

are $(A|C)$ and $(B|AC)$. This leads to a functional equation for F which under mild regularity assumptions can be solved. The solution, after being rescaled by a monotone transformation, gives the usual formula for the probability of an intersection. Further qualitative assumptions and another functional equation lead to finite additivity and Bayes' formula. Jaynes writes in the objective Bayesian tradition of Laplace and Jeffreys, but the approach should be of interest to subjectivists also.

Finally, I would like to comment briefly on countable additivity. The requirement of coherence does not imply countable additivity as de Finetti has often emphasized, nor do the Cox-Jaynes axioms. Even the objectivistic relative frequency interpretation of probability fails to support it. The axiom of monotone

continuity may be an appealing way to reformulate countable additivity, but in general I agree with Kolmogorov (1933) that the assumption of countable additivity, although expedient, is arbitrary.

ADDITIONAL REFERENCES

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Rejoinder

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It is a pleasure to thank the discussants for their contributions to issues of subjective probability and decision making in the face of uncertainty. I am especially grateful for their enhancement of the whole by their emphasis on topics covered only lightly if at all in my survey of axiomatics.

The diversity of the responses, ranging from Suppes's discussion of the foundational issues of uniqueness, exchangeability, and expectation to Stone's more pointed focus on finite versus countable additivity, is impressive. At the same time, three concerns were raised in common by several discussants, namely the problem of imprecision, the decisional bases of subjective probability, and the matter of finite versus countable additivity. I conclude with a few words on each of these primary issues.

The problem of imprecision or vagueness in judgment is raised by Berger, Good, Fine, and Seidenfeld. It has been a central theme in my own research, beginning with *Decision and Value Theory* (Wiley, 1964). Others who have mined it in past years include Good, Art Dempster, C. A. B. Smith, and, before them, John Maynard Keynes. A typical way of dealing with the problem is to admit a possibly convex family of representing measures (not necessarily additive), which might be characterized by intervals or upper and lower bounds on distributions. It remains a viable research topic as seen in the exciting work of Glen

Shafer and the research papers of Fine and his co-authors.

The decisional bases of subjective probability are discussed by Berger, Sudderth, and Seidenfeld. Berger emphasizes the interface with statistical practice, while Sudderth and Seidenfeld recall the important works of Bruno de Finetti, C. A. B. Smith, and others that phrase axioms for subjective probability in terms of preferences or choices in the face of uncertainty. This too has been one of my own preoccupations in the tradition pioneered by Frank P. Ramsey, Jimmie Savage, and de Finetti, although it was mentioned only briefly in the survey. I am indebted to the discussants for reminding us of its centrality.

Finally, the matter of finite versus countable additivity, made prominent by de Finetti and Savage, is raised by Berger, Stone, and Sudderth. The present wisdom seems to be that countable additivity can keep one out of trouble that might arise in its absence even if it is arbitrary, or at best un compelling, as a principle of rational choice. My own attitude toward the issue is pragmatic. Much like the Axiom of Choice in set theory, if I can do without countable additivity to get where I want to go, so much the better. But I will not hesitate to invoke it when its denial would create mathematical complexities of little interest to the topic at hand.