point property for the semigroup of norm-one positive linear functionals on the algebra. Their result answers an open question posed at the International Conference on Nonlinear and Convex Analysis at Keio University, Tokyo, in 1993. We also note that a characterization of amenable Banach algebras in terms of the Hahn–Banach extension property was obtained by Tony in [22, Theorem 1] (see also [15, Theorem 2.8, pp. 253, 265]).

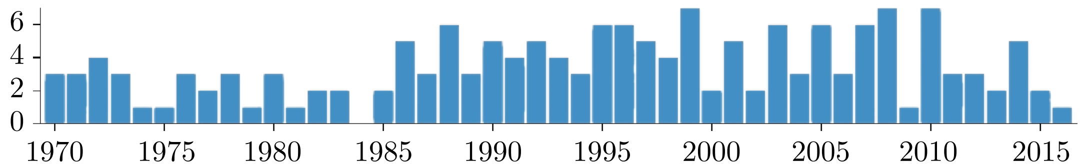

FIGURE 4. Lau's publications by year. *Source*: Zbl.

4. Bibliometrics

In this section, we give a quantitative analysis of Tony's publications using *MathSciNet* (MR), *Zentralblatt MATH* (Zbl), and *Web of Science* (WOS).

The following are Tony's top three most-cited publications in MR:

- H. G. Dales and A. T.-M. Lau, *The second duals of Beurling algebras*, Mem. Amer. Math. Soc. **177** (2005), no. 836. (72 citations)
- A. T.-M. Lau, *Analysis on a class of Banach algebras with applications to harmonic analysis on locally compact groups and semigroups*, Fund. Math. **118** (1983), no. 3, 161–175. (69 citations)
- H. G. Dales, A. T.-M. Lau, and D. Strauss, *Banach algebras on semigroups and on their compactifications*, Mem. Amer. Math. Soc. **205** (2010), no. 966. (68 citations)

The following are Tony's top three most-cited publications in WOS:

- A. T.-M. Lau, *Analysis on a class of Banach algebras with applications to harmonic analysis on locally compact groups and semigroups*, Fund. Math. **118** (1983), no. 3, 161–175. (72 citations)
- A. T.-M. Lau and V. Losert, *On the second conjugate algebra of $L_1(G)$ of a locally compact group*, J. London Math. Soc. (2) **37** (1988), no. 3, 464–470. (60 citations)
- A. T.-M. Lau, N. Shioji, and W. Takahashi, *Existence of nonexpansive retractions for amenable semigroups of nonexpansive mappings and nonlinear ergodic theorems in Banach spaces*, J. Funct. Anal. **161** (1999), no. 1, 62–75. (56 citations)

The number of Tony's publications recorded in MR and Zbl are 156 and 162, respectively. According to MR, they are cited 1,798 times by 358 authors. Abstract harmonic analysis is the subject in which Tony has published most of his articles and in which his works are most cited.

According to Zbl (Figure 4), the top three journals in which Tony's publications appear are *Journal of Functional Analysis* (20 papers), *Transactions of the American Mathematical Society* (19 papers), and *Proceedings of the American Mathematical Society* (16 papers). He has had 59 coauthors and has collaborated the most with Eberhard Kaniuth (23 papers), Wataru Takahashi (18 papers), and John B. Pym (13 papers). Four conference proceedings and one Springer lecture note are recorded in Zbl as his books.

WOS records 89 publications by Tony (Figure 5, left). The average citation per publication is 14.39, and Tony's h-index is 20 (Figure 5, right). Recall that the h-index is an index measuring the productivity and citation impact of an author.

