Statistical Science
1987, Vol. 2, No. 4, 484–503

Survey of Soviet Work in Reliability

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Abstract. This survey is a review of Soviet studies in reliability theory. The stress is on the theoretical work developed in the Soviet Union during the last two decades. Some related work, although not by Soviet scholars, is also described.

Key words and phrases: Coherent system, failure rate, confidence bounds, optimization method, censored data.

1. INTRODUCTION

In the present survey we give a review of Soviet studies in reliability theory. We do not intend to cover all research works of Soviet scholars. Our stress is on the mathematically rigorous theoretical work and not on applications. Also, an attempt was made to give more attention to the latest research.

An early respectable reliability-related research study in the Soviet Union is due to Gnedenko (1943), who formalized the fundamental result in the theory of extreme values discovered by Frechet (1927) and Fisher and Tippet (1928). The main result states that if $X_{(n)}$ is the largest among n independent and identically distributed (iid) random variables, and if there exist sequences of constants $\{a_n\}$ and $\{b_n, b_n > 0\}$ such that $(X_{(n)} - a_n)/b_n$ has a limiting distribution G, then G must have one of three possible forms. An analogous result also holds for the smallest variables. (See Barlow and Proschan (1975) and Mann and Singpurwalla (1985) for more detailed discussion of these results.) Much of the latter development of reliability theory was continued by Gnedenko himself and his collaborators and students (Solovyev, Ushakov, Belyayev, Kovalenko and many others).

In view of the so-called scientific technological revolution, reliability theory started to develop in the sixties when the Soviets began to adopt large-sized and operationally complex technical systems, such as communication systems, fuel energy complexes, automated control industrial systems, information computational systems (multicomputer complexes), etc. Unclassified Soviet research in reliability theory seems to be greatly influenced by western, mainly American, studies. All major western monographs on reliability theory have been translated into Russian, and typicallly are out of sale despite a substantial number of edition copies. Fundamental notions of reliability, developed in the western world, such as coherent systems, distribution with monotone failure rate, optimal maintenance and control, standby items, different types of reserve, etc., found quick response from Soviet applied probabilists already familiar with queuing theory and quality control. It seems, however, that sometimes the results of queuing theory were applied mechanically in reliability problems giving formally correct but practically useless answers.

A comprehensive review of reliability theory is given in the *Handbooks on Reliability* by Kozlov and Ushakov (1970, 1975), the first of which quoted here is an English translation. There are also several review papers by Belyayev, Gnedenko and Ushakov (1983), Gnedenko, Kozlov and Ushakov (1969) and Levin and Ushakov (1965) that are dedicated to the state of art of reliability theory. More mathematically oriented is the monograph of Gnedenko, Belyayev and Solovyev (1969); various aspects of reliability are covered in books by Polovko (1965), Shishonok, Repkin and Barvinski (1964) and Shor (1962). A huge bibliography can be found in Gnedenko (1983).

Many important results have been obtained by Soviet scholars. Their research, however, is not always familiar in this country, and the present authors hope to bring it to the attention of more people.

The main Soviet research in reliability theory can be found in the English translations of the journals *Izvestiya Akademii Nauk SSSR Tekhnicheskaya Kibernetika* [*Soviet Journal of Computer and System Sciences* (formerly called *Engineering Cybernetics*)], *Avtomatyika* (*Soviet Automatic Control*) and *Kibernetika* (*Cybernetics*). Some articles are published in *Kybernetes* (in English), an International Journal of Cybernetics and General Systems.

To help the readers to identify Soviet works, we have divided the references at the end of this paper into two parts. Part I contains works of Soviet authors, and Part II contains works of non-Soviet authors.

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2. COHERENT SYSTEMS

In this section we discuss the problem of determining the reliability of coherent systems introduced by Birnbaum, Esary and Saunders (1961) and developed in Barlow and Proschan (1975). This notion had an enormous impact on Soviet work in reliability, and there is a vast number of papers inspired by this idea.

Consider a system with N components which are labeled from 1 to N. We associate with the ith component a binary variable x_i such that, at any specified time, $x_i = 1$ if the ith component is functioning and $x_i = 0$ otherwise. Let $\mathbf{x} = (x_i, \cdots, x_N)$. The structure function of the system ϕ is a function of $\mathbf{x}$ such that $\phi(\mathbf{x}) = 1$ or 0 depending on whether the system is functioning or not functioning at that moment. A system is said to be coherent if its structure function ϕ is such that $\phi(0, \cdots, 0) = 0$; $\phi(1, \cdots, 1) = 1$; and $\phi(x_1, \cdots, x_N)$ is nondecreasing in each argument. In other words, a coherent system is a system that is not functioning if all of its components are not functioning, is functioning if all of its components are functioning and if it is initially functioning, remains functioning whenever some initially failed components are restored to functioning states.

If the lifetimes of the N components of a coherent system are statistically independent and T denotes the lifetime of the system, then the system reliability at time t is given by

$$R(t) = P[T > t] = \sum_{\{\mathbf{x}:\phi(\mathbf{x})=1\}} \prod_{i=1}^{N} [x_i p_i(t) + (1 - x_i) q_i(t)],$$

where $p_i(t)$ is the probability that the ith component is still functioning at time t, and $q_i(t) = 1 - p_i(t)$. In particular, if the ith component has an exponential life distribution with $p_i(t) = \exp(-\lambda_i t)$ for $t > 0$, then

$$R(t) = \sum_{\{\mathbf{x}:\phi(\mathbf{x})=1\}} \prod_{\{i\in A(\mathbf{x})\}} \exp(-\lambda_i t) \times \prod_{\{j\in B(\mathbf{x})\}} [1 - \exp(-\lambda_j t)],$$

where $A(\mathbf{x}) = \{i: x_i = 1\}$ and $B(\mathbf{x}) = \{j: x_j = 0\}$. Burtin and Pittel (1972a) performed an asymptotical study of this formula when $\lambda_i = \overline{\lambda}_i\theta$ and θ is small. They showed that if $\Lambda_\theta(t) = -R'(t)/R(t)$ is the system failure rate, then as $\theta \to 0$,

$$\Lambda_\theta(t) = \left[k\theta^k t^{k-1} \sum_{\{\mathbf{x}\in D\}} \prod_{\{j\in B(\mathbf{x})\}} \overline{\lambda}_j\right](1 + o(1)),$$

where $k = \min_{\{\mathbf{x}:\phi(\mathbf{x})=0\}} n(\mathbf{x})$, $n(\mathbf{x})$ is the number of 0's in $\mathbf{x}$, and $D = \{\mathbf{x}: \phi(\mathbf{x}) = 0 \text{ and } n(\mathbf{x}) = k\}$. The implication of the above result is that when θ is small, the system failure rate is approximated by the failure rate of a Weibull distribution with cumulative distribution function: $1 - \exp(-\mu t^\alpha)$ with $\alpha = k$ and $\mu = \theta^k \sum_{\{\mathbf{x}\in D\}} \prod_{\{j\in B(\mathbf{x})\}} \overline{\lambda}_j$.

The reliability of a coherent system can also be calculated through minimal paths or minimal cuts representations. A path set is a set of components such that the functioning of these components ensures the functioning of the system. A minimal path set is a path set such that all of its components must be functioning to ensure the functioning of the system. Similarly, a cut set is a set of components such that the failure of these components ensures the failure of the system. A minimal cut set is a cut set such that all of its components must fail to ensure the failure of the system. Let $P_1, \cdots, P_l$ denote all possible minimal path sets, and $C_1, \cdots, C_k$ denote all possible minimal cut sets of a coherent system with structure function ϕ. Then ϕ can be represented as a parallel-series or a series-parallel structure:

$$\phi(\mathbf{x}) = \begin{cases} 1 - \prod_{i=1}^{l} \left(1 - \prod_{j\in P_i} x_j\right) \\ \prod_{i=1}^{k} \left[1 - \prod_{j\in C_i} (1 - x_j)\right]. \end{cases}$$

The system reliability at time t is $E\phi(\mathbf{x})$, the expected value of $\phi(\mathbf{x})$. The expectation can be evaluated term-by-term with the help of the identity $x_j^2 = x_j$, after the right side is expanded into a sum of products of x_j's.

