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Comment

Peter C. Fishburn

Readers of *Statistical Science* owe a debt of gratitude to Glenn Shafer for his penetrating analysis of Jimmie Savage's views on the foundations of choice in the face of uncertainty and for his exposition of a constructive approach to subjective expected utility that is informed by research on individual choice behavior accumulated since the 1954 publication of *The Foundations of Statistics*.

Shafer's reconsideration of Savage's key axioms in the light of empirical evidence, his insistence on the practical difficulties of formulating decision problems in Savage's states-consequences mode and its effect on independence, and his analysis of small worlds are welcome and cogent. I am less comfortable, however, with Shafer's central claim that Savage's view was not constructive and will suggest below why I think he has misunderstood Savage. To do this I will summarize my understanding of Shafer's constructive approach and then say what I think Savage intended.

Some preliminary remarks will help to focus my viewpoint. As Shafer notes, it has become common to distinguish between descriptive (empirical, behavioral) and normative (prescriptive, recommendatory) interpretations of choices and decision theory. Several theorists, among them Bernoulli (1738) and Allais (1953, 1979), assert that their theories of rational choice accord precisely with actual behavior and hence they see no discord between the normative and descriptive interpretations. Others who advocate normative theories, including Savage (1954), are more

modest in their behavioral claims and suggest that their theories are descriptively valid only to a first approximation. Other theories, such as the prospect theory of Kahneman and Tversky (1979), are proposed as descriptive without claim to normative status.

A large number of empirical studies by Ward Edwards, Clyde Coombs, Duncan Luce, Sarah Lichtenstein and Paul Slovic, Amos Tversky and Danny Kahneman, Hillel Einhorn, and Ken MacCrimmon, among others, provide convincing evidence that proposed normative theories, including various versions of expected utility, are not descriptively valid. In particular, many people exhibit systematic and persistent violations of transitivity and independence (cancellation, substitution, additivity) axioms along with the reduction or invariance principle which says that preference or choice between acts depends only on their separate probability distributions over outcomes. A recent paper by Tversky and Kahneman (1986) argues persuasively that no adequate normative theory can be descriptively accurate and, although I take issue with their view of what is normative, I believe their conclusion is inescapable.

During the past several years, the gulf between the traditional expected utility theories of von Neumann and Morgenstern (1944) for risky decisions and Savage (1954) for decision under uncertainty, and the systematic empirical violations of these theories has led to a family of new theories designed to accommodate such violations. The new theories might be said to be generalized expected utility theories since they usually weaken one or more of the von Neumann-Morgenstern or Savage axioms and involve an expectation operation in their numerical representations of preference. In the von Neumann-Morgenstern setting, Machina (1982), Fishburn (1983), and Chew (1983)

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weaken the independence axiom, and Fishburn (1982) weakens both independence and transitivity. In Savage's setting of decision under uncertainty with traditional representation

$$(1) \quad f \succ g \Leftrightarrow \int_S u(f(s)) \, d\pi(s) > \int_S u(g(s)) \, d\pi(s),$$

where f and g are functions from the state set S into the consequence set, \succ denotes *is preferred to*, and u and π are utility and probability functions, respectively, Schmeidler (1984) and Gilboa (1985) weaken Savage's independence postulate P2 (Section 2.2) and replace his additive measure π by a monotonic but not necessarily additive measure. This generalization accommodates Ellsberg (1961) type violations of additivity while retaining transitivity. A different weakening (Fishburn, 1984, 1986; Fishburn and LaValle, 1987) retains P2 and the full force of Savage's sure thing principle but weakens transitivity (P1) to obtain the generalized representation

$$f \succ g \Leftrightarrow \int_S \phi(f(s), g(s)) \, d\pi(s) > 0,$$

where ϕ is skew symmetric [$\phi(y, x) = -\phi(x, y)$] and π is the same as in Savage's representation. A similar representation that interprets ϕ as a measure of riskless preference difference coupled with a concept of regret is discussed by Loomes and Sugden (1982, 1986).

The new theories cited in the preceding paragraph could be regarded as a blend of the normative and descriptive approaches since they retain many of the traditional normative features while accommodating systematic behaviors uncovered by empirical research. Indeed, it is sometimes unclear whether their authors see them as primarily normative or primarily descriptive. I believe, however, that they tend toward the normative interpretation. In consequence, the meaning of what is normative appears to be changing to include some behaviors not covered by the traditional expected utility theories. Allais (1953, 1979) in fact has advocated an empirically oriented view of the normative or rational for many years. At the same time others, including Edwards (1985), maintain a normative position quite similar to Savage's.