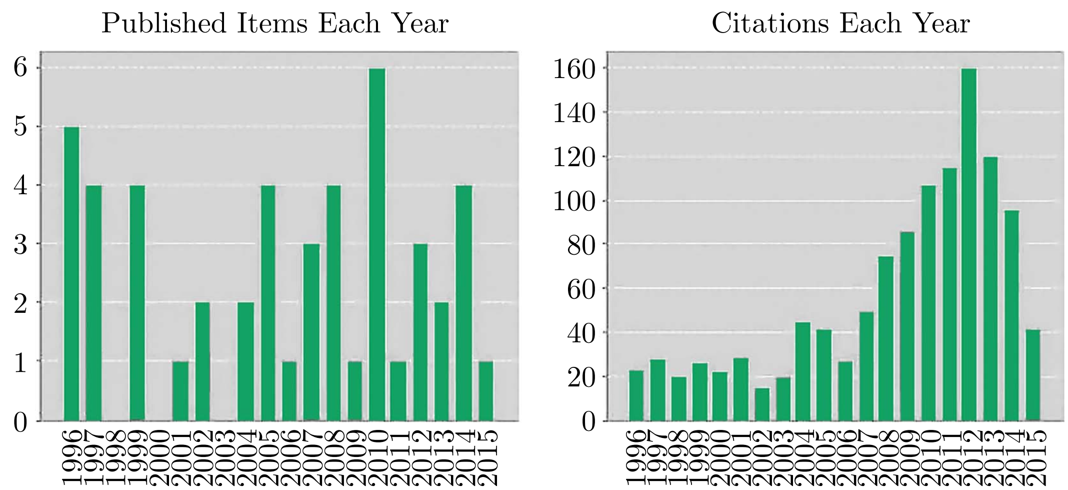

FIGURE 5. Lau's published items and citations of his work in the last 20 years. *Source*: Web of Science.

The h-index of a mathematician is n if he has n publications, each of which has at least n citations.

References

1. W. Bade, H. G. Dales, and Z. A. Lykova, *Algebraic and strong splittings of extensions of Banach algebras*, Mem. Amer. Math. Soc. **137** (1999), no. 656. Zbl 0931.46034. MR1491607. DOI 10.1090/memo/0656. 210
2. J. Baker, A. T.-M. Lau, and J. Pym, *Module homomorphisms and topological centres associated with weakly sequentially complete Banach algebras*, J. Funct. Anal. **158** (1998), no. 1, 186–208. Zbl 0911.46030. MR1641570. DOI 10.1006/jfan.1998.3280. 210
3. G. Choquet and J. Deny, *Sur l'équation de convolution $\mu = \mu * \sigma$*, C. R. Acad. Sci. Paris **250** (1960), 799–801. MR0119041. 211
4. C.-H. Chu and A. T.-M. Lau, *Harmonic Functions on Groups and Fourier Algebras*, Lecture Notes in Math. **1782**, Springer, Berlin, 2002. Zbl 1009.43001. MR1914221. DOI 10.1007/b83280. 212
5. I. C. Craw and N. J. Young, *Regularity of multiplication in weighted group and semigroup algebras*, Quart. J. Math. Oxford Ser. (2) **25** (1974), 351–358. Zbl 0304.46027. MR0365029. 210
6. H. G. Dales, A. T.-M. Lau, and D. Strauss, *Banach algebras on semigroups and on their compactifications*, Mem. Amer. Math. Soc. **205** (2010), no. 966. Zbl 1192.43001. MR2650729. DOI 10.1090/S0065-9266-10-00595-8. 210
7. H. G. Dales and A. T.-M. Lau, *The second duals of Beurling algebras*, Mem. Amer. Math. Soc. **177** (2005), no. 836. MR2155972. DOI 10.1090/memo/0836. 210
8. G. Fendler, A. T.-M. Lau, and M. Leinert, *Weak* fixed point property and asymptotic centre for the Fourier–Stieltjes algebra of a locally compact group*, J. Funct. Anal. **264** (2013), no. 1, 288–302. MR2995708. DOI 10.1016/j.jfa.2012.10.011. 211
9. B. Forrest, *Amenability and ideals in $A(G)$*, J. Austral. Math. Soc. Ser. A **53** (1992), no. 2, 143–155. Zbl 0767.43003. MR1175708. 211
10. B. Forrest, E. Kaniuth, A. T.-M. Lau, and N. Spronk, *Ideals with bounded approximate identities in Fourier algebras*, J. Funct. Anal. **203** (2003), no. 1, 286–304. Zbl 1039.46042. MR1996874. DOI 10.1016/S0022-1236(02)00121-0. 211