If a structure consists of a large number of components, the determination of all minimal path series structures and of all minimal cut parallel structures can be difficult. The knowledge of these substructures is however not needed to obtain lower and upper bounds for the reliability, so that it seems rather natural to eliminate minimal path series structures that are formed by a large number of components and that have a small probability of functioning. Similarly, it seems reasonable to exclude minimal cut parallel structures with a large number of components and high reliability. In this way, Litvak (1981) has obtained lower and upper bounds for the reliability of a complex system by using smaller sets of minimal path series structures and minimal cut parallel structures that do not have common components. These bounds turn out to be almost exact in many practical situations (for instance, they are exact for a bridge structure). Litvak (1979) and Ushakov and Litvak (1977) have used the mentioned bounds for reliability in the case of other physical characteristics such as capacity, resistance, transportation cost, etc. They considered using methods of graph theory, a two-pole network of an arbitrary structure with given mutually independent estimates of its parameters. In particular cases, the known Ford-Fulkerson theorem about the maximum flow in a two-pole network and earlier estimates of Esary and Proschan were obtained. Litvak and Ushakov (1984) have also considered different methods to estimate the characteristics of two-terminal networks of arbitrary physical nature. The main difficulty here is due to the fact that reliability calculations for structures, which cannot be represented as a superposition of simpler (parallel-series or series-parallel) substructures, are typically very difficult. Thus it is desirable to develop simple estimates of the parameters of complex networks that require less computations than optimal estimators. Some principles for obtaining these estimators are discussed by these authors with applications to estimation of the characteristics of networks with elements of two possible types. Litvak (1974) has used the technique of Boolean algebras to obtain an exact formula for the reliability of a system represented in parallel-series or series-parallel form.

Lomonosov and Polesskii (1971, 1972) considered an information network with n nodes and a set of channels connecting these nodes. Every channel can be in two states, independent of the others: operative (with probability p) and inoperative (with probability $q = 1 - p$). Inoperative channels may cause the network loss of connectivity, i.e., signals cannot be transmitted from one end of the network to the other. The network is said to be connected if signals can be transmitted successfully, through operative channels, from one end to the other. At a specified time, the reliability R of such a network is the probability that the network is connected. They obtained the following two-sided bounds on R:

$$n(1 - q^{r/2})^{n-1} - (n - 1)(1 - q^{r/2})^n \le R \le \sum_{k=1}^{n-1} (1 - q^{v_i}),$$

where r is the number of edges in the smallest cut set, $v_i = |C_i|$, and $\{C_i, i = 1, \cdots, n - 1\}$ is a collection of cut sets which form so-called cut basis. The bounds are also sharp in the sense that there are networks of special configurations for which the inequalities turn into equalities. Clearly, by replacing p by the channel reliability function $R(t)$ one can obtain useful asymptotic approximations to the network reliability when, for example, the channels are highly reliable.

Genis (1985a, 1985b) obtained two-sided bounds on the reliability of a renewable standby system, and considered an estimate of the failure rate of the same system. For other kinds of reliability bounds one can refer to Barlow and Proschan (1975).

Ushakov (1960) considered systems with several possible levels of functioning. The characteristic of functioning quality is a linear function of the reliabilities of all components, which can be used in optimization problems under maintenance restraints. Netes (1980) used the decomposition method to estimate the level of functioning (effectiveness) of a system. Gadasin (1973) and Gadasin and Lakaev (1978, 1979) have obtained formulae for various characteristics of reliability for a special class of coherent structures (so-called recurrent structures). In particular they considered systems with a grid structure where a connection between two elements is achieved by retranslation of a signal along communication channels through intermediate faulty elements. The problem is to determine the probability of connectedness of a fixed set of objects. This probability is a complicated function of the reliability parameters, and it was suggested to estimate it by taking several first terms in the multivariate Taylor expansion. Because there is no convenient analytical expression for this function, the derivatives are evaluated numerically at the point where the parameters vanish. A general study of recurrent structures (like railways or highways, oil or gas pipelines, electrical systems) can be found in the monograph of Gadasin and Ushakov (1975) where a number of new analytical and algorithmic results were obtained. In particular, isotropic systems with a network structure close to that of a planar graph were investigated.

Notice that for complex engineering systems even the notion of failure or reliability is difficult to formulate. A monograph of Dzirkal (1981) is dedicated to methods of assigning the operational efficiency of such systems.

Books of Cherkesov (1974) and Kredenster (1978) deal with so-called systems with time redundancy. Possessing essentially two states "up" or "down" with respect to functioning of the structure at a given time, these systems may or may not implement the preassigned task, depending on the nature of the whole trajectory of transitions from one state to another. For example, if there is time to carry out an operation then failures of the system during its operation process may require only that one implements once again either the whole operation or part of the operation. A similar situation may occur if the system becomes "inertial" in a sense and allows failure periods not exceeding a given time period. Such systems are most common for information and computational devices where restarting is possible during the operation, and storage (batteries) are available for electronic computers in the event of short-time interruptions of the power supply. The paper of Mikadze (1979) is dedicated to the analysis of reliability of computational systems with time redundancy in the presence of failures and malfunctioning.

Gnedenko (1964a, 1964b) had initiated the study of systems that have an idle backup device that is put into operation when the main device fails. The distribution of failure-free time has been studied under different assumptions. In the case of preventive maintenance, Gnedenko and Makhmud (1976) showed that there exists a unique period for preventive maintenance operation that leads to maximal failure-free duration. They considered a maintenance policy under which the operating element of a duplicated system continues to work until a failure of the second element while the first is undergoing preventive maintenance. The existence of the time period t_0 such that, for performance time t of preventive maintenance operations less than t_0, the use of preventive maintenance causes a decrease in the mean duration of failure-free operation of the system, generalizes previous results of Osaki and Asakura (1970).

Kabashkin (1984) considered systems, the behavior of which is described by a stationary Markov process, with the graph of state transitions decomposed into two subgraphs. He suggested a method to calculate the stationary probabilities without solving simultaneous equations. Kartashov and Shvedova (1983) offered an approximate method of evaluating the reliability of objects chosen from a control lot in a prescribed way. This method is based on the assumption that changes over time of the parameters of a system can be regarded as a Markov process.

The need to estimate the reliability of highly reliable systems along with the need for simpler tractable formulae resulted in the development of various asymptotic methods. An excellent review of these methods (based mainly on Soviet work) is given by Gertsbakh (1984). For instance, for practical purposes it is important to estimate the difference between the lower and upper bounds (the error of reliability evaluation). Kovalenko (1975) had shown that this difference is bounded by the tail probability of a Poisson distribution. This fact can be explained by the majorization of the process describing the evolution of the system by a Poisson process with parameter $\sum \lambda_i$.

Solovyev (1971) introduced a sequence of regenerating processes depending on a parameter that influences the behavior of the process in such a way that the probability of failure during a single regeneration period tends to zero. He obtained interesting theorems concerning the convergence of the distribution of properly normalized failure time to the exponential distribution. Similar results have been obtained by Gnedenko and Solovyev (1974, 1975) for various models of standby with renewal. Their methods allow estimation of the reliability of complex systems with possible interaction between components. A different method for describing complex systems with renewal is the consolidation of states (see Korolyuk and Turbin, 1978). Besides reducing the dimensionality, this method provides a very convenient representation of a complicated semimarkovian process with a large number of states by a Markov process with a smaller number of states.