Shafer's constructive viewpoint follows the recent trend of adapting the traditional normative interpretation to empirical realities. He suggests that constructive may be a more suitable descriptor than normative in such cases. My understanding of his constructive interpretation can be summarized in four parts.

First, the very act of formulating a decision problem under uncertainty is itself a decision process that reflects situationally specific factors of preference,

belief, economy, convenience, and the purposes and needs for decision in the first place.

Second, probabilities that enter into subjective expected utility calculations, or perhaps some other decision rule appropriate to the problem at hand, should be based on available evidence. I presume that this follows the spirit of Shafer (1981) and, to a lesser extent, Good (1950). For reasons that hinge on vagueness of preference and the practical problem of separating belief from value, Shafer finds the preference-oriented willingness to bet view of personal probability forwarded by Ramsey (1931), de Finetti (1937), and Savage unsuitable.

Third, utilities are deliberately adopted as a constructive measure of value, and not derived from preferences per se. In fact, the decision problem itself is likely to have arisen from a situation of indecision and vacillation in which preferences are vague or initially meaningless. In Shafer's view, people do not have preferences, they construct preferences.

Finally, the constructive measurement of probability and utility precedes the determination or computation of preferences between decision alternatives according to the subjective expected utility model or another model. (It is not clear to me what other models Shafer has in mind.) Indeed, one might use several models and perhaps even several formulations of the problem to test the robustness of derived preferences or best alternatives. As in the dialectic method, one might examine the problem from different perspectives before settling on a decision that seems right.

Aspects of Shafer's constructive approach are not altogether new. For example, in monetary situations Bernoulli proposed to measure the utility of wealth or return in an intensity of preference (or value) manner completely separate from considerations of risk or probability. He then combined this with probabilities to compute expected utilities, regarding it as obvious that a best alternative is one that maximizes expected utility. He does not talk explicitly about preferences between alternatives. A somewhat similar view is embraced by Allais, who assesses utility separately from probability through comparisons of preference differences between outcomes. Probability and utility are then merged in a holistic value function, but not by Bernoulli's expectation operation since Allais finds its independence implications normatively and descriptively untenable.

To begin a sketch of my understanding of Savage's intentions, it should be said first that he speaks eloquently for himself in *The Foundations* as well as in later work, among which I feel that Savage (1967) most accurately reveals his mature views. However, by way of commentary on Shafer's interpretations, I shall proceed.

Savage presented his formulation and axioms as a

normative ideal that hopefully might be approximated for realistic decision problems. Despite objections by Shafer and others, including Drèze (1961) and Jeffrey (1965), I believe that his separation of beliefs and values through states and consequences has been enormously useful in our attempt to understand decision under uncertainty.

Savage invented the notion of small worlds as a way of approaching practical application of his normative theory, but I believe he was never completely comfortable with his own analysis and hoped that others would work on the small worlds problem. It is to Shafer's great credit that he has tackled the problem, and one hopes that it will be pursued further.

My discomfort with Shafer's interpretations can be focused around Savage's representation theorem. In a formal vein, the theorem says that if a preference relation on a suitably rich set of acts satisfies certain postulates, then there is a utility function u on consequences and a probability measure π on events in S that satisfy (1) for all acts f, g, \dots in Savage's act set. In Section 2.3, Shafer gives the impression that Savage was only interested in the direction of going from preference between acts to utilities and probabilities, and while this is true in part, I think it misses important aspects of Savage's approach.

In an axiomatization such as Savage's, it is customary to treat the preference relation as an undefined primitive, endowed with certain extramathematical interpretations supplied by the author. One might, for example, view Savage's representation theorem as a means of discovering one's preferences by an approach not unlike that suggested by Bernoulli or Shafer that first measures utilities and probabilities and then applies the expectation operation. If the preference relation is then defined from (1) it will satisfy all of Savage's axioms (except perhaps P6, which requires an infinite number of states).

