FIENBERG, S. E. (1971). Randomization and social affairs: the 1970 draft lottery. *Science* **171** 255–261.

GELBART, S. (1975). *Automorphic Forms on Adele Groups.* Princeton Univ. Press, Princeton, N. J.

HOPF, E. (1934). On causality, statistics and probability. *J. Mathematical Phys.* **17** 51–102.

HOPF, E. (1937). Ein Verteilungsproblem bei dissipativen dynamischen Systemen. *Math. Ann.* **114** 161–186.

KELLER, J. (1985). The probability of heads. *Amer. Math. Monthly*, in press.

KEMPERMAN, J. (1975). Sharp bounds for discrepancies (mod 1) with application to the first digit problem. Unpublished manuscript.

LÉVY, P. (1961). Quelques problèmes non résolus de la theorie des functions caractéristiques. *Ann. Mat. Pura Appl.* **53** 315–332.

MACKEY, G. W. (1978). *Unitary Group Representations in Physics, Probability, and Number Theory.* Benjamin/Cummings, Reading, Mass.

SAVAGE, L. J. (1973). Probability in science: A personalistic account. In *Logic, Methodology and Philosophy of Science* (P. Suppes et al., eds.) 417–428. North Holland, Amsterdam.

VON PLATO, J. (1983). The method of arbitrary functions. *Br. J. Philos. Sci.* **34** 37–47.

VULOVIC, V. Z. and PRANGE, R. E. (1985). Is the toss of a coin really random? Univ. of Maryland. Preprint.

WOLFRAM, S. (1985). Origins of randomness in physical systems. *Phys. Rev. Lett.* **55** 449–452.

YUE, Z. and ZHANG, B. (1985). On the sensitive dynamical system and the transition from the apparently deterministic process to the completely random process. *Appl. Math. Mech.* (English ed.) **6** 193–211.

Comment

Herbert Solomon

I. J. Good has done a service in highlighting some of Simeon Denis Poisson's (1781–1840) contributions to statistics and probability. While his name is a commonplace to us, the breadth and variety of Poisson's work is neglected in formal courses undertaken by even the most advanced students. It is good to view Poisson as a member of the French school of probabilists who thrived from the late 18th century to the middle of the 19th century. Laplace overwhelms this group but the contributions of Condorcet, Cournot, and Bienaymé, as well as Poisson, among others, must receive attention. This is especially so in model building and estimation in the behavioral sciences. Applying the calculus of probabilities to important societal problems such as jury behavior was not beneath them. All lived in dramatically changing times in France where individual rights had assumed an importance that did not exist before the late 18th century.

Unfortunately, this kind of endeavor was frowned upon in the latter half of the 19th century by European mathematicians and it was not until the middle of this century that we found this activity again receiving the attention it deserves. For some reason or other, R. A. Fisher also refers in a rather negative manner to that earlier era when probabilists concerned themselves with the veracity of witnesses and group decisions. Yet one of Poisson's most important works was his 1837 volume on the *Calculus of Probability Applied to Civil and Criminal Proceedings*—the book in which what we now call the Poisson distribution first appears; albeit as a mathematical approximation artifice.

Much as we wish that Good would have elaborated more on the themes in his paper, editorial constraints and his own tastes no doubt limited the size of his effort. Let us now look into the Poisson jury model in some detail to catch the flavor of the statistical thinking and the concern with moral and societal values demonstrated by Poisson.

It is important to note that Poisson in developing his model paid heed to the data available in his day. For the period 1825–1830, jury decisions were based on 7 or more out of 12 jurors favoring either conviction or acquittal. Cases based on verdicts of exactly 7 out of 12 went to a higher court which could change the verdict. For each year, the number of trials and number of convictions were registered and listed for crimes against persons and crimes against property. Note here that this distinction in crimes is definitely drawn over 150 years ago. In the period 1831–1833, listings were also available, except the majority required was 8 or more out of 12. In 1832 and 1833, the jury could find extenuating circumstances in a conviction that would then lead to a lighter penalty.

What impressed Poisson was the stability of the conviction ratios over each of the years 1825–1830 and 1832–1833. He felt this was a basis for developing a model that in some parsimonious way could reproduce the data, and if so, lead to the computation of the probabilities of the two kinds of errors important in judging the effects of size and decision-making rules of a jury, namely, the probability of acquitting a guilty defendant, and the probability of convicting an innocent defendant.

Herbert Solomon is Professor of Statistics, Department of Statistics, Stanford University, Sequoia Hall, Stanford, California 94305.