Kalashnikov (1969) used the direct Lyapunov method to estimate the reliability of a redundant system with constant repair time of failed items giving special consideration to the cases in which the number of repair positions is smaller than the total number of items. He obtained some asymptotic estimates of reliability as time increases.

The reliability of a "k-out-of-n" system has been well studied for identical element reliabilities, but little work has been done for the case in which the elements have different reliabilities. Zhegalov (1986) developed a method of calculating the reliability of this kind of system which avoids complete enumeration of all possible states of the system.

The problems of statistical simulation of the functioning of complex systems have been initiated in the USSR by Buslenko (1976, 1978). Reliability aspects of these simulations have been investigated by Gorskiy (1970) and by Groysberg (1981). In particular these authors compared the existing methods of confidence estimation of reliability characteristics (the plane method, the substitution method and the heuristic method) and determined conditions under which each method gives the best results. For instance, the heuristic method works best for reliability block diagrams of the series type. To increase the efficiency of reliability estimation for redundant systems in which failures are rare, a combined method based on joint use of known procedures was suggested.

Simulations of the operational process of highly reliable systems evoked the need to develop accelerated simulation (Monte Carlo) methods. Kovalenko (1976, 1980) contributed to the solution of this problem by proposing a version of a small parameter method for calculation of the characteristics of a Markov chain. Lubkov (1980) gave recurrent formulae for an algorithm to simulate failures of components of technological systems. This algorithm is effective in estimating the reliability of highly reliable standby systems by simulation.

Summing up we see that this branch of reliability theory is very well developed in the USSR with a number of important and original contributions.

3. SYSTEMS WITH MONOTONE FAILURE RATE

For many engineering systems, especially for those having mechanical components, certain reliability characteristics decrease due to the deterioration or "aging" of elements. In the Soviet Union, the notion of aging has been introduced by Solovyev (1965), Ushakov (1966) and Solovyev and Ushakov (1967). The effect of aging can be formulated in terms of the behavior of the hazard function, namely the conditional probability density of failure given that no failure has occurred up to the moment under consideration. For aging elements the hazard function is monotonically nondecreasing. This property of the hazard function imposes restrictions on all the moments of the distribution; this fact can be used to obtain useful bounds for the whole family of distributions. However, except for the above-mentioned authors, there are relatively few Soviet specialists working in this field. There are no reports of whether a given complex system possesses a monotone failure rate. The assumption of an exponential distribution of lifetime seems to be too widely accepted.

Ushakov (1966) has constructed the following upper and lower bounds for the probability P that the system is functioning during a given time period t_0:

$$K(1 - t_0/T) \leq P \leq K \exp(-t_0/T).$$

Here T is the mean time between failures, t is the mean time of regeneration and $K = T/(T + t)$. For small values of t_0, these bounds are close to one another, and for the evaluation of P in this case, it suffices to know only the expected time between the failures, not its distribution.

One of the basic questions of finding the reliability of a system with components whose failure rate is monotone was addressed by Solovyev and Shakhbazov (1981). These authors have obtained lower and upper bounds for the average working time of series and parallel systems formed by aging components with given expected values of lifetimes.

In the book of Bolotin (1971) some probability models of the crossing of a certain level by a stochastic process were used to describe the failure processes of mechanical parts (see also Konenkov and Ushakov, 1975). A number of mathematical models of failure are considered in the monograph of Gertsbakh and Kordonskiy (1969), which has more references to (rather inaccessible) Soviet papers.

A related subject is the study of accelerated testing. These methods are especially important when direct verification of reliability under normal working conditions is practically impossible because of the large number of tests needed or for some other technical reasons. A number of different models for accelerated failure processes and corresponding mathematical techniques are given by Perrote, Kartashov and Tsvetayev (1968) and Kartashov (1979).

We also mention here some physical principles suggested by Russian scholars as the basis of reliability theory. They are the "Sedyakin principle," the "heredity principle" and the "least action" principle.

The Sedyakin principle, suggested by Sedyakin (1966), states that the reliability of a system under conditions ε depends only on the amount of resource (cumulated hazard) developed by it in the past, and not on how this resource was developed. Let $\lambda(z, \varepsilon)$ denote the failure (or hazard) function of a system operated under conditions ε, and let x_1 and x_2 be the lengths of the intervals of operation time for the system operated under conditions of ε_1 and ε_2. According to Sedyakin's principle, if x_1 and x_2 satisfy the integral expressions

$$r = \int_0^{x_1} \lambda(z, \varepsilon_1)\, dz = \int_0^{x_2} \lambda(z, \varepsilon_2)\, dz,$$

then the probability $p(t)$ of trouble-free operation of the system during $[0, t]$ depends on x_1, ε_1, x_2 and ε_2 only through r, and

$$p(t) = p^{(1)}(t \mid x_1) = p^{(2)}(t \mid x_2),$$

where $p^{(i)}(t \mid x_i)$, $i = 1, 2$, is the conditional probability of trouble-free operation of the system during $[x_i, x_i + t]$ given that the system remains operative at time x_i under conditions ε_i. This principle was shown to be in accord with experimental data obtained from an accelerated life-testing of light bulbs. It also can be used to construct reliability models for a broad class of systems.

The "heredity principle" was suggested by Kartashov and Perrote (1968), and states that the production process can change the values of the internal parameters of a system, but cannot disturb the functional relationships between them. This principle makes it possible to formulate many reliability problems in a rigorous mathematical language and to obtain recommendations for constructing methods of accelerated testing. On the basis of the heredity principle, Kartashov and Perrote also concluded that Sedyakin's principle can take place for products of one type and may not be satisfactory for products of another type. Using the "heredity principle," Kartashov (1971) studied accelerated testing in the case of an unstable production process. He obtained conditions under which it is possible to estimate, from the results of accelerated testing, certain numerical characteristics of the tested units.

The "least action" principle was suggested by Perrote and Yavriyan (1978). It is formulated in connection with the problems of resource consumption of a system. This principle says that the motion of a system along any coordinate of the complete set of parameters takes place along the trajectory, corresponding to the minimum of an integral obtained from the Lagrange function of the system. The "least action" principle can be used to predict a number of properties that objects operating in variable regimes can possess.

4. OPTIMIZATION METHODS OF RELIABILITY

Accounting for various restrictions on resources (for instance, cost limitations) results in a number of problems of conditional (linear or nonlinear) optimization. As a rule the common features of these problems are the complexity of structural restrictions, large dimensionality and complicated target functions. The Soviet research in this field seems to be influenced greatly by dynamic programming, optimal inventory problems and queuing theory.

For example, the traditional problems of optimal inventory are treated now as problems of supplying spare parts for complex systems. The supply process has a hierarchical structure and involves a complex rule of replenishing at each stage (see Ushakov, 1969; Shura-Bura and Topolskiy, 1961; Rubalsky and Ushakov, 1976). These problems are closely related to classical problems of inventory theory when the items are assumed to be withdrawn continuously while the supply is discrete (Rubalsky, 1977). Paramonov and Savvin (1978) examined the problem of determination of an assigned standby for objects on the basis of experimental results. They offered an adaptive method to calculate the necessary time of service. Dzirkal and Shura-Bura (1980) have suggested a model of functioning of standby units, which was used to obtain a computational scheme to calculate the reliability of the standby group with uncertain reswitching under general assumptions about monitoring of the state of the standby components. Rubalsky (1984) has obtained a procedure that optimizes the standby stock when a part of the rejected product is duplicated and a part is repaired.

