He wrote this memorandum to the Government of India on a visit to India at the invitation of Professor P. C. Mahalanobis:

"Research in statistical theory and technique is necessarily mathematical, scholarly, and abstract in character, requiring some degree of leisure and detachment, and access to a good mathematical and statistical library. The importance of continuing such research is very great, although it is not always obvious to those whose interest is entirely in practical applications of already existing theory. Excepting in the presence of active research in a pure science, the applications of the science tend to drop into a deadly rut of unthinking routine, incapable of progress beyond a limited range predetermined by the accomplishments of pure science, and are in constant danger of falling into the hands of people who do not really understand the tools that they are working with and who are out of touch with those that do. . . . It is in fact rather absurd, though quite in line with the precedents of earlier centuries, that scientific men of the highest talents can live only by doing work that could be done by others of lesser special ability, while the real worth of their most important work receives no official recognition.—Harold Hotelling, Memorandum to the Government of India, 24 February 1940."

An analytic study is one in which the problem is to make changes in a process or in a practice, with the aim to improve quality or yield in the future, next week or in next year's crop. Inference from an experiment is an example.

An enumerative study, in contrast, is to estimate what the result might be of the equal complete coverage of a frame. The 10-yr census is an example. The present worth of accounts receivable is another. The dollar value of a shipload of iron ore is another.

Statistical theory, as used in an enumerative study, does not help us in an analytic study. The theory of estimation and statistical tests of significance as commonly taught and used—e.g., the t test, z test, χ^2, goodness of fit—do not provide measures of degree of belief in a prediction. Tomorrow's run, or next year's crop, will be governed by conditions different from those that governed the data from a study of the past. Confidence limits are useful in enumerative problems, not in analytic problems.

Harold Hotelling was often critical of use of the wrong distribution for calculation of a probability in an analytic problem. Now, after many years, one might rightly question whether any distribution whatever should be used for an analytic problem. It may be that Hotelling, along with others, used the theory for enumerative studies to make calculations of probability in an analytic problem.

One must remember that in an analytic problem, there is no frame; hence, no complete coverage. There is no sample, no distribution of samples. There is no operational definition of a confidence interval. A test of significance has no meaning, is no aid to prediction, hence conveys no knowledge.

Harold Hotelling was a very kind man, always ready to help anyone. He helped other people to finish their books, but never took time to finish his own.

Comment

Shanti S. Gupta

I would like to express my appreciation to Professor Morris DeGroot for inviting me to comment on the two papers of Professor Harold Hotelling. During my 3 years (1953–1956) as a graduate student at the University of North Carolina at Chapel Hill, I took several courses from Professor Hotelling and attended several of the famous Sunday tea parties given by him and Mrs. Susanna Hotelling in their lovely home on

Shanti S. Gupta is Professor and Head, Department of Statistics, Purdue University, West Lafayette, Indiana 47907.

the first Sunday of each month (the atmosphere at these gatherings at Hotellings' T"s used to be, indeed, quite stimulating and, of course, very very sociable).

INTRODUCTION

It is close to half a century since the first of these two papers by Hotelling was published. The second paper, published nearly a decade after publication of the first, re-examines the issues raised in the first and amplifies on the organizational and teaching personnel issues. For one who reads (or rereads) these papers, the striking fact is the thoroughness with which

Hotelling has analyzed the many issues that could and would arise in the teaching of statistics. In 1940, statistics was not yet thought of as an independent discipline of study. The subject was still very young. There were only a handful of pioneers—Hotelling one among them—doing research that can be classified as theoretical or mathematical statistics as we know it today. Many of the personalities that would come later and leave their impact on the history and development of the subject were yet to embark or had hardly embarked upon their careers in statistics. It is in this context that Hotelling's discussion and analysis of the issues involved in the teaching of statistics stand out so distinctively for the foresight and clarity exhibited.

In analyzing the problems associated with the teaching of statistics in universities, Hotelling has viewed the topic as consisting of three inseparably connected issues: teaching, research and consulting. Many of the problems discussed here are typically still with us, some have been resolved the way he foresaw or recommended, and some would come back in different contexts as the subject itself were to grow. It is in this light that we re-examine these papers.

PROLIFERATION OF ELEMENTARY STATISTICS COURSES

One of the main problems still faced on many university campuses across the country is that of the duplication of basic statistics courses and the much debated question of who is best qualified to teach these. The nature of this problem is clearly stressed by Hotelling who sees it as a fact that is "often concealed by including the teaching of statistical theory in a course whose title and prospectus are more suggestive of applications." His example (given in both papers) of how a course in price forecasting ended up as a course in statistical theory with virtually nothing about forecasting should sound familiar to many of us. The last straw for him is "putting the Department of Education in charge of an essential part of the training of scientific students." This problem of overlapping and duplicating of elementary statistics courses can be solved by understanding the reasons (as put forth by Hotelling) why it is in the interests of all users of statistics that the *elementary* teaching of statistics should be "conducted by those with a thorough and critical knowledge of current research in statistical theory." This is *no less* true today than when Hotelling recommended it as "perhaps the greatest reform needed in teaching of statistics today."