Although Savage did not advocate this reverse constructive direction, it is not entirely absent from his thinking. To justify this claim, I will first provide a brief quote from *The Foundations* (page 20) and then suggest that Savage maintained *two* interpretations of preference that must be considered if his intentions are to be clear. The quote: "...the main use I would make of P1 and its successors is normative, to police my own decisions for consistency and, where possible, to make complicated decisions depend on simpler ones." Shafer puts emphasis on the first part of this statement but virtually ignores its final clause. In so doing he misses a constructive theme in Savage's approach. One way the final clause could work for Savage is through direct application of his sure thing dominance principle. More to the point of Shafer's claims is Savage's frequent acknowledgment of vagueness or indecision in preference between complex acts,

and I have no doubt that Savage would not hesitate to use his representation in the constructive direction to clarify such preferences if he were first satisfied that the necessary pieces of u and π had been assessed accurately from simpler comparisons.

The two interpretations of preference that Savage maintained might be called *casual preference* and *consistent preference*. Casual preferences are intuitive first-impression judgments of the type described by Savage (1954, page 103, lines 1–4) for his initial reactions to an example from Allais (1953). On the other hand, consistent preferences are simply preferences that obey Savage's postulates. Since consistent preferences are the norm for Savage, he uses the term error in describing casual preferences that are inconsistent. Moreover, he recognized that casual preferences (including no casual preference by indecision) are often inconsistent and, as in the quote above, would use his theory to weed out inconsistencies. (Savage, 1967, is helpful on this point.)

It seems to me that this use of his theory has a constructive edge even if it differs from Shafer's use of the term. One begins with rather ill-defined preferences, and, by refinement and clarification based on the postulates as guidelines, attempts to arrive at a set of consistent preferences. If that ideal is in fact attained, u and π follow as in Savage's representation theorem. But even if it is not, aspects of u and π might be assessed that will help to discover consistent preferences for more complex comparisons.

Finally, one might note that the separation between beliefs and values that Savage attained as an ideal through his postulates for consistent preferences is achieved by Shafer through his constructive approach in the separate assessment of probabilities and utilities. If preferences between acts in Shafer's approach are then constructed (defined) from the subjective expected utility representation, one arrives at consistent Savage preferences.

It is a pleasure to thank the executive editor for this opportunity to comment on "Savage Revisited" and to express my gratitude to Glenn Shafer for his stimulating analysis.

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Comment

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I agree completely with Shafer that a coherent normative system of choice must be compatible with a realistic description of how people choose. “Ought” implies “is.” We do not recommend the impossible. But the observation that certain particular choices may be in conflict with a set of normative decision making principles (or ethical ones) does not lead us to abandon these principles automatically; to do so would be to identify the “ought” and the “is.” Instead, we look at the world of conflicting—and often confusing and incoherent—choice to determine whether there are empirical patterns consistent with the normative system we propose. I believe that by a rather selective choice of example Shafer has managed to obscure these empirical regularities; in particular, by treating choosing individuals as if they were “of one mind” about their decisions and decision making process, he has ignored the degree to which we *do* seek to make “policy choices,” the degree to which we experience conflict and attempt to resolve it by subordinating isolated desires and modes of thought to more general ones, and, most importantly, the empirically demonstrable degree to which we achieve our broad goals when we in fact succeed in making these policy

judgments, which he questions. I have five basic disagreements with his characterization of our decision making behavior.

1. In Section 2.3 Shafer writes: “It is almost always more sensible to construct preferences from judgments of probability and value than to try to work backward from choices between hypothetical acts to judgment of probability and value.” I agree. But why is that “sensible?” His advice is sensible due to the *empirical* findings (Dawes, 1979) that expert and nonexpert predictions made in that “decomposed” manner are superior to those made wholistically. And because preference is in part a prediction (of one’s future state of mind), then it is reasonable to suppose (Dawes, 1986) that preference judgments made in this manner will be superior as well—as a general rule, certainly subject to exceptions.

But the success of the decomposition procedure hinges on an ability to make such component judgments *across* individual choices, an ability the empirical research implies we possess. My hypothesis for explaining the empirical finding is that wholistic judgments in a context of implicitly comparing psychologically incomparable dimensions or aspects are much more difficult than are judgments about what dimensions and aspects predict and in which direction. (The decision analyst would include weighting them, but that goes beyond the empirical results.) We can be consistent and accurate if we ask ourselves the right question. It is the commitment and ability to make

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