A study of different maintenance and replacement models in the reliability theory of engineering systems has been performed in a series of books by Raykin (1967, 1971, 1978). The optimality of spare parts allocations for electronic devices was studied by Kulback (1970). The Lagrange multiplier method has been used by these authors to obtain exact and approximate formulae for the optimal number of spare parts. Under different forms of budget restraints, the best allocations of spare parts were found, and some constructive algorithms (essentially of Kettele type) for their practical implementation were suggested. In the case of a problem with a large number of restraints, Demin and Malashenko (1974) suggested the use of the dual problem of linear programming. In a sense the dual problem of optimal spare parts allocation with several restraints is the optimization problem of multifunctional systems under one-sided conditions (see also Karshtedt and Kogan, 1971).

Ushakov and Yasenovets (1978) studied two limiting cases. In the first case, the functioning time of the system is considerably smaller than the average functioning time of any components. In the second case this functioning time is much larger. Under a budget constraint in the first case, one should distribute spare parts uniformly among the components. In the second case, if T_i is the average lifetime of the ith unit, the optimal number x_i of spare elements of type i, is approximately equal to

$$x_i \sim t_0/T_i + 1,$$

where t_0 is a large fixed number.

Brodetskiy (1978, 1980, 1984) has studied systems with two types of failure (with and without erasing information). To improve the quality of such systems, intermediate results are stored in a device that prevents the erasure of data. If a failure leading to erasure occurs, the process is resumed from the point of the last data storage. In this problem it is found that for optimal control of system operating time, the intermediate results must be stored periodically. These results are extended to the case when the initial task can be interrupted in order to execute another task with higher priority.

Ushakov and Yasenovets (1977) considered a new version of the optimal maintenance policy on the basis of limited funds. Assume that the spare items can be obtained until a given sum C_0 is spent. During this stage, one observes the moments of replacement and the types of failed items. Under resource restrictions, an optimization of some functional (like mean time of functioning) is solved by methods of integer programming. One of the methods of integer programming (so-called branch-and-bound method) has been used by Tatashev and Ushakov (1981) to find an optimal algorithm of switching standby elements according to a given timetable.

Barzilovich and Kashtanov (1975) considered the problem of optimal preventive replacement when the information concerning reliability is incomplete. The latter circumstance causes drastic complications of the mathematical formulation and solution of the problem. Churkin (1984) considered the problem of the optimal turning on of a symmetric standby device, when the strategy of turning on and off is such that the length of the queue, when the standby device is disconnected, is one unit shorter than that at which it is turned on. He found an optimal procedure that takes into account the expenditures associated with turning the standby device on and off, the operation cost and the transition time from one state to another.

The passage times from one state to another by a birth and death process is of interest in reliability theory where the behavior of storage systems with replacements are described by these processes. These times can be interpreted then as the periods of functioning without a failure. Solovyev (1972) obtained the exact distributions for the moment of first crossing. These formulae are rather cumbersome, so the asymptotic distribution of properly normalized first passage moments is shown to be of the form $1 - ae^{-x}$, $x > 0$, $0 < a \leq 1$. The theory of random processes also has been used by Burtin and Pittel (1972b) for a joint study of the aging and functioning process of an unreliable system. Genis and Ushakov (1983) offered a simple method for the optimal choice of the number of standby units in multipurpose systems. They solved the minimization problem of weighted expenditures of resources under a restriction on reliability characteristics and the maximization problem of weighted overall performance under limitations on the expenditure.

Pashkovskiy (1981) has obtained some results concerning engineering diagnostics. In particular, he developed the so-called recursive method that permits the construction of optimal diagnostic procedures for the case of a complex structure of inspection tests and also allows one to select a battery of tests out of a collection of all possible tests to carry out the inspection.

Related to the problems of optimal inventory are the problems of dynamic inventory (see Mandel and Raykin, 1967; Konev, 1974; Ushakov, 1981). In these problems it is assumed that a number of spare items is provided and these spare items function under lightened conditions so that their failure rate is smaller. It is possible to switch the spare items on into normal working conditions. Clearly the switching on of all items at the beginning causes all of them to begin using up the resource. On the other hand, excessively economical switching on of the elements in the first stages of the operation may lead to failure in these stages and to curtailment of further functioning of the system. Thus, heuristically there must be an optimal switching on strategy.

Gertsbakh (1966, 1970) studied standby control problems in the situation when the true condition of the system is unknown and can be tested only at given time moments. The number of elements included in the hot reserve after each check-up is selected so as to minimize the probability of the system failure during the given operation time. Related problems along with a good list of references can be found in the book of Gertsbakh (1977).

5. STATISTICAL AND MATHEMATICAL PROBLEMS OF RELIABILITY THEORY

The direction of Soviet work in the statistical aspects of reliability theory has been determined by the book of Gnedenko, Belyayev and Solovyev (1969) mentioned earlier. The statistical methods used here are mainly confidence intervals of rather complicated parametric function and classical (unbiased or maximum likelihood) estimators of such functions. A study of the robustness of these procedures by and large has not been undertaken, although Chepurin and Dugina (1970) have studied stable estimators for Weibull and log normal distributions. Methods of applied multivariate analysis or data analysis have not been used in statistical reliability problems.

The work on confidence intervals has been initiated in Mirnyi and Solovyev (1964) and in Belyayev, Dugina and Chepurin (1967) and was developed further by Pavlov (1982), Sapozhnikov (1970), Sudakov (1974, 1980) and Tyoskin (1979). Confidence limits for reliability have been obtained for different models. Confidence sets for the parameters of monotone failure rate distribution are constructed by Pavlov (1980), who gave conditions under which interval estimates of several unknown parameters can be substituted in the parametric function to be estimated. Interval estimation of the reliability of complex systems by results of reliability testing of its components is a particular case of this problem. The practical importance of these problems is due to the fact that complex systems cannot be accurately tested for their reliability because of the constant change in their structure.

Sonkina and Tyoskin (1984) constructed confidence limits for the probability of failure-free operation of a series system with Weibull distribution of the failures. Kaminsky (1984) obtained nonparametric confidence intervals for quantiles of the aging distributions with the aim of choosing the optimal number of first failures so as to maximize the mean value of the lower confidence limit. Related parametric problems were investigated earlier by Paramonov (1975), who considered the estimation of a percentile on the basis of a small number of life tests. The best invariant estimates were derived for several location-scale parametric families including Weibull, log normal and normal distributions. Notice that the practical importance of the latter distribution in reliability theory is not really significant. Yet there are many papers in the Soviet literature where statistical reliability motivated problems are studied under normality assumption.

Various statistical problems arise in the processing of data on reliability of items after special experiments or operations. Belyayev (1982) proposed a class of experimental designs for censored data for which one can use methods of sequential analysis. He also suggested the use of the Bayesian approach to deal with data on the operation of items with "aging" distribution.

In the textbook of Belyayev and Chepurin (1983) one chapter is devoted to the construction of isotonic estimators of the hazard function that could be used for the analysis of reliability data based on results of shock testing.

Belyayev and Khafid (1984) studied the behavior of the posterior density of reliability parameters. Exponential and Poisson distribution models have been investigated in detail. It was shown that the convergence of the Bayes modal estimator to the true parametric value is described by a random process close to a Gaussian one. A different Bayes approach was taken up by Penskaya (1984), who considered an empirical Bayes estimation of the reliability function when the prior density is unknown but can be estimated on the basis of previous experiments. In a recent paper, Savchuk (1986) suggested a method for nonparametric Bayesian estimation of an aging-in-the-mean distribution $F(t)$. The method is based on piecewise linear approximation of the cumulative hazard function

$$\Lambda(t) = \int_0^t \lambda(u)\, du = -\ln[1 - F(t)],$$

where $\lambda(u)$ is the failure rate. This method is applicable to both complete and censored data.

Groysberg (1980), on the other hand, considered the fiducial approach to reliability estimation problems. This approach reduces the problems to that of finding a distribution function having random arguments with known probability distributions, a solution of which can be obtained by a statistical simulation on a computer.