To check on the homogeneity of the annual proportions of conviction over the years 1825–1830, Poisson divided the 6 years into two groups, 1825–1827 and 1828–1830, and tested the difference of the proportions of conviction in each period employing the normal approximation to the binomial. He concluded there was no difference. Good refers to Poisson's work on this statistical test in Section 3. Since the χ^2 goodness of fit test over the 6 years is now available in our statistical armory, the homogeneity hypothesis was tested in this manner employing a χ^2 with five degrees of freedom. We computed $\chi_5^2 = 14.04$ from the data. Thus homogeneity is rejected at the .05 level of significance but accepted at the .01 level of significance. The principal contribution to statistical significance comes in 1830 and Poisson, in his work, remarks that possibly the proportion of convictions in that year may be a little out of line. If we omit 1830, we compute $\chi_4^2 = 4.85$ which indicates no significance at the .05 level and thus homogeneity over the 5 years 1825–1829.

Poisson had criticized Laplace's work on juries as not being meaningful. We are aware of Poisson's concern to include more meaningful parameters than Laplace did and he chose two parameters: θ, the probability that the accused is guilty before the evidence is presented to the jury, and μ, the probability that a juror will not make an error. The fact that data were available for two differing jury situations meant that two equations with two unknowns would result.

The first parameter, θ, is a commentary on the society and its law enforcement procedures and the second, μ, relates to how well a selected juror can sift through and assess evidence. We now list θ, μ, and the following definitions to develop the model: P_C, probability of conviction; P_A, probability of an acquittal; $P_{G/A}$, probability of guilt given an acquittal; $P_{I/C}$, probability of innocence given a conviction. For a jury of size n, Poisson derives $\gamma_{n,i}$, the probability of conviction given i votes for acquittal in a jury of size n as

$$\gamma_{n,i} = \binom{n}{i} [\theta\mu^{n-i}(1-\mu)^i + (1-\theta)\mu^i(1-\mu)^{n-i}];$$

and $P_C = \sum_{i=0}^{5} \gamma_{12,i}$ or $P_C = \sum_{i=0}^{4} \gamma_{12,i}$. Note that the two terms in the brackets are, respectively, the guilty component where the i votes for acquittal are in error and the not guilty component where the i votes for acquittal are not in error.

Over all trials, Poisson obtained the estimates $\theta = .64$, $\mu = .75$; for crimes against persons, the estimates are $\theta = .54$, $\mu = .68$; for crimes against property, the estimates are $\theta = .67$, $\mu = .78$. This demonstrates how θ and μ can easily vary with the criminal charge.

Poisson is quite aware of the complementary nature of θ and μ, namely that $(1-\theta)$ and $(1-\mu)$ will produce the same probability of conviction in his model. He comments that the high proportion of convictions during the period of the French Revolution cannot be employed to suggest fairness, equity, or reasonableness since values, $\theta = .36$ and $\mu = .25$ yield the same values for P_C as $\theta = .64$, $\theta = .75$ that were derived from his model and the data of 1825–1830 and 1831–1833. Thus bringing to trial an individual whose prior probability of guilt is about ⅓ where jurors can be in error ¾ of the time, gives (in the 7 or more out of 12 situations), $P_C = .61$ just as in the case where the probability of juror error equals ¼ and probability of guilt before trial is about ⅔. If we assume $\mu \geq$ ½, $\theta \geq$ ½, which is realistic in any rational society, then the μ, θ solutions are unique.

Armed with the results of his model, Poisson proceeds to estimate the two kinds of jury errors. For the period 1825–1830, he estimates the probability of convicting an innocent defendant is $P_{I/C} = .06$ (over person and property crimes) and the probability of acquitting a guilty defendant is $P_{G/A} = .18$. Poisson gives more results but the figures just cited do not differ drastically from estimates from models of the American experience in the mid 20th century with juries of size 12 and unanimity required for decision.

To summarize, the Poisson jury model is an excellent example of the development of models in the behavioral sciences. First, it seems to serve the French jury experience quite well. There are only two parameters to produce a rather parsimonious accounting of French jury decisions in the period 1825–1833. The data on hand, e.g., proportion of convictions by 7 or more out of 12, and by 8 or more out of 12, permit the development of two equations with two unknowns under the implicit assumption that one ballot is required. However the required plurality of 7 or 8 out of 12 jurors to produce a verdict, essentially leads to only one ballot. The parameters, θ (the probability the defendant is guilty before the trial begins and evidence is presented) and μ (the probability a juror will not make an error), are latent parameters. Estimates of these parameters are produced from the data on proportion of jury convictions. Poisson computed these values by solving equations of high degree and he essentially employed the method of moments to obtain these estimates.