11. F. Ghahramani and R. J. Loy, *Multipliers and ideals in second conjugate algebras related to locally compact groups*, J. Funct. Anal. **132** (1995), no. 1, 170–191. Zbl 0832.22007. MR1346222. DOI 10.1006/jfan.1995.1104. 211
12. F. Ghahramani, R. J. Loy, and G. A. Willis, *Amenability and weak amenability of second conjugate Banach algebras*, Proc. Amer. Math. Soc. **124** (1996), no. 5, 1489–1497. Zbl 0851.46035. MR1307520. DOI 10.1090/S0002-9939-96-03177-2. 210
13. J. E. Gilbert, *On projections of $L^\infty(G)$ onto translation-invariant subspaces*, Proc. London Math. Soc. (3) **19** (1969), 69–88. Zbl 0176.11603. MR0244705. 211
14. E. Granirer and A. T.-M. Lau, *Invariant means on locally compact groups*, Illinois J. Math. **15** (1971), 249–257. Zbl 0212.14904. MR0277667. 210
15. A. Ya. Helemskii, *The Homology of Banach and Topological Algebras*, Math. Appl. (Soviet Ser.) **41**, Kluwer, Dordrecht, 1989. MR1093462. DOI 10.1007/978-94-009-2354-6. 212
16. N. Işik, J. Pym, and A. Ülger, *The second dual of the group algebra of a compact group*, J. London Math. Soc. (2) **35** (1987), no. 1, 135–148. MR0871771. DOI 10.1112/jlms/s2-35.1.135. 210
17. E. Kaniuth and A. T.-M. Lau, *Extension and separation properties of positive definite functions on locally compact groups*, Trans. Amer. Math. Soc. **359** (2007), no. 1, 447–463. Zbl 1120.43003. MR2247899. DOI 10.1090/S0002-9947-06-03969-9. 211
18. E. Kaniuth, A. T.-M. Lau, and A. Ülger, *Multipliers of commutative Banach algebras, power boundedness and Fourier–Stieltjes algebras*, J. Lond. Math. Soc. (2) **81** (2010), no. 1, 255–275. Zbl 1196.46037. MR2580464. DOI 10.1112/jlms/jdp068. 210
19. E. Kaniuth, A. T.-M. Lau, and A. Ülger, *Power boundedness in Fourier and Fourier–Stieltjes algebras and other commutative Banach algebras* J. Funct. Anal. **260** (2011), no. 8, 2366–2386. Zbl 1215.43001. MR2772374. DOI 10.1016/j.jfa.2010.11.012. 211
20. E. Kaniuth, A. T.-M. Lau, and A. Ülger, *Power boundedness in Banach algebras associated with locally compact groups*, Studia Math. **222** (2014), no. 2, 165–189. Zbl 1306.43004. MR3223323. DOI 10.4064/sm222-2-4. 211
21. A. T.-M. Lau, *Topological semigroups with invariant means in the convex hull of multiplicative means*, Trans. Amer. Math. Soc. **148** (1970), 69–84. Zbl 0201.02901. MR0257260. 207
22. A. T.-M. Lau, *Characterizations of amenable Banach algebras*, Proc. Amer. Math. Soc. **70** (1978), no. 2, 156–160. Zbl 0384.46031. MR0492065. DOI 10.2307/2042079. 212
23. A. T.-M. Lau, *Analysis on a class of Banach algebras with applications to harmonic analysis on locally compact groups and semigroups*, Fund. Math. **118** (1983), no. 3, 161–175. Zbl 0545.46051. MR0736276. 207
24. A. T.-M. Lau, *Continuity of Arens multiplication on the dual space of bounded uniformly continuous functions on locally compact groups and topological semigroups*, Math. Proc. Cambridge Philos. Soc. **99** (1986), no. 2, 273–283. Zbl 0591.43003. MR0817669. DOI 10.1017/S0305004100064197. 210
25. A. T.-M. Lau and V. Losert, *On the second conjugate algebra of $L_1(G)$ of a locally compact group*, J. London Math. Soc. (2) **37** (1988), no. 3, 464–470. Zbl 0608.43002. MR0939122. DOI 10.1112/jlms/s2-37.3.464. 210
26. A. T.-M. Lau and J. Ludwig, *Fourier–Stieltjes algebra of a topological group*, Adv. Math. **229** (2012), no. 3, 2000–2023. Zbl 1236.22001. MR2871165. DOI 10.1016/j.aim.2011.12.022. 210
27. A. T.-M. Lau and P. F. Mah, *Normal structure in dual Banach spaces associated with a locally compact group*, Trans. Amer. Math. Soc. **310** (1988), no. 1, 341–353. Zbl 0706.43003. MR0937247. DOI 10.2307/2001126. 211
28. A. T.-M. Lau, N. Shioji, and W. Takahashi, *Existence of nonexpansive refractions for amenable semigroups of nonexpansive mappings and nonlinear ergodic theorems in Banach spaces*, J. Funct. Anal. **161** (1998), 62–75. MR1670206. DOI 10.1006/jfan.1998.3352. 212
29. A. T.-M. Lau and W. Takahashi, *Fixed point properties for semigroup of nonexpansive mappings on Fréchet spaces*, Nonlinear Anal. **70** (2009), no. 11, 3837–3841. Zbl 1219.47082.