In many physical models of the accumulation of breakdowns, it is reasonable to assume that the breakdowns occur randomly and independently of the previous history of breakdowns in an object. In this case the process of accumulation of breakdowns is an additive Markov process. With a suitable choice of internal parameters, the degradation of the object can be described by the parametric failure whose process of change will be an additive process. Matveyev (1978) considered methods of estimating the probability of parametric failure when the degradation of the internal parameters can be regarded as an additive Markov process with continuous realization. The methods discussed there include the maximum likelihood, the least squares and the weighted least squares estimates.

Some results on recurrent estimation of reliability from one experiment to another are obtained in Barzilovich (1983). Karapenev (1978) had studied a system with components and a repair device in which a failure of a component is not detected immediately but only after some lag. Under the assumption that the rate of repairing is much greater than the rate of component failure, he obtained an asymptotic estimate of the system reliability. The statistical aspects of optimal control have been investigated by Rosenblit (1973) under the assumption that the model contains unknown parameters. Explicit formulae for estimates of optimal strategies were obtained.

Klebanov (1978) has considered the following characterization problem: Let us consider two systems A and B in series, where A has n components and B has k components, $n > k$. If the lifetime distribution of each component is $F(t)$ then the reliabilities of A and B at time t are given by $(1 - F(t))^n$ and $(1 - F(t))^k$, respectively. Let $T = T(t, F)$ denote the time such that the reliability of B at T is equal to the reliability of A at t. What can be said about F if T is given? A characterization of the exponential distribution by the property that $T(t, F) = at$, $a > 1$ is derived.

Genis (1978) otained the rate of convergence to an exponential distribution that is the limiting distribution of a random variable with rational Laplace transform. This situation occurs in systems with fast servicing, for example, in automatic control systems. Obretnev (1977) characterized the exponential distribution in the class of all increasing failure rate distributions by the property that $(EX)^2 = \mathrm{Var}(X)$ or, equivalently $\mu_2 = 2\mu_1^2$. Azlarov and Volodin (1981) have studied the stability of this characterization and obtained the following estimate that holds for any IFR distribution function $F(x)$:

$$\sup_{x>0} | 1 - F(x) - e^{-x/\mu_1} | \leq [2(1 - \mu_2/(\mu_1^2))]^{1/2}.$$

Various other characterizations of the exponential distribution related to the lack of memory property or some other reliability motivated properties were obtained by Klebanov (1980).

Another important area of research in reliability theory is the problem of dealing with censored data. Censored data arise often in clinical trials and in machine lifetime testing experiments. For example, in a medical follow-up study, some patients of the study may drop out unexpectedly, or the study itself must conclude at a prespecified date. In a machine-testing experiment, some of the objects may be removed from testing according to already established rules or on the basis of information regarding the results of the test. The problem here is to estimate lifetime distributions based on incomplete (censored) data. Serious western studies in this area started when Kaplan and Meier (1958) reported their product-limit estimator, and the research became even more active after Efron (1967) and Breslow and Crowley (1974) established the consistency and asymptotic normality of the estimator. In the Soviet Union, there have also been some theoretical developments in this area.

Artamonovskii and Kordonskii (1970) considered maximum likelihood estimators (MLEs) of distributions with location and scale parameters, on the basis of k-grouped samples obtained from the simple inspection of machines. It is assumed there that all the machines in question have iid lifetimes, and each has distribution of the form $G((x - a)/b)$. In the jth sample, there are N_j machines. One observes only the number m_j of machines that failed before an independently determined time T_j, $j = 1, \cdots, k$. Based on the realizations $\{m_j, T_j, N_j\}$, they have obtained a criterion for the existence and the uniqueness of the MLEs.

Ushakov (1980), on the other hand, did not assume the form of the distribution and suggested a nonparametric estimate of the distribution function—the same problem studied by Kaplan and Meier (1958). Here we have n machines, which are put on life testing. Let x_j be the actual lifetime of the jth machine, $j = 1, \cdots, n$. The x_j's are iid, and have the same cdf F. Associated with the jth machine, there is an independently determined censoring variable y_j, $j = 1, \cdots, n$, where the y_j's are iid and have the same known (or unknown) cdf G. For the jth machine one observed only $\min(x_j, y_j)$, and knows whether the minimum is the actual failure time x_j or the censoring time y_j. Ushakov's approach and formulae for estimating F are different from that of Kaplan and Meier's product-limit estimator. It is interesting to note, based on the present authors' few calculations, that these two methods produce the same numerical results. Kaplan and Meier's work was not mentioned in Ushakov's paper.

In a similar problem concerning censored data, Pavlov and Ushakov (1984) obtained unbiased estimators, also in a nonparametric setting, for some sampling schemes. Recently, Belyayev (1985) used the methods of martingale theory developed by Aalen (1978), Gill (1980, 1983) and others to prove the consistency of the Kaplan-Meier estimator, a property that was rigorously proved by Földes and Rejtö (1981). This author also developed confidence bounds for the reliability functions of systems under repair or the replacement maintenance policy.

6. FURTHER REMARKS

Soviet scholars have made many important and original contributions in reliability theory, yet many important areas of statistical analysis have not received much of their attention. They seem to have more interest in reliability estimation problems, and less interest in statistical properties of their suggested estimators or in hypothesis testing problems.

The weakest point of the Soviet reliability research, from the authors' point of view, is the lack of statistical motivation for mathematical models that allow the evaluation of reliability characteristics. Such models are typically accepted before the system is actually constructed and their reliability is calculated on the basis of the model without checking it against actual failure data. Problems of model identification, goodness-of-fit tests and model selection procedures were rarely published in Soviet journals.

Some Soviet researchers seem to be unaware of western studies related to their work. In many of the Soviet papers that we surveyed, the authors failed to discuss or even mention related influential results that had been previously published in major western (English language) journals.

ACKNOWLEDGMENTS

The article is an expanded version of "Survey of Soviet Work in Reliability," written by Andrew L. Rukhin, Technical Report No. 373, April 29, 1986, Department of Statistics, Stanford University, Stanford, California. The original technical report was prepared under Contract N00014-76-C-0475 (NR-042-267) for the Office of Naval Research. The authors wish to express their gratitude for many helpful remarks to Ilya Gertsbakh and Boris Pittel. They also wish to thank Herbert Solomon, Ingram Olkin and Morris H. DeGroot for their encouragement to write and revise the technical report.

REFERENCES

Part I: Soviet Authors

ARTAMONOVSKII, V. P. and KORDONSKII, KH. B. (1970). Estimate of maximum likelihood for simplest grouping of data. *Theory Probab. Appl.* **15** 128–132.

AZLAROV, T. A. and VOLODIN, N. A. (1981). On closeness to exponential distribution with monotone failure rate (in Russian). *Teor. Veroyatnost. i Primenen.* **26** 665.

BARZILOVICH, E. YU. (1983). *Problems of Mathematical Reliability* (in Russian). Radio i Svyaz Press, Moscow.

BARZILOVICH, E. YU. and KASHTANOV, V. A. (1975). *Organization of Service Under Limited Information about System Reliability* (in Russian). Soviet Radio Press, Moscow.

BELYAYEV, YU. K. (1982). *Statistical Methods in Reliability Theory* (in Russian). Znaniye Press, Moscow.

BELYAYEV, YU. K. (1985). Multiplier estimates of the probability of failure-free operation. *Soviet J. Computer Systems Sci.* **23** (5) 107–121.

BELYAYEV, YU. K. and CHEPURIN, YE. V. (1983). *Foundations of Mathematical Statistics* (in Russian). Moscow State Univ. Press, Moscow.