TEACHERS OF STATISTICS AND ORGANIZATIONAL SETUP

In both papers, Hotelling has discussed in detail the qualifications necessary for teachers of statistics and the organizational aspects of carrying out the teaching. In the 1949 (First Berkeley Symposium) paper, he has listed in decreasing order of desirability five different ways that the teaching of statistics can be organized. Perhaps, there are still a few places today with the last (fifth) setup in which the teaching is disorganized among a heterogeneous group of departments of application.

The fourth possibility of having it under the exclusive jurisdiction of the Department of Mathematics exists in quite a few places in a modified form in the sense that a small part of the elementary courses are taught by some departments of application such as psychology, business and economics. However, this still requires that teaching of statistics even at lower levels is entrusted to qualified specialists who have "a genuine sympathy and understanding for applications." As we all know today, there is still the problem of the perception by others, as Hotelling feared, that "courses in statistics given by a department consisting largely of pure mathematicians are unsuitable for their purposes."

The third setup is to have an interdepartmental committee in charge of statistics. This was not viewed (then) by Hotelling and is not considered now by many as anything more than a makeshift until a proper Department of Statistics is organized.

The second setup of a Department of Statistics is what has largely evolved in the intervening decades. This has provided for a smooth transition in many universities and has helped the healthy growth of graduate programs in statistics as well as fostering fundamental research in theoretical statistics. It has also provided the right kind of environment to gather teachers of statistics. The importance generally accorded to teaching, research and university-based consulting in a typical department is just what Hotelling is describing. A Department of Statistics should have "full time professors engaged in the teaching of and research in statistical theory and methods, without spending time over applied statistical problems excepting insofar as such problems might present novel features calling for the development of new statistical methods or theoretical extensions having interest going beyond the immediate case." He rightly visualizes the academic statistician as a toolmaker who "must not put all his time on using the tools he makes" but must focus his attention on the tools themselves.

The top preference is for a two-department Institute of Statistics, one department dealing with theoretical statistics and the other with applied statistics, with migration of faculty from one to the other made easy and frequent, but not rigid, with periodic transfers. Such a setup is generally possible only in larger institutions. It is desirable to have a general setup that

serves as an umbrella for persons working in different areas who have common interests and a possible desire to work on interdisciplinary projects. A Center for Statistical Decision Sciences such as the one recently started at Purdue University provides such a forum and working arrangement.

UNDERGRADUATE AND GRADUATE CURRICULA

Some of the deficiencies seen by Hotelling have been corrected but some others do persist in some form or other, partly because of the constant growth of the subject itself. Until the early seventies, there were not many statistical courses to speak of at the sophomore level. Generally, courses at the senior level with a calculus prerequisite were available. A graduate student in sociology, for example, never had any elementary statistics course as an undergraduate. Now many undergraduates do take elementary courses in statistics. However, "a better integration of undergraduate with graduate work" is still a far cry as far as statistics is concerned.

The important point that is emphasized by Hotelling without missing a single opportunity is the sound mathematical basis needed for pursuit of statistics. Such a need has never been greater than today, given the recent developments and current trends. It is certainly debatable whether every PhD student in theoretical statistics is equipped enough mathematically. To function well as a consultant to research workers in other fields in the university, a statistician needs "a somewhat wider diffusion of studies among various fields than is customary in many institutions, especially in graduate work." This need as well as the advantage of familiarity with empirical data deserve the attention of educators.

Related closely with the problem of proliferation of basic courses in statistical methods is the problem of many textbooks written that propagate errors. What we really need are textbooks that "abandon the idea of telling what statistical methods are used, and say instead what methods ought to be used."

CONCLUDING REMARKS

As pointed out above, Hotelling has examined, very critically, the problem of teaching of statistics in colleges and universities and offered several recommendations. As far as the curricula are concerned, he did not anticipate the extent to which computers would come to influence the teaching of statistical methods. The problems pointed out by Dr. Hotelling still remain in many universities and colleges. The disorganization of their statistical programs, the proliferation of statistics courses and deficiency of selecting suitable teachers to teach statistics courses are just some of the areas where improvements are needed. Many universities still have not established the Institute of Statistics or the Department of Statistics envisaged by Hotelling. If students in this country had had calculus at an early stage, say, in high school, "statistical teaching could be made more efficient."

Although the 1940 paper was Hotelling's invited address at the Hanover meeting of the Institute of Mathematical Statistics that year, he did not address himself to the possible role of professional societies such as the Institute of Mathematical Statistics and the American Statistical Association in influencing the teaching of statistics in universities. For a discussion of this role, see Tanur (1982) and Gupta (1982).

ACKNOWLEDGMENT

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