MR2515303. DOI 10.1016/j.na.2008.07.041. 212

30. A. T.-M. Lau and W. Takahashi, *Weak convergence and nonlinear ergodic theorems for reversible semigroups of nonexpansive mappings*, Pacific J. Math. **126** (1987), no. 2, 277–294. Zbl 0587.47058. MR0869780. 212
31. A. T.-M. Lau and A. Ülgar, *Some geometric properties on the Fourier and Fourier–Stieltjes algebras of locally compact groups, Arens regularity and related problems*, Trans. Amer. Math. Soc. **337** (1993), no. 1, 321–359. Zbl 0778.43003. MR1147402. DOI 10.2307/2154325. 211
32. A. T.-M. Lau and Y. Zhang, *Fixed point properties of semigroups of non-expansive mappings*, J. Funct. Anal. **254** (2008), 2513–2533. MR2406686. DOI 10.1016/j.jfa.2008.02.006. 212
33. A. T.-M. Lau and Y. Zhang, *Finite-dimensional invariant subspace property and amenability for a class of Banach algebras*, Trans. Amer. Math. Soc., published electronically 1 July 2015. DOI 10.1090/tran/6442. 212
34. T. C. Lim, *Asymptotic centres and nonexpansive mappings in conjugate Banach spaces*, Pacific J. Math. **90** (1980), 135–143. MR0599326. 211
35. V. Losert, M. Neufang, J. Pachl, and J. Steprans, *Proof of the Ghahramani–Lau conjecture*, to appear in Adv. Math., preprint arXiv:1510.05412 [math.FA]. 211
36. M. S. Monfared, *On certain products of Banach algebras with applications to harmonic analysis*, Studia Math. **178** (2007), no. 3, 277–294. Zbl 1121.46041. MR2289357. DOI 10.4064/sm178-3-4. 207
37. M. Nemati and H. Javanshiri, *Some homological and cohomological notions on T-Lau product of Banach algebras*, Banach J. Math. Anal. **9** (2015), no. 2, 183–195. MR3296113. DOI 10.15352/bjma/09-2-13. 207
38. M. Neufang, *On a conjecture by Ghahramani–Lau and related problems concerning topological centres*, J. Funct. Anal. **224** (2005), no. 1, 217–229. Zbl 1085.43002. MR2139110. DOI 10.1016/j.jfa.2004.11.007. 211
39. J.-P. Pier, *Amenable Locally Compact Groups*, Pure Appl. Math. (N.Y.). Wiley, New York, 1984. MR0767264. 206
40. B. M. Schreiber, *Measures with bounded convolution powers*, Trans. Amer. Math. Soc. **151** (1970), 405–431. Zbl 0202.14202. MR0264335.

211

[1]Department of Pure Mathematics, Center of Excellence in Analysis on Algebraic Structures (CEAAS), Ferdowsi University of Mashhad, P.O. Box 1159, Mashhad 91775, Iran.

E-mail address: moslehian@um.ac.ir

[2]School of Mathematics and Statistics, Carleton University, 1125 Colonel By Drive, Ottawa, Ontario, K1S 5B6, Canada; and Laboratoire de Mathématiques Paul Painlevé (UMR CNRS 8524), Université Lille 1–Sciences et Technologies, UFR de Mathématiques, 59655 Villeneuve d'Ascq Cedex, France.

E-mail address: mneufang@math.carleton.ca; matthias.neufang@math.univ-lille1.fr