BELYAYEV, YU. K., DUGINA, T. N. and CHEPURIN, YE. V. (1967). Calculation of lower confidence bounds on the probability of faultless performance of complex systems. *Engrg. Cybernetics* **5** (2) 52–59.

BELYAYEV, YU. K., GNEDENKO, B. V. and USHAKOV, I. A. (1983). Mathematical problems in queuing and reliability theory. *Engrg. Cybernetics* **21** (6) 62–69.

BELYAYEV, YU. K. and KHAFID, M. S. A. (1984). Asymptotic properties of the posterior densities of parameters in tests for reliability. *Engrg. Cybernetics* **22** (1) 1–16.

BOLOTIN, V. V. (1971). *Applications of Methods of Probability Theory and Reliability Theory in Construction Calculations* (in Russian). Stroyizdat Press, Moscow.

BRODETSKIY, G. L. (1978). The efficiency of entering intermediate results in systems with failures that destroy information. *Engrg. Cybernetics* **16** (6) 75–81.

BRODETSKIY, G. L. (1980). The question of optimal organization of the entering of intermediate information when there are random systems errors. *Engrg. Cybernetics* **18** (2) 61–65.

BRODETSKIY, G. L. (1984). Optimization of the information storage time in case of random task interrupts. *Soviet Automat. Control* **16** 64–68.

BURTIN, YU. D. and PITTEL, B. G. (1972a). Asymptotic estimates of the reliability of a complex system. *Engrg. Cybernetics* **10** (3) 445–451.

BURTIN, YU. D. and PITTEL, B. G. (1972b). Semimarkov decisions in a problem of optimizing a checking procedure for an unreliable queuing system. *Theory Probab. Appl.* **17** 472–492.

BUSLENKO, N. P. (1976). Complex systems and simulation models. *Cybernetics* **12** 862–870.

BUSLENKO, N. P. (1978). *Simulation of Complex Systems* (in Russian). Nauka, Moscow.

CHEPURIN, YE. V. and DUGINA, T. N. (1970). On a class of stable statistical estimates in reliability theory. *Engrg. Cybernetics* **8** (1) 65–72.

CHERKESOV, G. N. (1974). *Reliability of Engineering Systems with Temporal Redundancy* (in Russian). Soviet Radio Press, Moscow.

CHURKIN, V. I. (1984). The problem of optimal instants for turning on a standby device. *Engrg. Cybernetics* **22** (6) 23–28.

DEMIN, V. K. and MALASHENKO, YU. E. (1974). Obtaining bounding solutions in optimal standby problems. *Engrg. Cybernetics* **12** (1) 94–99.

DZIRKAL, E. V. (1981). *Assignment and Verification of Requirements for Reliability of Complex Items* (in Russian). Radio i Svyaz Press, Moscow.

DZIRKAL, E. V. and SHURA-BURA, A. E. (1980). Calculation of the reliability of a redundant group with unreliable reswitching and incomplete monitoring. *Engrg. Cybernetics* **18** (6) 84–90.

GADASIN, V. A. (1973). The reliability of information grids represented by semioriented graphs. *Engrg. Cybernetics* **11** (5) 767–775.

GADASIN, V. A. and LAKAEV, A. S. (1978). Estimation of the reliability of retranslation grids by the method of decomposition. *Engrg. Cybernetics* **16** (4) 71–76.

GADASIN, V. A. and LAKAEV, A. S. (1979). Method of linear complexity for estimation of reliability characteristics of redundant communication networks (in Russian). *Upravlyaushchiye Systemy i Mashiny* **3** 23–31.

GADASIN, V. A. and USHAKOV, I. A. (1975). *Reliabililty of Complex Information Control Systems* (in Russian). Soviet Radio Press, Moscow.

GENIS, YA. G. (1978). The convergence of a certain class of distribution functions to an exponential distribution function in reliability and queuing problems. *Engrg. Cybernetics* **16** (6) 94–101.

GENIS, YA. G. (1985a). Reliability of renewable standby systems under general distributions of the time of failure-free operation of its elements. *Soviet J. Computer Systems Sci.* **23** (3) 103–109.

GENIS, YA. G. (1985b). The failure rate of a renewable system with arbitrary distributions of the duration of failure-free operation of its elements. *Soviet J. Computer Systems Sci.* **23** (5) 126–131.

GENIS, YA. G. and USHAKOV, I. A. (1983). Optimization of the reliability of multipurpose systems. *Engrg. Cybernetics* **21** (3) 54–61.

GERTSBAKH, I. B. (1966). Optimum use of reserve elements. *Engrg. Cybernetics* **4** (5) 73–78.

GERTSBAKH, I. B. (1970). Dynamic reserves: Optimal control of spare items (in Russian). *Automat. i Vychisl. Tekhn.* **1** 57–63.

GERTSBAKH, I. B. (1977). *Models of Preventive Maintenance.* North-Holland, Amsterdam.

GERTSBAKH, I. B. (1984). Asymptotic methods in reliability theory: A review. *Adv. in Appl. Probab.* **16** 147–175.

GERTSBAKH, I. B. and KORDONSKIY, KH. B. (1969). *Models of Failure.* Springer, New York.

GNEDENKO, B. V. (1943). Sur la distribution limite du terme maximum d'une série aléatoire. *Ann. Math.* **44** 423–453.

GNEDENKO, B. V. (1964a). Idle duplication. *Engrg. Cybernetics* **2** (4) 1–9.

GNEDENKO, B. V. (1964b). Duplication with repair. *Engrg. Cybernetics* **2** (5) 102–108.

GNEDENKO, B. V., ed. (1983). *Mathematical Aspects of Reliability Theory* (in Russian). Radio i Svyaz Press, Moscow.

GNEDENKO, B. V., BELYAYEV, YU. K. and SOLOVYEV, A. D. (1969). *Mathematical Methods of Reliability Theory.* Academic, New York.

GNEDENKO, B. V., KOZLOV, B. A. and USHAKOV, I. A. (1969). The role of reliability theory in the construction of complex systems (in Russian). In *Reliability Theory and Queuing Theory* (B. V. Gnedenko, ed.). Soviet Radio Press, Moscow.

GNEDENKO, B. V. and MAKHMUD, I. M. (1976). The duration of failure-free operation of a duplicated system with renewal and preventive maintenance. *Engrg. Cybernetics* **14** (3) 65–70.

GNEDENKO, B. V. and SOLOVYEV, A. D. (1974). A general model for standby with renewal. *Engrg. Cybernetics* **12** (6) 82–86.

GNEDENKO, B. V. and SOLOVYEV, A. D. (1975). Estimation of reliability of complex renewal systems. *Engrg. Cybernetics* **13** (3) 89–96.

GORSKIY, L. K. (1970). *Statistical Algorithms of Reliability Study* (in Russian). Nauka, Moscow.

GROYSBERG, L. B. (1980). A fiducial approach in the estimation of the characteristics of complex systems. *Engrg. Cybernetics* **18** (4) 73–80.

GROYSBERG, L. B. (1981). On increasing the efficiency of confidence estimation of the reliability of systems. *Engrg. Cybernetics* **19** (2) 102–112.

KABASHKIN, I. V. (1984). A method of calculating the reliability indices of a class of Markov systems models. *Engrg. Cybernetics* **22** (5) 18–21.

KALASHNIKOV, V. U. (1969). Estimating the reliability of redundant systems with constant restoration time by means of an analogy to the direct Lyapunov method. *Engrg. Cybernetics* **7** (3) 66–72.

KAMINSKY, M. P. (1984). Nonparametric confidence estimation of the quantiles of the duration of failure-free operation in classes of continuous and aging distributions. *Engrg. Cybernetics* **22** (1) 10–14.

KARAPENEV, KH. K. (1978). Estimation of the reliability of a standby system with renewal and lag in the detection of failures. *Engrg. Cybernetics* **16** (3) 88–93.