Perhaps others will expand on the other themes presented by Good. I believe we are indebted to him for bringing Poisson's works to a wide audience who know the name very well but not the diversity of interests of that scholar. Good refers to his participation at a Poisson Bicentennial Commemoration held at George Washington University in 1982 (201 years after Poisson's birth). I also had the privilege of participating in that meeting. Also at that meeting were several descendants of Poisson who had been located by the Smithsonian Institution. The maiden name of

Madame Pompadour, alleged mistress of Louis XV, was Poisson. All the Poissons at the meeting claimed some descent from her but would argue about her relationship with the king.

I cannot let Good's comments on the pronunciation of Poisson pass without comment. While spoken English at the movies, theater, radio, and TV might indicate that Brooklynese for "person" would lead to the pronunciation "Poyson," he might find that pronunciation more common in his region of the country. In fact, I recall hearing more than one prominent U.S. Senator from the South producing those sounds.

Comment

C. C. Heyde

Good's matter of fact treatment of some of his topics suppresses the human interest and controversy surrounding much of Poisson's work. Poisson was ambitious and competitive and often provoked violent debate. Accounts of the controversy over his work on probability and the law and over the law of large numbers appear in Heyde and Seneta (1977), Chapters 2 and 3, respectively.

The curious title of Poisson's major work in probability and statistics (1837), not all of which is quoted by Good, bears some explanation. Poisson's motivation is evident from the title although from a modern point of view the work contains so much important preliminary material of a purely theoretical kind that it should be regarded as a treatise on probability with applications to the legal system.

The background to Poisson's investigation of the jury law is interesting. This was a subject which had already received considerable attention, dating back to the work of Condorcet (1785), which had apparently been stimulated by the administrator Turgot, and flowing from the concern for justice which was widespread at the time. Laplace had been a major contributor and results in the first supplement, added in 1816, to his treatise on probability (1812) became a source of argument after the jury system was changed in 1831. Ammunition for this argument was readily available since the publication by the government of annual statistical reports, *Comptes généraux de l'administration de la justice criminelle*, had commenced in 1825.

In the period from 1825–1830, criminal trials were heard before a jury of 12 members and a majority of at least 7 to 5 was required for conviction. This was changed to 8 to 4 in 1831 and these arrangements were strongly, but ultimately unsuccessfully, defended in the Chamber of Deputies in 1835 by the famous astronomer Arago, then Deputy from Pyrénées-Orientales. In this argument, Arago drew heavily on a formula of Laplace in which (under questionable conditions) the probability of a correct verdict is given as a function of the majority obtained. Nevertheless, the law of 9 September 1835 returned the required majority to at least 7 to 5.

Laplace had been Poisson's teacher and patron but the use of his formula was sharply criticized by Poisson (1835) in a paper read to the Academie des Sciences on 14 December 1835, and it was here that he first applied his law of large numbers as described in Section 4 of Good's paper. This did not settle the matter, however, and Poisson in turn came in for serious criticism from Poinsot, Dupin, Navier, and others as the argument continued. Some of this is reported in the *Comptes Rendus* following the papers Poisson (1836a, 1836b) (read, respectively, on 11 April and 18 April 1836) which, together with the 1835 paper cited above were the basis of his work of 1837. A detailed account of the changes to the jury law and the contributions of probability to the debate, including the work of Poisson, can be found in Cournot (1984), Chapter XVI and editorial notes thereon.

The first paragraph of Section 6 of Good's paper, which concerns the Poisson distribution, is not entirely accurate. Poisson limits of the first few binomial probabilities were established in the context of a particular gambling problem by de Moivre (1967, Problem V, p. 45) and he states that "... the law of continuation of these equations is manifest." For further background on the history of the Poisson approximation to the binomial see Seneta (1983). Finally, a useful biographical account of Poisson appears in Costabel (1978).

C. C. Heyde is Professor and Chairman, Department of Statistics, University of Melbourne, Parkville, VIC 3052, Australia.

ADDITIONAL REFERENCES

CONDORCET, M. (1785). *Essai sur l'application de l'analyse à la probabilité des decisions rendus à la pluralité des voix.* Imprimerie Royale, Paris.

COURNOT, A. A. (1984). *Exposition de la théorie des chances et des probabilities.* Oeuvres Complètes, Tome 1. (B. Bru, ed.). Li-