KARSHTEDT, I. M. and KOGAN, L. M. (1971). Optimal standby with a choice of setups. *Engrg. Cybernetics* **9** (2) 231–245.

KARTASHOV, G. D. (1971). Accelerated testing in the case of an unstable production process. *Engrg. Cybernetics* **9** (4) 664–669.

KARTASHOV, G. D. (1979). *Methods of Forced (Augmented) Experiments* (in Russian). Znaniye Press, Moscow.

KARTASHOV, G. D. and PERROTE, A. I. (1968). On the principle of "heredity" in reliability theory. *Engrg. Cybernetics* **6** (1) 78–83.

KARTASHOV, G. D. and SHVEDOVA, I. G. (1983). A problem of selecting objects. *Engrg. Cybernetics* **21** (3) 62–67.

KLEBANOV, L. B. (1978). Some problems of characterizing distributions arising in reliability theory. *Theory Probab. Appl.* **23** 798–801.

KLEBANOV, L. B. (1980). Some results connected with a characterization of the exponential distribution. *Theory Probab. Appl.* **25** 617–622.

KONENKOV, YU. K. and USHAKOV, I. A. (1975). *Problems of Reliability of Radioelectronic Equipment under a Mechanical Load* (in Russian). Soviet Radio Press, Moscow.

KONEV, U. V. (1974). Optimal inclusion of standby elements. *Engrg. Cybernetics* **12** (4) 56–62.

KOROLYUK, V. S. and TURBIN, A. F. (1978). *Mathematical Foundations of Phase Consolidation of Complex Systems* (in Russian).

Naukova Dumka Press, Kiev.

KOVALENKO, I. N. (1975). *Investigations in Analysis of Reliability of Complex Systems* (in Russian). Naukova Dumka Press, Kiev.

KOVALENKO, I. N. (1976). Analytic-statistical method for calculating the characteristics of highly reliable systems. *Cybernetics* **12** 895–907.

KOVALENKO, I. N. (1980). *Analysis of Rare Events when Estimating Efficiency and Reliability of Systems* (in Russian). Soviet Radio Press, Moscow.

KOZLOV, B. A. and USHAKOV, I. A. (1970). *Reliability Handbook.* Holt, Rinehart and Winston, New York.

KOZLOV, B. A. and USHAKOV, I. A. (1975). *Handbook of Reliability Computation* (in Russian). Soviet Radio Press, Moscow.

KREDENSTER, B. P. (1978). *Forecasting Reliability with Temporal Redundancy* (in Russian). Naukova Dumka Press, Kiev.

KULBACK, L. I. (1970). *Methods of Calculation of Necessary Supply of Spare Components for Electronic Devices* (in Russian). Soviet Radio Press, Moscow.

LEVIN, B. R. and USHAKOV, I. A. (1965). Some aspects of the present state of reliability (in Russian). *Radiotekhnika* **4** 17–30.

LITVAK, YE. I. (1974). Reliability functions of discrete systems (in Russian). *Avtomat. i Vychisl. Tekhn.* **5** 111–114.

LITVAK, YE. I. (1979). A new method of resistance determination of complex electrical networks (in Russian). *Electrichestvo* **12** 35–40.

LITVAK, YE. I. (1981). A generalized triangle-star transformation in the investigation of the properties of complex networks. *Engrg. Cybernetics* **19** (1) 158–162.

LITVAK, YE. I. and USHAKOV, I. A. (1984). Estimation of the parameters of structurally complex systems. *Engrg. Cybernetics* **22** (4) 35–50.

LOMONOSOV, M. V. and POLESSKII, V. P. (1971). An upper bound for the reliability of information network. *Problems Inform. Transmission* **7** 337–339.

LOMONOSOV, M. V. and POLESSKII, V. P. (1972). Lower bound for network reliability. *Problems Inform. Transmission* **8** 118–123.

LUBKOV, N. V. (1980). A recurrent procedure of simulating failures in the estimation of reliability of complex systems. *Engrg. Cybernetics* **18** (3) 58–61.

MANDEL, A. S. and RAYKIN, A. L. (1967). The optimal plan of switching on spare units (in Russian). *Avtomat. i Telemekh.* **5** 15–19.

MATVEYEV, A. V. (1978). Estimation of the probability of parametric failure. *Engrg. Cybernetics* **16** (4) 76–82.

MIKADZE, I. S. (1979). Probabilistic characteristics of a digital computer taking its reliability into account (in Russian). *Avtomat. i Telemekh.* **17** 77–81.

MIRNYI, R. A. and SOLOVYEV, A. D. (1964). An estimate of reliability after tests of components (in Russian). In *Cybernetics in the Service of Communism: Reliability Theory and Queuing Theory* (A. I. Berg, N. G. Bruevich and B. V. Gnedenko, eds.) **2** 213–218. Energia Press, Moscow.

NETES, V. A. (1980). Expected value of effectiveness of discrete systems (in Russian). *Avtomat. i Vychisl. Tekhn.* **11** 85–90.

OBRETNEV, A. (1977). Convergence of IFR-distribution to the exponential one. *Dokl. Bulgarian Akad. Sci.* **30** 1385–1387.

PARAMONOV, YU. M. (1975). Determination of an assigned resource from a small number of preliminary observations. *Engrg. Cybernetics* **13** (5) 93–97.

PARAMONOV, YU. M. and SAVVIN, A. L. (1978). Determination of an assigned reserve with allowance for the number of objects in use. *Engrg. Cybernetics* **16** (5) 84–88.

PASHKOVSKIY, G. S. (1981). *Problems of Optimal Detection and Search for Failures in Radioelectronic Equipment* (in Russian). Radio i Svyaz Press, Moscow.

PAVLOV, I. V. (1980). Confidence bounds for convex functions of many unknown parameters. *Theory Probab. Appl.* **25** 388–393.

PAVLOV, I. V. (1982). *Statistical Methods of Estimation of Reliability of Complex Systems Based on Experimental Results* (in Russian). Radio i Svyaz Press, Moscow.

PAVLOV, I. V. and USHAKOV, I. A. (1984). Unbiased estimation of a distribution function from a multiply truncated sample. *Theory Probab. Appl.* **29** 617–620.

PENSKAYA, M. YA. (1984). Estimation of the a priori distribution density in reliability problems on the basis of Bayesian approach by the method of statistical regularization. *Engrg. Cybernetics* **22** (3) 34–42.

PERROTE, A. I., KARTASHOV, G. D. and TSVETAYEV, K. N. (1968). *Foundations of Accelerated Reliability Testing of Radio Parts* (in Russian). Soviet Radio Press, Moscow.

PERROTE, A. I. and YAVRIYAN, A. N. (1978). The principle of least action in reliability theory. *Engrg. Cybernetics* **16** (6) 101–106.

POLOVKO, A. M. (1965). *Foundations of Reliability Theory* (in Russian). Nauka, Moscow.

RAYKIN, A. L. (1967). *Reliability Theory in Planning of Engineering Systems* (in Russian). Soviet Radio Press, Moscow.

RAYKIN, A. L. (1971). *Probability Models of Functioning of System with Reserves* (in Russian). Nauka, Moscow.

RAYKIN, A. L. (1978). *Reliability Theory of Engineering Systems* (in Russian). Soviet Radio Press, Moscow.

ROSENBLIT, P. YA. (1973). Estimating optimal preventive maintenance strategies. *Engrg. Cybernetics* **11** (5) 758–762.

RUBALSKY, G. B. (1977). *Inventory Control under Random Demand* (in Russian). Soviet Radio Press, Moscow.

RUBALSKY, G. B. (1984). Mixed renewal and restocking models for product of a single type. *Engrg. Cybernetics* **22** (2) 59–66.

RUBALSKY, G. B. and USHAKOV, I. A. (1976). Multiechelon inventory control system with dense branches. *Kybernetes* **5** (2) 101–105.

SAPOZHNIKOV, P. N. (1970). Estimating the reliability of parallel and series circuits from trials of individual elements. *Engrg. Cybernetics* **8** (4) 683–687.

SAVCHUK, V. P. (1986). Approximate Bayesian estimation of the probability of failure-free operation for a class of aging distributions. *Soviet J. Computer Systems Sci.* **24** (2) 37–43.

SEDYAKIN, N. M. (1966). A physical reliability theory. *Engrg. Cybernetics* **4** (3) 295–303.

SHISHONOK, N. A., REPKIN, V. F. and BARVINSKI, I. L. (1964). *Foundations of Reliability Theory and Use of Radioelectronic Equipment* (in Russian). Soviet Radio Press, Moscow.

SHOR, Y. B. (1962). *Statistical Methods for Analyzing Control of Quality and Reliability* (in Russian). Soviet Radio Press, Moscow.

SHURA-BURA, A. E. and TOPOLSKIY, M. V. (1961). *Methods of Organization Calculation and Optimization of Batches of Spare Parts of Complex Engineering Systems* (in Russian). Znaniye Press, Moscow.

SOLOVYEV, A. D. (1965). Theory of aging elements. *Proc. Fifth Berkeley Symp. Math. Statist. Probab.* **3** 313–324. Univ. California Press.

SOLOVYEV, A. D. (1971). Asymptotic behavior of the time of the first occurrence of a rare event in a regenerating process. *Engrg. Cybernetics* **9** (6) 1038–1048.

SOLOVYEV, A. D. (1972). Asymptotic distribution of the moment of first crossing of a high level by a birth and death process. *Proc. Sixth Berkeley Symp. Math. Statist. Probab.* **3** 71–86. Univ. California Press.

SOLOVYEV, A. D. and SHAKHBAZOV, A. A. (1981). Nonhomogeneous standby with renewal. *Engrg. Cybernetics* **19** (5) 30–38.

SOLOVYEV, A. D. and USHAKOV, I. A. (1967). Some bounds on a system with aging elements (in Russian). *Avtomat. i Vychisl. Tekhn.* **6** 37–41.

SONKINA, T. P. and TYOSKIN, O. I. (1984). Confidence limits for the reliability of systems of elements connected in series. *Engrg. Cybernetics* **22** (4) 105–113.

SUDAKOV, R. S. (1974). A problem of interval estimation of the reliability index of a series system. *Engrg. Cybernetics* **12** (3) 55–63.

SUDAKOV, R. S. (1980). On interval estimation. *Engrg. Cybernetics* **18** (1) 54–64.

TATASHEV, A. G. and USHAKOV, I. A. (1981). A problem of reswitching standby elements after a schedule. *Engrg. Cybernetics* **19** (5) 57–65.

TYOSKIN, O. I. (1979). Precise confidence limits for the reliability of redundant systems in the case of failure-free tests of their elements. *Engrg. Cybernetics* **17** (4) 116–123.

USHAKOV, I. A. (1960). An estimate of effectiveness of complex systems (in Russian). In *Reliability of Radioelectronic Equipment* (O. Poslavskii, ed.). Soviet Radio Press, Moscow.

USHAKOV, I. A. (1966). An estimate of reliability of systems with renewal for a stationary process (in Russian). *Radiotekhnika* **5** 43–50.

USHAKOV, I. A. (1969). *Methods for Solving Simplest Problems of Optimal Inventory under Restrictions* (in Russian). Soviet Radio Press, Moscow.

USHAKOV, I. A. (1980). Estimation of reliability from the results of truncated observations. *Engrg. Cybernetics* **18** (5) 76–78.

USHAKOV, I. A. (1981). Method of approximate solution of problems of dynamic standby problems. *Engrg. Cybernetics* **19** (2) 94–101.

USHAKOV, I. A. and LITVAK, YE. I. (1977). Upper and lower bounds on parameters of two-terminal networks. *Engrg. Cybernetics* **15** (1) 59–67.

USHAKOV, I. A. and YASENOVETS, A. V. (1977). Statistical methods of solving problems of optimal standby. *Engrg. Cybernetics* **15** (6) 8–16.

USHAKOV, I. A. and YASENOVETS, A. V. (1978). Two limit problems of optimal standby (in Russian). *Nadezhn. Kach.* **3** 77–82.

ZHEGALOV, S. I. (1986). A method of calculating the probability of failure-free operation of a system with different elements. *Soviet J. Computer Systems Sci.* **24** (1) 147–149.

Part II: Non-Soviet Authors

AALEN, O. O. (1978). Nonparametric inference for a family of counting processes. *Ann. Statist.* **6** 701–726.

BARLOW, R. E. and PROSCHAN, F. (1975). *Statistical Theory of Reliability and Life Testing.* Holt, Rinehart and Winston, New York. (Reprint with corrections, 1981, by To Begin With Publishers, Maryland.)

BIRNBAUM, Z. W., ESARY, J. D. and SAUNDERS, S. C. (1961). Multicomponent systems and structures and their reliability. *Technometrics* **3** 55–77.

BRESLOW, N. and CROWLEY, J. (1974). A large sample study of the life table and product limit estimates under censorship. *Ann. Statist.* **2** 437–453.

EFRON, B. (1967). The two sample problem with censored data. *Proc. Fifth Berkeley Symp. Math. Statist. Probab.* **4** 831–853. Univ. California Press.

FISHER, R. A. and TIPPETT, L. H. C. (1928). Limiting forms of the frequency distribution of the largest or smallest member of a sample. *Proc. Cambridge Philos. Soc.* **24** 180–190.

FÖLDES, A. and REJTÖ, L. (1981). Strong uniform consistency for nonparametric survival curve estimators from randomly censored data. *Ann. Statist.* **9** 122–129.

FRECHET, M. (1927). Sur la loi de probabilité de l'écart maximum. *Ann. Soc. Polon. Math. Cravovie* **6** 93.

GILL, R. D. (1980). *Censoring and Stochastic Integrals.* Mathematical Centre Tracts **124**. Mathematical Centre, Amsterdam.

GILL, R. D. (1983). Large sample behavior of the product-limit estimator on the whole line. *Ann. Statist.* **11** 49–58.

KAPLAN, E. L. and MEIER, P. (1958). Nonparametric estimation from incomplete observations. *J. Amer. Statist. Assoc.* **53** 457–481.

MANN, N. R. and SINGPURWALLA, N. D. (1985). Extreme value distributions. In *Encyclopedia of Statistical Sciences* (N. L. Johnson and S. Kotz, eds.) **2** 606–613. Wiley, New York.

OSAKI, S. and ASAKURA, T. (1970). A two-unit standby redundant system with repair and preventive maintenance. *J. Appl. Probab.* **7** 641–648.

Comment

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Rukhin and Hsieh should be thanked for preparing this survey because it must have involved considerable time and effort to dig out and interpret such a large volume of research work. However, this survey article could be by no means a complete review of Soviet work in reliability theory. An excellent short survey of Soviet work in asymptotic methods in reliability theory appeared in *Advances in Applied Probability* by Gertsbakh (1984). Rukhin and Hsieh have expanded their survey to include additional topics in reliability theory. It would have been helpful had they also mentioned the related excellent work of Brown (1987), Brown and Ge (1984) and Keilson (1979, 1986) in this country, because their work is very close and overlaps in many respects the work of the Gnedenko school of reliability at Moscow University.

One of us (Khalil) was a Ph.D. thesis student of Gnedenko at the beginning of the era of the Moscow school of reliability theory. He studied at Moscow University from 1963–1969 and has kept in contact with Gnedenko. Hence, we will first give a short

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